A N

INTRODUCTION

To ſo much of the

ARTS and SCIENCES,

More immediately concerned in an

Excellent Education for Trade

In its lower Scenes and more genteel Profeſſions,

AND FOR

Preparing YOUNG GENTLEMEN in Grammar Schools
to attend LECTURES in the UNIVERSITIES.

In FOUR PARTS.

PART I.

The Theory and diffuſive Practice of ARITHMETIC, VULGAR and DECIMAL; the *Arithmetic* of ALGEBRA by Tranſpoſition introduced, as it gives Rules to the Accomptant; Annuities for Time; the Principles of Mr. *De Moivre*, Mr. *Simpſon*, and of the AUTHOR, in eſtimating Annuities for Single Lives, conſider'd from the Equity in ſinking Money. Digeſted into Syſtems, as Trade, young Ladies, or the Concerns of the Gentry may require.

PART II.

An extenſive Courſe of GEOMETRY; Menſuration; an Introduction to Gauging and Surveying; Plain Trigonometry applied to the Merchant's and Surveyor's Uſe; the Aſtronomical Principles of Geography; the Conſtruction and Uſe of Maps; the Menſuration of the Globes of the Solar Syſtem, and of their Orbits; the Eſtimations of Artificers; and the Debates of the Globes concerning the Earth's two Motions, to effect the Seaſons, &c.

Expreſly deſigned to remove that general Complaint of not effectually inſtructing YOUTH, while at School, in what may be of Importance in their future Stations, and enlarging their narrow Conceptions and ſcanty Views of Nature.

The various Subjects are ſo digeſted and expreſſed, as to *aſſiſt* the *Maſter*, and to ground, forward, and encourage the *Scholar*, and to make him of immediate Uſe when he is put to the Teſt.

With Eight COPPER-PLATES.

To which is prefix'd a LETTER on EDUCATION.

By J. RANDALL, R.
Formerly Maſter of the Academy at Heath *near* Wakefield, *but now at* York.

LONDON: Sold by W. NICOLL, in St. *Paul's* Church-Yard.

[Price bound Three Shillings and Sixpence.]

TO THE

Rt. Rev. Dr. DODGSON,

Lord Biſhop of *OSSORY*.

My Lord,

A Treatiſe of this complex Nature, wherein the common Path is often left, and many Things appear to have a Caſt of being Originals, points out your Lordſhip as it's Patron, an excellent Judge of the various Subjects herein treated of, and of their Importance to Youth, who are to move in their deſtined Spheres, in the awful Machine of Divine Providence.

I need not trouble the Reader with telling him my private Sentiments of your Lordſhip's Conduct from your earlier Years; your whole Behaviour, from your leaving the Univerſity to the Time of my being diſunited from you, was of ſuch a Nature, as to leave a lovely Impreſſion of yourſelf on the Minds of Abundance of Youth of

Fortune, who had the Happineſs of receiving a Claſſical Education, of deriving a refined Taſte for thoſe Authors of Antiquity, from your Lordſhip, and of forming their Hearts to an unaffected Love of Sincerity and Benevolence, by your Example. Thoſe Impreſſions are now, with the Youth, ripened into Maturity, and are ſo many living Witneſſes, throughout the King's Dominions, that I only repeat what they themſelves do on all Occaſions.

That your Lordſhip may long continue to deſerve the Eſteem of the Public, to be an Ornament to the Church, and to the BRITISH Literati; an Encourager of whatever may promote Learning and Piety, and the Scourge of Impudence and deſtructive Principles, is the unfeigned Wiſh of,

My Lord,

Your Lordſhip's

moſt humble and

moſt obedient Servant,

York, July 14,
1765.

J. RANDALL.

THE

PREFACE.

I Should have been extremely glad if ſo much of the Arts and Sciences, *more immediately concerned in* Trade, *and the genteeler* Employments *and* Profeſſions, *had been, by ſome able Author, thrown into a narrow Compaſs, and properly adapted to the Wants of Schools. I have, for above twenty Years paſt, wiſhed to ſee this Piece of Service to the Public performed; but the different Stations for which Youth committed to my Care are deſigned, have abſolutely obliged me, at laſt, to write ſuch an Introduction, inſtead of uſing any longer the beſt Authors on the various Subjects, whereby too great an Expence, and an intolerable Waſte of Time, are the natural Conſequences, not to mention the Difficulty of meeting with Books adapted to the Capacities of Youth.*

The Reader is not to look upon this Treatiſe as a Collection, but a Performance ariſing from the different Diſpoſitions and Abilities of Youth, their Infelicities; and the great Difficulty there generally is in fixing any Subject on their giddy Minds, and making them fit for immediate Uſe in their deſtined Spheres. I cannot preſume ſo far as to think the Performance does not ſtand in Need of the kindeſt Indulgence from thoſe who are Maſters of the ſeveral Subjects herein treated of; for as there is a Sort of Novelty running *through the whole, and ſome Attempts every now and then to make Improvements in the different Branches of the Arts and Sciences introduced, it will be a Happineſs if there is not too much Room for Reprehenſion.*

Arithmetic makes a conſiderable Part of this Treatiſe; for if there is not an excellent Foundation laid in this, and the Youth extremely perfect therein, he only gropes in the Dark when put upon higher Acquirements. To avoid that Confuſion which Youth are uſually thrown into as they paſs through numerous Rules, making at laſt a Chaos, I have all along endeavoured to make them underſtand the Reaſon of what they are about, when within their Reach, which ſerves as a Light to conduct and give them a Preciſion on all uſeful Occaſions, and thereby anſwer the End of Education.

When a Book is properly adapted to the Wants of a public School, Youth have the great Advantage of hearing their Maſter continually lecturing, as it were, on the various Subjects, and giving out freſh Sums and Propoſitions, inſtead of the Scholars being obliged to ſit ſtill for Want of Queſtions, when the Teacher is otherwiſe engaged in the Execution of his Office. Thoſe Books which only give the Rules and freſh Queſtions for Employment, are of ſome Service in a public School; but, for Want of putting a Light into the Youth's Hand, and properly talking to him, as it were, at the ſame Time, the End of Education is, by theſe Books, but poorly anſwered, and yet the Maſter has had more than his Share of the Fatigue. Of this Nature are ſcanty Manuſcripts uſed in public Schools, as they ſeldom or never are diffuſive enough to anſwer the great End of ſilently lecturing Youth into their Education, even with all the Elocution of an *an ingenious Maſter, unleſs in a very private School. But, in public Schools, to uſe neither Books nor Manuſcripts, is to throw the whole Weight on the Teacher; and the neceſſary Conſequence is, the Scholars muſt wait till his Maſter has done equal Juſtice to all in dividing his Inſtruction. The Remedy to this loſing of Time, and retarding the Youth's Progreſs, is to allow him proper Helps till his Maſter is at Leiſure to examine him in his Turn.*

In this Treatiſe the Reader will find what more immediately concerns the Merchant's Counting-Houſe, the Attorney's and Steward's Offices, &c. *the Wholeſale Dealer, the Counter, and Mechanical Trades in general, and whatever ſhould adorn a Gentleman's Claſſical Education, to make him uſeful to himſelf: And this may be truly ſaid, that whoever reads the Book over, will find more in it than the* Title *promiſes, or perhaps is expected.*

THE

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THE LETTER TO A MERCHANT in *BRISTOL*,

Whereof a few Copies were lately printed by the Author's Conſent, under a fictitious Name.

SIR,

I Received your obliging Letter, requeſting my Opinion concerning the Education of your Sons, and ſhall undertake the Taſk with Pleaſure. You are pleaſed to lay a great Streſs on my long Experience in educating Youth; but I do not preſume to dictate to the World; I only deſire that you would carefully weigh every Circumſtance I ſhall offer for your ſerious Conſideration, and requeſt you will pardon any Freedom I may take in expreſſing myſelf.

Your Circumſtances, Sir, enable you to give your Sons a genteel Education, and as I find you intend them for Buſineſs, I ſhall ſpeak particularly to that Point: But firſt of the Choice of a School, on which Succeſs in general depends.

The Manner of Education for Trade being, within theſe hundred Years, changed conſiderably for the better, I muſt give you the general Characteriſtic of theſe Schools; that is, what ought to be taught in them, to keep up the evident Preference to the old *Latin* and Writing Schools, otherwiſe People will be nearly where they were. The *Engliſh Language* ought to be taught in high Perfection, and the Youth continued in the Practice thereof from Six Years of Age, to the Time he goes to Buſineſs. *Latin* alſo muſt not be omitted; for tho' theſe Schools profeſſedly point at Commerce, yet *Latin* is a Part of a genteel Education, and may be of Importance to Youth on the Change of a deſtined Station, and in particular for Trade, in giving him a Taſte of Propriety in writing our Language, which otherwiſe cannot, with Succeſs, be done from an *Engliſh* Education only, as Boys are fond of ſome Dignity in their Manner of Education. This Propriety ariſes from the conſtant Practice of making *Latin*, which cannot be done without the Knowledge of the *Latin* Rules; and it is the Teacher's Buſineſs to ſhew the Youth how the two Languages differ in Conſtruction, the Tenſes and Caſes: And from hence you will perceive, that though a Youth, when he leaves School, ſhould quite neglect his *Latin*, yet it is worth while to learn it for the Sake of the Grammatical Inſtruction, which will never leave him, becauſe ſo deeply impreſſed upon his Mind, and will always ſhew he has had ſomething of a learned Education. The Manner of Teaching *Latin* began about a hundred Years ago to take alſo a more favourable Turn, by allowing a Boy ſome Help, till he is better able to go on with the Aſſiſtance only of his Grammar and Dictionary; inſtead of the rigid old Method, ſtill obſerved in many Schools, eſpecially in ſome of the greater. The new Method I have found to ſucceed very well with Boys in the lower Claſſics, even that could not otherwiſe be brought forward: Beyond this they ought to have no other Help than their Dictionary and Grammar in reading an Author, as it is very evident where the Preference lies in this Caſe.

The *Hands for Buſineſs* ought of Courſe to be well taught in theſe Schools, by Specimens of real Buſineſs, not by conſtant ſingle-lined Copies. *Arithmetic* in all its Parts, with Vulgar and Decimal Fractions, applied to all the Concerns of Commerce: And ſo much of Algebra as to perfect the Acomptant, and underſtand the difficult Buſineſs of the compound Arbitration of Exchanges, and the general Nature of Annuities for Time and Lives. The Principles of the *Italian Method of Book-keeping* muſt be here taught, and effectually impreſſed, whereby a Youth will more readily fall into any Variation in Practice amongſt Merchants; but without this Foundation he long continues, during his Apprenticeſhip, to grope in the Dark, and will be eſteemed dull for Want of this Inſtruction; or, being ignorant thereof, may take no Delight in the Counting-Houſe. The *Terreſtrial Globe* muſt alſo be here taught, not only for its own delightful Excellency, but more particularly as a general Map of the Earth, to impreſs the Situations, and Diſtances of Countries and Places, on the Minds of young Merchants: Nor ſhould they be unacquainted with ſo much of Geometry, Menſuration, and Gauging, as come within the Sphere of a Man of Buſineſs; nor with ſo much of the Application of Geometry to Navigation, as will enable them to examine a Sea Journal when they are concern'd in Shipping, as they frequently find it their Intereſt to be. The *French Language* ought not by any Means to be omitted, for this unanſwerable Reaſon, becauſe remarkably uſeful to foreign Dealers, and is often the Means of procuring a handſome Living, if a Youth ſhould act in the Capacity of a Book-Keeper, before he commences Merchant for himſelf; I do not mean any other Acquiſition in this Language, than that of writing *French* Letters; the Talking Part requires too much Time,

This, Sir, is the Sort of School that I wiſh for your Sons. The profeſſed Academies in *London*, indeed, are upon a more extenſive Plan, embracing all Parts of Knowledge, and every Accompliſhment. The Parts of Education I have inſerted are quite ſufficient for you, and are eaſily acquired by a Youth of a tolerable Genius, as his Time may be proportionally adapted, without letting one Branch break in too much upon, or be conducted to the Prejudice of, another. In theſe Schools a Youth, by ſeeing Varieties daily carried on, is often led to fix his Genius upon what may prove his favourite Study; and this is no mean Advantage. And you will be very happy if you can meet with an honeſt faithful Head of ſuch a School in *Briſtol*, that will ſee all theſe Things properly carried into Execution; not one whoſe Learning conſiſts in *Latin* and *Greek* only, but one who is well acquainted with the World, and a competent Judge of the Branches of Education proper for a Merchant; otherwiſe you may be deceived by Subſtitutes, or at beſt diſappointed in your Expectations.

Your own virtuous Turn of Mind, and Love to your Children, will make you anxious to have them *Good:* And, in order to this, you will think all Returns of Kindneſs too little to the chief Maſter, for his conſtant Endeavours to impreſs a Senſe of Goodneſs on the Minds of your Children. This is better done in ſuch a School, under one chief Maſter, and for Reaſons which you will eaſily aſſign, particularly, as the Holy Scripture is one of the Books your Sons will or ſhould be conſtantly engaged in; and a well-diſpoſed Maſter may give proper Hints on the Subject of the Day, which, with a ſhort Form of Prayer read Morning and Evening to the Boys, will give them, giddy as they generally are, a conſtant Remembrance, or Senſe of their Creator. A little Diſcourſe between the Maſter and his young Diſciples, concerning the Bulk, Situation, and Diſtance of the Earth from the Sun, will have good Effect; the Uſe of the latter to us; Who could poſſibly place all thoſe ſhining Bodies where they are? Theſe little Intervals of accidental Converſation, when a Claſs is ſtanding round the Maſter, after the Leſſon is ſaid, will wonderfully operate upon their inquiſitive Minds, and diſpoſe them to ſome Attention on the Approach of Prayer-Time. A Lye detected will enable him to ſhew what an ill Preparation to Devotion ſuch a Vice is, and how odious to the Object of their Prayers. An idle Conduct will be a Hint to remind his little Circle, that He, they ſo often pray to, expects ſome Improvement of their Time and Talents. Diſobedience to Parents will be the Opportunity to tell the young Audience, that He, with whom they are to ſpend their Eternity, will not only be highly diſpleaſed at them, but, ſooner or later, will make them ſenſible of their Error: That Diſreputation follows Curſing, Swearing, Injuſtice, or any Immorality, and will certainly hurt their Intereſt in Heaven, unleſs timely left off. A Neceſſity of puniſhing an Offender will give him a juſt Opportunity to declare to them all, that his Duty to God, who has thus impowered him; to Parents, who have intruſted him; to the Public, who expects it of him, will not ſuffer him to wave the Correction which is deſigned to make an Offender better.

And here, Sir, let me freely tell you the Difficulties an honeſt Teacher labours under, when Honour and Conſcience tell him a Youth muſt be corrected. Some Boys look upon Puniſhment as really deſigned for their Good, or at leaſt are not enraged at a Teacher behind his Back. Others ſhew the utmoſt Reſentment, and lay their little Schemes how to ſacrifice their Maſter to their Parents' Revenge, which too often ſhews itſelf to his Diſadvantage. This is an ill Return. Some Teachers, more mindful of their Eaſe, and leſs conſcientious, make their Court to the Boys, in order to ſecure both them and their Parents, by letting Education go on at an eaſier Rate. Dull Boys too generally occaſion a great Coldneſs from the Parents to a diligent Maſter; but it is very unjuſt to be blamed for what is not in a Man's Power to effect, for not poliſhing a Pebble into a Diamond, after his conſtant Efforts, and which is thus appointed by the Creator for wiſe Ends. Parents ſhould conſider theſe Things, that it may go the better with the World, by uſing Teachers candidly, and then there will be more Encouragement, than there is at preſent, for Men of Ability, Honour, and Conſcience to undertake the painful Taſk.

The Buſineſs of conducting Youth has been extremely injured by the Writings of ſpeculative Men; who, without the requiſite Experience, have declared to the World, that *if a Boy will not learn without the Rod, he never will with it,* This raſh Aſſertion of the *Spectator* and others, Hundreds of very worthy Men can contradict from their own Experience. It is much to be wondered at, that ſuch Names as *Locke, Addiſon, &c.* ſhould thus expoſe themſelves to the unwilling Cenſure of many learned and humane Teachers.

If you fix on a Maſter, who both knows and does his Duty, I muſt obſerve to you, that it will be an Injuſtice both to him, abſolutely ſo to your Sons, to remove them to any other School in *Briſtol*; for ſurely a new Phyſician muſt have much Time to diſcover a new Patient's Conſtitution: Nor will it appear free from the Imputation of Levity, chiefly to pleaſe Boys, who commonly wiſh to change, for the Happineſs of being idle or inſolent with Impunity; or of enjoying a playful and riotous Diſpoſition; or for the Sake of Novelty.

If Parents apprehend they have a Right to detain their Children from their Attendance on Inſtruction, it is extremely unjuſt to complain that Improvements go but ſlowly on: It is doing and undoing; the Maſter pours in, and the Parents let it out again, when thoſe Avocations are too general; and, when but ſeldom, there is more Injury done than Parents will credit; it throws a Youth off his Bias, and leaves Chaſms in that Day's Inſtruction, which are ſeldom or never cloſed up. If Parents will, therefore, ſuffer theſe Breaches to be made, they ſhould conſult their Honour, and bluſh when they complain of Defects in their Children's Education. But Parents ſhould conſider that it is invading the Maſter's Right, when they detain their Children from their Attendance, as they are given up to the Care of another, who, from the Duties of his Station, is accountable to the Public till the Truſt is diſſolved, or Illneſs ſuſpends it.

I think your Lady's Obſervation on a Country Education, in a private Boarding-School, not far from the Mark; unleſs you intend one of theſe Schools as a Nurſery, and deſign to have your Children ſpend the two or three laſt Years of their Education in a more general School. As good Teachers require pretty high Salaries, it is quite reaſonable to expect that the chief Maſter of one of theſe private Schools ſhould keep very inſufficient Uſhers: It would require, at leaſt, fifty Boarders, in the ſame Houſe, to afford right good ones in the Country. Beſides, it does not lie within the Sphere of a retired Clergyman to ſuperintend an Education for general Commerce, even were his Uſhers properly qualified.

But, Sir, if you can find ſuch a genteel School as I have deſcribed, and can depend upon the ſteady Conduct of your Sons, or that they will liſten to the Advice of the Maſter, I can eaſily pardon that laſting Ruſticity, rambling Diſpoſition, and Impatience of Reſtraint, which uſually accompany thoſe brought up in the Country. But I believe you will be inclined to think, that there are none more faithful, or more proper, to become Tutors to Boys, or to take the Direction of them at the leiſure Hours of the School, than their affectionate Parents, if they have any Diſcretion in them. Yet if a Boy, of a wild Turn of Mind, is ſuffered to intermix in his Diverſions with Town-Boys, and to do the ſame with his School-Fellows in the Country, I cannot ſee how the Caſe is altered as to his Morals; becauſe, in either Caſe, ſome of his Companions are not to be ſuppoſed uncapable of communicating ſome Sort of Infection, which will give a virtuous Parent Uneaſineſs.

And now, Sir, I ſhall take a voluntary Taſk on myſelf, and that is, to ſay ſomething concerning your two Daughters Education. And this occurred to me, on ſeeing the Progreſs a young Gentlewoman has made with a Neighbour, who undertakes to teach young Ladies privately in his own Houſe. His profeſſed Deſign is to finiſh their Education in a much higher Degree than uſually falls to their Share. They are made to read in a very genteel Manner, and to ſpell during the whole Time they continue with him. This is a great Point obtained. Writing, and tranſcribing conſtantly, make them very expert at inditing. The more uſeful Parts of Arithmetic are well impreſſed on their Minds. The young Lady, I mentioned, would make many Gentlemen bluſh in her Readineſs at caſting up every Thing, or computing Intereſt,

tereſt, Simple and Compound, and buying and ſelling Eſtates, and knowing what Eſtate any Sum of Money will purchaſe; and all this without Trouble of learning Decimal Fractions. The Principles of *Engliſh* Grammar they are taught while they learn the general Conſtruction, regular Verbs, and Declenſions, in the *French* Grammar. But, Sir, would you believe that he likewiſe gives them a ſhort, but pretty, Notion of digeſting and poſting Accounts by Single Entry, as if he intended them to be their Huſbands' Book-Keepers, or to ſee their Accounts are kept in Order, that a Gentleman may know at one View how his Affairs ſtand? And, that nothing may be left ſhort, they improve their Underſtandings by the Inſtructions he gives them on the Globe; the modeſt Repetitions they make in Company of theſe Leſſons, and of the Manners and Cuſtoms of different Nations, are really very entertaining. The Enlargement of their Ideas appears truly noble and remarkable, in the diſtinct and pretty Account ſome of theſe young Ladies, who have been under his Inſtruction, give of the Solar Syſtem. It is a plain Demonſtration, that a Man of an eaſy Expreſſion may make theſe ſublime Truths very familiar to young Ladies, without any more Trouble or Difficulty to themſelves, than the Pains of giving Attention to be agreeably and rationally entertained.

I deteſt the common Nonſenſe of ſome Parents, who for Fear, as they pretend, that their Daughters ſhould make bad Wives, are prevailed on to keep this lovely Part of the Creation in total Ignorance of what contributes to exalt their Natures. Another common Reaſon aſſigned is, that if the Wife is more knowing in theſe genteel Accompliſhments than the Huſband, his Ignorance will be the more conſpicuous: But is not this a ſorry Motive for neglecting the Education of the Fair Sex? Let the Men adorn their Minds with theſe rational Accompliſhments, and then they will be happy in the Converſation of the Ladies. Madam *Dacier*'s Huſband, among Variety of Inſtances, was not leſs happy becauſe his Wife was an excellent Judge of Learning; they loved one another for their Acquirements, and ſo will any Man and Woman of Senſe.

Miſs *C. B.* married a Gentleman at *F——m*, who ruined his Family; but, by Means of their excellent Education, they conduct a Boarding-School for young Ladies, with great Reputation, at *W——m*, and are making a handſome Proviſion for their Children. Miſs *L. W.* of *S——n*, married a Gentleman of a ſufficient Eſtate at *A——r*, and ſhe helped to reduce this Family; but as ſhe had no higher Knowledge than to neglect her Houſe, in order to lead a faſhionable Life, ſhe was forced into Place to learn a young Child to walk; and her Huſband is *Gentleman* to a young extravagant *Fox-hunter*, and the Houſe-keeper is now teaching him Accounts to qualify him to be the Houſe-Steward. My dear Sir, all I mean by theſe Examples is only to determine your Choice with regard to the finiſhed Education of your two Daughters; its own Excellency is what I plead for, without paying much Attention to the Situation of the Lady that was Miſs *C. B.*

I come now, Sir, to your Friend's Queſtion, *Whether a public or private Education be more proper to prepare a Youth of Fortune for the Duties of Religion and Society?* Eſſays on this long-conteſted Queſtion have hitherto fallen to the Share of mere ſpeculative Writers, both antient and modern; and yet the Doubt remains to this Day as they found it, and every one left to go with that Party on each Side of the Queſtion, as his Fancy or Prepoſſeſſion leads him. I ſhall not trouble your Friend with Quotations of great Names on each Side of the Queſtion, but tell him what the Nature of Man and Experience have ſuggeſted to me, from Time to Time, towards ſatisfying myſelf in this Affair.

If his Son is a ſteady prudent Youth, one of the three public Schools *, *when he is fourteen Years of Age, is beſt for him; but if he is of a Temper readily to yield to the Advice and Examples of vicious Boys, a private Education may preſerve him from Deſtruction.* There is ſomething ſo lovely in being Good, and ſo forbidding in being Wicked, that a prudent Boy (and ſuch many are) will not eaſily miſtake his Way. This is the Principle by which to determine this Point. He is not long, when he is ſent to a public School, before he ſingles out Companions of his own happy Turn of Mind. You need not ſort Boys, they ſoon do this of themſelves. Here is a Set of promiſing young Gentlemen walking or playing together; there of young Bucks; here of chaſte innocent Converſation; there Lewdneſs in Miniature. In ſhort, when a Youth is well inclined, from the happy Example of his Parents, he will generally continue ſo; when another is vicious, from the careleſs Conduct, or bad Example, of his indiſcreet Parents, of whom he will, ſooner or later, demand his Heaven; he ſeldom, at beſt, comes home more reformed, becauſe the Good will not admit him into their Parties, and the Bad delight to be daring in their Enterprizes. This is generally the Caſe with both; but we muſt not forbid Exceptions. I have no Mind to tire your Friend with the convincing Arguments, that a prudent Youth, when ſent to a public School, enters into Society in Miniature, is urged on by a noble Ambition to excel, forms his Taſte from Emulation, and cements future Connections of the higheſt Importance. I am,

---

* *Wincheſter*, *Weſtminſter*, or *Eaton*.

SIR,

You moſt obedient Servant.

AN

[illegible]

[illegible] to the Greek, with a Prose Translation [illegible] striking Parts of Ovid's Metamorphoses [illegible] sufficient to employ a Youth in their [illegible] consistently with his Commercial [illegible] Improvement of the English [illegible] neatly placed in the Margin; on the same [illegible] Young Gentleman may have the Pleasure left him [illegible] the Text by himself. And in the Notes there [illegible] Remarks on the State of the Arts and Sciences; and where a [illegible] Instruction in the Duties of civil Life occurs; and a general Attempt to render the Heathen Theology at least consistent with itself, and to guard the Hearts of Youth, when a Heathen Deity is introduced. Price 6s.

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He undertakes the private Tutorage of about ſix young Gentlemen at a Time, in his own Houſe, which is large and commodious, with Gardens adjoining; and as any of theſe Pupils go off, others boarded in the City are admitted under his own more immediate Care, if deſired, on the following Terms:

	l.	s.	d.
Boarding (Proviſions being much dearer) the ſame as in private Houſes, without Waſhing.	12	12	0
General Inſtruction in the School, which is a public Building. *	2	2	0
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per Annum	17	17	0

Entrance One Guinea. Each young Gentleman boarded with him finds a Pair of Sheets. For the above two Guineas are taught, in the public School, Latin, Greek, French, the beſt Engliſh Authors, Writing, Arithmetic, Merchants's Accompts, the Terreſtrial Globe, Geography, and to lay down the common Geometrical Figures and to find their Contents. The additional Expences are, dropping the Two Guineas where this Mark * is, per Annum,

Dancing, Entrance Half a Guinea	2	8	0
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N. B. Mr. RANDALL begs of thoſe Gentlemen, who ſend him Queſtions relating to Annuities, Reverſions, Leaſes, Livings, &c. or Matters of intricate Accounts, that they will, to avoid Inconveniencies, endeavour to be as particular as poſſible to which they require an Anſwer.

AN
INTRODUCTION
TO THE
ARTS *and* SCIENCES, &c.

PART I.

ARITHMETIC, &c.

SYSTEM I.

NUMERATION.

THE *Arabians*, by a wonderful Contrivance, expreſſed any Number, how large ſoever, by a very eaſy Method; for the happy Inventor having introduced theſe ſimple Characters, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9; and inſtead of uſing nine other Marks, as the Eaſtern Nations did, to expreſs the Number of Tens, from one to nine, happily hit on this Character 0, to ſignify nothing, or a Cypher, which was to be inſignificant of itſelf, but its Place was to alter the Value of the preceding Figures, on the Left Hand of it, in a tenfold Proportion.

It is very remarkable, that almoſt every Nation ſhould, unknown to each other, agree in making Uſe of juſt ten Characters, of ſome Sort or other, for the Expreſſion of the ten firſt Numbers, when any other Number would have ſerved; for the *Thracians* had only theſe three ſimple Characters, *viz.* 1, 2, 3; and without deſigning any other Mark to expreſs the Number *Four*, uſed alſo the Cypher 0 in the ſame Manner as the *Arabians* or *Indians*. Thus 10, the *Thracian* Number, ſignifying 4 by the *Arabian* Method, 20 being 8, 30 making 12, 100 being four Times four, or 16, &c.

The Simplicity and Eaſe attending the *Thracian* Notation are every Way well adapted to ordinary Capacities, and to a Nation not engaged in extenſive Commerce, but would by no Means do where many Millions of Money are frequently uſed in Accounts and Converſation. But the *Arabian* Notation, and that of almoſt every People, might poſſibly take its Riſe from the common Way of reckoning on the Fingers, and ſtopping when they counted ten. However, great is the Antiquity of Arithmetic, and we find Mention is made, in the Writings of the Antients, of ſome who were eminent for their Skill in Figures. And ſuch is the Benefit ariſing from a thorough Knowledge thereof, that a Perſon, who writes a good Commercial Hand, ſhall frequently, with theſe two Qualifications, and a virtuous Conduct, raiſe a Fortune, and make a conſiderable Figure in the World.

Among the Variety of Ideas that preſent themſelves to the Minds of Children, there is none more ſimple than that of Unity, or One; every Object their Senſes are employed about brings this Idea along with it: By repeating this Unit in their Minds they come by the Number Two; and ſo proceeding on, ſtill adding one more to the laſt Number, and giving to it the new Name, which our Language affords, they count or have Ideas for ſeveral Units. But many Children, who are otherwiſe of quick and rational Parts enough, do not readily acquire a diſtinct Notion of the higher complex Modes of Unity; neither have they much Inclination to ſtruggle with the common Parts of Arithmetic, unleſs made extremely eaſy to them, by lowering every Thing, in order to raiſe their Underſtandings; and this is what I ſhall attempt in the following Treatiſe, and which will oblige me frequently to depart from the common Method of Teaching Arithmetic, that I may the better aſſiſt and enlarge the ſcanty Conceptions of Children, and to overcome their heedleſs, giddy, Humour, if I poſſibly can.

GENERAL PROBLEM.

To find the Value of each Figure in any Rank, how large ſoever.

A Number is ſaid to have ſo many Places as there are Figures in it; thus, 1, 2, 3, &c. in the firſt Place, on the Right Hand, is 1, 2, 3, &c. but in the ſecond Place it will be *ten*, *twenty*, *thirty*, &c. and in the third Place it will be *one hundred*, *two hundred*, *three hundred*, &c. Now, from hence the Learner may ſee the Value of each Place is *ten Times* the Value of the next before it.

The NUMERATION TABLE.

Trillions.	Billions.	Hundreds of Thouſands of Millions.	Tens of Thouſands of Millions.	Thouſands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Unit's Place.
987, 657,	341, 687,	6	7	8,	9	4	3,	7	8	5,	4	6	5

Which is to be read and got by Heart thus; Nine hundred eighty-ſeven thouſand, ſix hundred and fifty-ſeven *Trillions*; Three hundred forty-one thouſand, ſix hundred and eighty-ſeven *Billions*; Six hundred ſeventy-eight thouſand, nine hundred forty-three *Millions*; Seven hundred eighty-five thouſands, four hundred and ſixty-five.

Now the Learner ſhould take Notice, and get by Heart, that when there are only *three* Places of Figures the higheſt Name will be *Hundreds*; when there are *ſix* Places of Figures, the higheſt Name will be *Hundreds of Thouſands*; and when there are *nine* Places, the higheſt Value is *Hundreds of Millions*; when there are *twelve*, the higheſt Name is *Hundreds of Thouſands of Millions*; and when there are *fifteen* Figures, the higheſt Name is *Hundreds of Billions*; where you meet with *eighteen* Places, the higheſt Name is *Hundreds of Thouſands of Billions*; and when there are *twenty-one* Places of Figures, the higheſt Name is *Hundreds of Trillions*; and whenever you ſee *twenty-four* Places of Figures, the higheſt Value will be *Hundreds of Thouſands of Trillions:* And if you ſhould meet with more than *twenty-four* Places, you may call them Units, Tens, Hundreds, Thouſands, Tens of Thouſands, Hundreds of Thouſands, of *Quadrillions*, by which Names you will be able to manage *thirty* Places of Figures, inſtead of being a Loſs to read more than *nine* as uſual.

But Children, when they begin Numeration, are frequently at a Loſs to call the Figure, in the Place of Tens, by its proper Name, as the doing it right ſuppoſes theſe Children can tell that ſix Tens make Sixty, that eight Tens make Eighty, and ſo on for the reſt between one and ten. Now, in order to remove this firſt Difficulty a Youth meets with, he muſt get by Heart, 2 *Tens are Twenty*; 3, *Thirty*;

4, *Forty*; 5, *Fifty*; 6, *Sixty*; 7, *Seventy*; 8, *Eighty*; *and* 9, *Ninety*. There is another Diſcouragement when Children begin to learn Arithmetic, and that is, their not being able readily to tell how much 9 and 8, or 7 and 9, *&c.* make.

The pretty Youth will therefore now willingly get this by Heart alſo, ſaying, 3 and 4 are 7, *&c.* as in the Side Table.

Explication of Notes alſo to be got by Heart.

Theſe Notes and Axioms can be no where ſo convenietly placed as here, becauſe when the Learner is at a Loſs, he may readily turn to this Place, and ſee what he wants to know.

Plus + ſignifies *Addition*; thus, 3 + 4 ſignifies 3 is, or to be, added to 4; and 9 + 8 ſignifies 9 is, or to be, added to 8.

Minus — ſignifies *Subtraction*; thus, 9—3 means that 3 is, or to be, taken from 9.

Into × ſignifies *Multiplication*; thus, 9 × 3 ſignifies that 9 is, or to be, multiplied into 3, or 3 into 9.

By ÷ ſignifies *Diviſion*; thus, 9 ÷ 3 ſignifies that 9 is, or to be, divided by 3. And ſometimes *By* ÷ is expreſſed thus, $\frac{9}{3}$.

Equal = ſignifies *Equality*; thus, 9 = 6 + 3 ſignifies that 9 is equal to 6 added to 3.

Raiſed ⊚· ſignifies *Involution*; thus, 7⊚·2 ſignifies that 7 is raiſed to the Square, 7⊚·3 that 7 is raiſed to the Cube.

Extract √ ſignifies *Evolution*; thus, $\sqrt[2]{9}$ implies that the Square Root of 9 is, or to be, extracted; and $\sqrt[3]{27}$ means the Cube Root of 27.

So is : : ſignifies *Proportion*; thus, 2 : 8 : : 6 : 24 mean as 2 is to 8, ſo is 6 to 24.

Table.

3	and	4	are	7
3		5		8
3		6		9
3		7		10
3		8		11
3		9		12
4		5		9
4		6		10
4		7		11
4		8		12
4		9		13
5		6		11
5		7		12
5		8		13
5		9		14
6		7		13
6		8		14
6		9		15
7		8		15
7		9		16
8		9		17

Axioms, or very uſeful Truths, to be got by Heart.

1. IF to equal Numbers, the ſame Addition be made, their Sums will be equal; thus *John* has 15*s.* and *William* has the ſame Sum, and if we add 5*s.* to each, they will be both equally rich.

2. If from equal Numbers, the ſame Figure be taken, the Remainder will be the ſame; thus, if we take 5*s.* from *John* and *William*'s 15*s.* they will each have only 10*s.*

3. If equal Numbers are multiplied by the very ſame Number, the Products will be equal; thus *John* and *William*'s 15*s.* being multiplied by 3, they will each have 45*s.*

4. If equal Numbers are divided by the very ſame Number, the Quotients will be equal; thus, *John* and *William*'s 15*s.* being divided by 3, they will each have 5*s.*

5. Thoſe Numbers which are equal to one and the ſame Value, are alſo equal to one another; thus, 12*d.*=1*s.* and 12*d.*=48 Farthings, then ſurely 1*s.*=48 Farthings.

6. The Whole is equal to all its Parts; thus, 12*d.* the Whole, is equal to 8*d.* and 4*d.* its Parts; thus, 20*s.*= 12+8*s.* 24 Men of War in a Fleet = 12+8+4 ſeparated in a Cruiſe.

Theſe ſix Axioms are placed here for the Youth to turn to them when he is deſired; and however plain they may now appear, they are Truths which will enable him to underſtand what may hereafter be propoſed.

ADDITION.

THIS Rule, in Arithmetic, teaches to find how much two or many Perſons owe us; how many Sheep many Flocks make when they are added together; what is the whole Number of Oxen in a County, by adding together what each Farmer has; how many Soldiers and Sailors belong to the King of *Great-Britain*, by adding thoſe in different Places together; what Number of Pariſhes there are in the Kingdom, by adding thoſe of every County together; what Number of People there are now living upon the Face of the Earth, by adding thoſe in every Nation together; and ſo on in like Inſtances.

Rules for writing down Figures.

Place Units under Units, Tens under Tens, Hundreds under Hundreds, Thouſands under Thouſands, and ſo on according to the Numeration Table.

Manner of Operation, with Examples as in the Margin.

THE Learner ſees that Units ſtand under Units, and Tens under Tens, Hundreds under Hundreds, and Thouſands under Thouſands, *&c.* Now let the Column of Figures marked £. be the Farms belonging to the Learner, and he is deſirous of knowing what Eſtate he has. Begin and reckon up the Units firſt, by ſaying, *three, four, eleven, thirteen, ſixteen, twenty-one, twenty-five, thirty-one, thirty-ſix, forty-two Units.* Now remember, above *ten*, you muſt carry *one* to the *Tens*; above *twenty*, you carry *two*; above *thirty*, you carry *three*; above *forty*, you carry *four*; and ſo on how long ſoever the Column of *Units* may be: So here the *Units* making *forty-two*, you muſt ſay, *two* and carry *four, ten, fifteen, ſixteen, eighteen, twenty-two, tenty-five, thirty-one, thirty-eight, forty-ſix, forty-eight Tens*, that is, *eight* and carry *four*. Then you will have *two* under the Units, *eight* under the Tens, and *four* ſtanding out on the Left Hand becauſe you have no Hundreds to add them to: And now you will find you have an Eſtate of Four hundred and eighty-two Pounds a Year, only for the Trouble of caſting it up. If you have a Mind to be ſatisfied that I have not deceived you in adding up your Farms, you may leave the uppermoſt Farm of twenty-ſix Pounds a Year out, and reckon up the reſt, which will amount to Four hundred and fifty-ſix Pounds a Year; and then adding this Sum to the Farm you left out, you have your whole Eſtate again, for which I hope you will, after you have filled up the Blanks, take Pains to caſt up by yourſelf the Sums under *Yards, Miles, Men*, and *Sheep:* Remembering to avoid the ungenteel Cuſtom of ſaying (under *Yards* for Inſtance) ſeven (being ſuppoſed) and five is twelve, twelve and ſix is eighteen, eighteen and ſeven is twenty-five, and ſo on, which not only loſes Time, but alſo tires the Memory, and is like keeping a Weight longer than neceſſary on a Perſon's Back: It is for this Reaſon I would have you leave out the *ands*, inſtead of ſaying, when you caſt up the Farms of your Eſtate, three *and* one is four, *and* ſeven is eleven, *and* two is thirteen, *and* three is ſixteen, *&c.* becauſe as you

£.
26
85
76
64
35
43
22
17
51
63
———
———

Yards.
378
295
476
393
242
607
566
785
* 7
———
———

Miles.
4760
1783
2820
4723
5867
9343
6742
3885
* 7
———
———

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*Theſe Blanks, at this Mark * in this and the following Pages, are to be filled up by the Scholar, who, upon Pain of his Maſter's Diſpleaſure, is never to uſe the ſame Figures to be found in any other Youth's Book in the School.*

have this pretty Income you muſt talk like a Gentleman to your Figures.

When you have caſt up the Sums in the Margin, ſeparate every Line, according to the Meaning of *Axiom* 6, as the Whole is equal to all its Parts, and add up all the Parts ſeparately, which will give you an excellent Notion of Addition, and of the Value of the Places in which the Figures ſtand. Thus, beginning with the Sum under the Mark £. 20 + 6, 80 + 5, 70 + 6, *&c.* then add up the *Units*, and let the 42 ſtand under *Units*; then add up the *Tens*, and let the 440 ſtand under the *Tens*, meaning the 0 under 2, and the 4 under the 4, and then the other 4 will ſtand out on the Left Hand: Then you will have 440 + 42 = 482 as before. So do with the Sum under *Yards*, by writing on your Slate, 300 + 70 + 8, 200 + 90 + 5, *&c.* and caſting up the *Units*, and ſetting what they make underneath, without carrying any Figures forwards; then caſting up the *Tens* and ſetting them down in like Manner; then the *Hundreds*, putting the Amount underneath. When this is done, make a little Sum of the *Hundreds*, *Tens*, and *Units*, putting them properly under one another, and caſt them up, and you will prove what you did before. In like Manner go on with the Sums under *Miles*, *Men*, *Sheep*, which will afford you a rational Entertainment, as well as an enlarged Idea of Addition.

Men.
57632
34581
27862
43217
82130
76392
63781
45862
98754
* 3

Sheep.
274538
868497
737864
656758
545963
939625
826386
798298
467572
* 3

Examples to be wrote down.

1. William *Johnſon* owes me two Pounds, and of ten Perſons, each owes me two Pounds more than any one to be named among them: Thus, the ſecond is four Pounds, the third is ſix Pounds, and ſo on to ten Perſons. What do they all owe me?

2. There are ten Pariſhes; in one there are twelve hundred and fifty Sheep, and in each of the reſt there are twenty-five more than in any one to be named among them: Thus, in the ſecond there are twelve hundred and ſeventy-five Sheep, in the third Pariſh there are thirteen hundred, and ſo on to ten Pariſhes. How many Sheep are there in all?

3. In ten Counties, one contains two thouſand five hundred Oxen, and in each of the reſt there are ſeventy-four Oxen more than in any one County to be named among the ten: Thus, in the ſecond County there are two thouſand five

hundred and ſeventy-four; in the third, two thouſand ſix hundred and forty-eight Oxen, and ſo on, ſtill adding ſeventy-four to the laſt before this Number is placed under the others. How many Oxen are there in all?

4. In twelve different Parts of the World we have, in the firſt, ſuppoſe, five thouſand eight hundred and ſeventy-five Sailors and Soldiers, and in each of the reſt of the Places there are eighty-ſeven Men more than in any one Place to be named: Thus, in the ſecond there will be five thouſand nine hundred and ſixty-two; in the third, ſix thouſand and forty-nine, and ſo on for the reſt. How many Men are there in all?

In theſe four Queſtions, when the Numbers are formed, any one in each Queſtion may be entirely altered, before they are added up, to prevent any Deceit in ſtealing the Total from other Boys' Books. And the ſame may be done in the following Queſtions.

5. There were twenty Cities of *Europe*; in the firſt the Riches of the foreign Merchants amounted to 576,378*l.* and in the ſecond there were 1769*l.* more than in the firſt; in the third 1769*l.* more than in the ſecond; and ſo on to the laſt City. What was the whole Amouut of all the Riches?

6. In the firſt of theſe Cities there are 58,765 Inhabitants; in the ſecond 3757 more than in the firſt; and in the third there are 3757 more than in the ſecond City; and ſo on, for every one of the above twenty Cities, each having 3757 Inhabitants more than the laſt you come to. How many Inhabitants are there in all theſe Places?

7. There are, ſuppoſe, ſomewhere upon the Face of the Earth, thirty Empires, Kingdoms, and States; in the firſt there are, by Suppoſition, three Millions ſeven Hundred ninety-five Thouſand eight Hundred and ſixty-five Men, Women, and Children; in the ſecond there are twenty-ſeven Thouſand nine Hundred and thirty-four more than in the firſt; in the third there are twenty-ſeven Thouſand nine Hundred and thirty-four more than in the ſecond; and ſo on, ſtill adding the ſame Number to the Inhabitants of every Country you come to. How many Inhabitants are there in all theſe thirty Empires, Kingdoms, and States?

I hope, if the Learner goes through the above Examples, there will be little Danger of his not being well grounded before he proceeds to the next Thing neceſſary in Addition.

AD-

ADDITION of MONEY.

The TABLE.

Of theſe Marks.
¼ ſignifies 1 Farthing
½ ——— 1 Halfpenny
¾ ——— 3 Farthings
(qrs. Quarters)

And
4 qrs. make 1 Penny *(d)*
12 d. ——— 1 Shilling *(s)*
20 s. ——— 1 Pound *(£)*

YOU may conſider the Sums in the Margin as the Rents of Farms and Houſes belonging to your Family; you are firſt to read the Figures, beginning at 20*l.* 17*s.* 6¾*d.* and going down in that Manner to the Bottom; and to read every Rent ſo often up and down till you are extremely perfect, and ſo avoid that ſad Defect of going forwards in Accounts, and not be able readily to read what you have on your Slate or in your Book. And now to prevent your deceiving yourſelf, and that you may be well grounded in caſting up Pounds, Shillings, and Pence, I muſt beg of you to obſerve what follows, when you will be the Delight of your Maſter, and of Uſe to yourſelf.

1. Make ſeveral little Sums of all the Rents, and find what each little Sum amounts to.

2. But you muſt be ſure to fill up the Blank * in the laſt Rents, in the laſt little Sums, ſo that the Pounds, Shillings, and Pence, put in, ſhall differ widely from the Pounds, Shillings, and Pence, put in the Book of any Scholar's in the School, and this you muſt do, if you would avoid the Imputation of a deceitful worthleſs Boy.

£.	*s.*	*d.*
20	17	6¾
5	18	9
13	13	3½
7	8	7
15	14	5¼
8	17	3
25	18	7¾
69	10	10
38	12	6¼
6	13	9
53	8	11½
93	14	9
19	16	5¾
6	9	10
34	15	7½
82	19	11
98	8	8¼
4	17	7
67	3	10¾
71	18	6
28	11	8½
50	17	10
87	13	7½
95	9	3
23	10	10¾
8	16	6
17	12	2½
85	13	11
20	15	9¾
79	7	8
63	14	10¼
8	9	6
*8		

3. When you have found what each little Sum amounts to, add all theſe Amounts together; and if you have acted like a worthy Youth, you will not find the whole Amount of the Rents to agree with any Cyphering Book in the School.

We muſt now proceed to the beſt Manner of caſting up theſe Sums in Pounds, Shillings, and Pence; but it will be of great Uſe to get by Heart the Table in the Margin, for the ready diſcovering how many Shillings are contained in any Column of Pence.

The Manner of caſting up the Rents.

1. WRITE down on your Slate the firſt five Rents: Then begin with the Farthings, and ſay 1, 3, 6, Farthings, which is Three-Halfpence, that is, ½ and carry 1 (to the Pence) 6, 13, 16, 25, 31, Pence; 30 Pence is 2*s.* 6*d.* by the Table, 1 Penny is 2*s.* 7*d.* that is, 7*d.* and carry 2*s.* (to the Units of the Shillings) 6, 14, 17, 25, 32; that is, 2*s.* and carry three Tens (to the Tens of Shillings) 4, 5, 7, the Half of 7 is 3*l.* and 1 Ten to be ſet on the Left of 2*s.* and carry 3*l.* (to the Units of the Pounds) 8, 15, 18, 23; that is, 3*l.* and carry 2 (to the Tens of Pounds) 3, 4, 6. And thus the Buſineſs is ended, and I find the Amount of this little Sum is 63*l.* 12*s.* 7½*d.* containing theſe particular Rents.

2. And in this Manner you may caſt up any other little Sum of five or ſix Rents, and find all their Amounts; then you may make two Sums of all the Rents, and add thoſe two Amounts together. When this is done, the laſt Buſineſs muſt be to caſt up the whole Column of Rents into one Sum, which, if rightly performed, will agree, by *Axiom* 6, with the Total of the little Sums, and the Total of the whole in two Sums. Now then, in order to caſt up the Farthings, we muſt know either how to take the Half of all the Farthings for Halfpence, and the Half of theſe for Pence, or get by Heart, 4 *Farthings* 1 *Penny*; 8, *Twopence*; 12, *Threepence*; 16, *Fourpence*; 20, *Five-*

Pence		*s.*	*d.*
20	make	1	8
30	——	2	6
40	——	3	4
50	——	4	2
60	——	5	—
70	——	5	10
80	——	6	8
90	——	7	6
100	——	8	4
110	——	9	2
120	——	10	—
130	——	10	10
140	——	11	8
150	——	12	6
160	——	13	4
170	——	14	2
180	——	15	—
190	——	15	10
200	——	16	8
210	——	17	6
220	——	18	4
230	——	19	2
240	——	20	—
24	——	2	—
36	——	3	—
48	——	4	—
60	——	5	—
72	——	6	—
84	——	7	—
96	——	8	—
108	——	9	—
120	——	10	—

pence; 24, *Sixpence*; 28, *Sevenpence*; 32, *Eightpence*; 36, *Ninepence*; 40, *Tenpence*; 44, *Elevenpence*; 48, *Twelvepence*. When you come to 48 Farthings make a Point: We will ſuppoſe there are 69 Farthings; then 48 Farthings make 12 Pence, and 20 more 5 Pence, and both make 17 Pence, and ¼ over, which put down, and proceed as in the other Sums of the Rents.

The Uſe of the Pence Table is this: When you reckon up 120 Pence, you may make a Point for 10 Shillings; and go forwards, or you may go on till you have 240 Pence, at which make a Point for 20 Shillings, and go forwards, if you have ſtill more to do. Thus, ſuppoſe I come to 125; here I make a Point, and go on with the 5; or if I come to 247, here I make a Point, and go on with the 7 over.

Examples to be wrote down.

1. TEN Perſons owe me the following Sums; the firſt 2*l.* 17*s.* 6*d.* the ſecond 2*s.* 4¾*d.* more than the firſt, the third the ſame more than the ſecond, the fourth the ſame more than the third, and ſo on, adding 2*s.* 4¾*d.* to the laſt, when the tenth will be 3*l.* 19*s.* 0¾*d.* Required the Amount of the Debts due to me. This is to be divided into two Sums, and the Amounts added together, and then the whole caſt up in one Sum.

2. A Merchant has owing him from twenty Perſons, as follows; the firſt owes him 15*l.* 18*s.* 6¾*d.* the ſecond 5*l.* 17*s.* 5¾*d.* more than the firſt, the third the ſame more than the ſecond, the fourth the ſame more than the third, and ſo on, adding 5*l.* 17*s.* 5¾*d.* to the laſt, when the twentieth will be 127*l.* 10*s.* 8*d.* Required the Total of all the Debts. This is to be divided into four Sums, and the Amounts added together; then it is to be divided into two Sums, and the Amounts added together; and, laſtly, the whole is to be caſt up in one Sum.

In both theſe Queſtions, when the Numbers are formed, and properly placed one under another on the Slate, then to prevent any copying from other Cyphering Books, any one of the middle Numbers may be entirely altered, that the Amounts may be very different throughout the School.

SUBTRACTION.

THIS Rule, in Arithmetic, teaches a Youth to take one Number from another, and to name the Remainder, or *Difference* between two ſuch Numbers. Thus, if we ſay 9 + 3 = 12, we mean Addition, and that 12 is the Amount; but if we ſay 9—3 = 6, we mean that 3 is taken from 9, and the Remainder, or *Difference*, is 6. Indeed the Youth cannot but ſee a wide Alteration between putting 3 *d.* to the 9 *d.* he had before in his Pocket, and taking 3 *d.* out of it from the 9 *d.* So then, in ſubtracting or taking 3 *d.* from 9 *d.* we ſay 3 from 9 and there remains 6. Now let the greater Number be called the *Minuend*, and let the leſſer Number be called the *Subducend*; then we have three Numbers, the *Minuend*, *Subducend*, and *Difference*. The Proof of Subtraction is, that the *Subducend* + the *Difference* = the *Minuend*; for when we take 3 *d.* from 9 *d.* the Difference or Remainder is 6; then if we put or add the 3 *d.* ſo taken away, to the 6 *d.* remaining, you will again have the 9 *d.* in your Pocket. Let all the Numbers from 8 to 1 be taken from 9, by Way of eight Examples.

But young Beginners are often at a Loſs, when one of the Figures in the *Subducend* is greater than that over it in the *Minuend*, to know the Reaſon of *borrowing* and *paying*, as it is called. Thus, if I want to perform 95—28, that is, ſubtract 28 from 95, I ſay, as uſual, 8 from 5 I cannot, but 8 from 15, that is 7 and carry 1 and 2 is 3 from 9, there remains 6. Now remember *Axiom* 6, that the *Whole* is *equal* to all its *Parts*, that is, 80 + 15 = 95; ſo then, we have 80 + 15 the *Minuend*, and 20 + 8 the *Subducend*. Take theſe down on your Slate, and ſay 8 from 15, and there remains 7; and 20 from 80, and there remains 60; ſo that the *Difference* is 60 + 7, or 67. You ſee when you talk of *borrowing*, it was only ſeparating 10 from 90; but if you let 90 remain as it was inſtead of 80, you are obliged to add 10 to 20, otherwiſe the *Difference* or Remainder would be 70, which is wrong.

Now, ſince the Nature of *borrowing* and *paying* in Subtraction is inſtanced in the Number 95, I muſt deſire the pretty Youth to cut himſelf out ſeveral Examples from it; thus, let there be ten, twenty, thirty, or more Sums formed from 95, by taking ten, twenty, thirty, or more different Numbers from it; but I beg he will alter the 95, ſo as to differ from any Cyphering Book in the School, and do them the two Ways mentioned before; that is, in theſe Forms 80 + 15 and 20 + 8, and 95 and 28, that the Learner may be not only very ready at *Subtraction*, but underſtand what he is about. Let ten, twenty, thirty, or more Examples be alſo formed both Ways from this *Minuend* 357,583, but the

Subducends to differ from other Boys' Sums; that is, by *Axiom* 6, when ſeparated into Parts, 300,000 + 50,000 + 7,000 + 500 + 80 + 3.

SUBTRACTION *of* MONEY.

WE will now proceed to *Subtraction* of *Pounds*, *Shillings*, *Pence*, and *Farthings*: But let us firſt ſee the Nature of *borrowing* and *paying* in them alſo. Suppoſe I would take 3*s.* 9*d.* from 4*s.* 6*d.* I ſay 9 from 6 I cannot, but borrow 12*d.* and 6*d.* is 18*d.* then 9*d.* from 18*d.* that is 9*d.* and carry 1*s.* and 3*s.* is 4*s.* from 4*s.* there remains 0; that is, 3*s.* + 18*d.* — 3*s.* + 9*d.* as was explained before. Again, take 3*l.* 17*s.* 6¾*d.* from 5*l.* 12*s.* 5½*d.*; put theſe on your Slate, and read them always thus, 3 from 6, 3 and carry 1 and 6 is 7 from 17, 10 and carry 1 and 17 is 18 from 32, 14 and carry 1 and 3 is 4 from 5, 1 making a Remainder of 1*l.* 14*s* 10¾*d.*; that is, by *Axiom* 6, ſeparating them thus on your Slate, 4*l.* 31*s.* 16*d.* 6 *qrs.* —— 3*l.* 17*s.* 6¾*d.* which ſhews how you *borrow* and *pay*, and gives the ſame *Remainder*. Now from the *Minuend* 5868*l.* 3*s.* 2¼*d.* form ten, twenty, or thirty Sums, by taking ten, twenty, or thirty leſſer Sums from it, but altering all the *Subducends*, every now and then, uſing both Ways, as 5867*l.* 22*s.* 13*d.* 5 *qrs.* which will richly reward the Learner for his Pains; and ſome reaſonable Pains a Learner muſt take, as God has ordained that, in order to exert the various Talents and improve them, nothing of Conſequence in the Concerns of the World can be acquired by only barely viewing what is to be learned; were this the Caſe, the whole Œconomy of Divine Providence would be totally ſubverted, as he has given various Talents for the different Scenes of Life, but expects every Youth ſhould take Pains to fit himſelf for ſome Rank in Society according as his Talent is adapted.

MULTIPLICATION.

THIS Rule ſhews us how to make any Number, two, three, or a great many Times more than it is, and is a ſhort Way of performing the ſame by Addition; thus, by Addition we can make 3 four Times more than it is, by adding four threes together, as 3 + 3 + 3 + 3; but by *Multiplication* we can do it at once, by ſaying 3 Times 4 is 12. So if we want to make 8, or any higher Number, nine Times more, we can do it by Addition, ſaying 8 + 8 + 8, &c. which would be extremely tedious, but by *Multiplication* we need only ſay 8 Times 9 is 72. However, we cannot enjoy this ſhort Method of performing, at once, many Additions, unleſs we take Pains to learn the *Multiplication Table* in the Margin.

Now, let the Learner take down on his Slate 24, and make it 3 Times more; then you muſt ſay 3 Times 4 is 12, 2 and carry 1, 3 Times 2 is 6 and 1 that I carry is 7; ſo that the whole is 72, which is 3 Times more than 24, and is the ſame by adding $24+24+24=72$. As the Whole is equal to all its Parts, by *Axiom* 6; ſo to multiply the *Whole* or its *Parts*, is the very ſame Thing; thus, $20+4\times3=60+12=72$. The Youth will be pleaſed to remember that the Number (24) to be multiplied, is called the *Multiplicand*, the Number (3) that multiplies it, is called the *Multiplier*, and the Amount of what is done (72) is called the *Product*.

Write down 987, and let the Multipliers be 3, 4, 5, 6, 7, 8, 9, ſo as to make ſeven diſtinct Sums of them: Then by 12, making in all eight Sums; thus, for Inſtance, when 12 is the Multiplier, ſay 12 Times 7 is 84, 4 and carry 8, 12 Times 8 is 96, and 8 is 104, 4 and carry 10, 12 Times 9 is 108, and 10 is 118, 8 and carry 11; hence $987\times12=11844$; or $900+80+7\times12=10800+966+84=11844$ as before. The Learner is deſired to take all this down upon his Slate and work it over again.

Write down 5,987,659, but alter it ſo as to differ from any Book in the School, and form ten, twenty, or thirty Sums of it, by multiplying by 21, 22, 23, &c. only remember when you multiply by the Figure in the ſecond Place, which you know are *Tens*, to ſet the firſt Figure of the ſecond Line under the *Tens* of the firſt Line, or *Product*, and then you cannot miſs placing the other properly.

3	times	3	is	9
3	—	4	—	12
3	—	5	—	15
3	—	6	—	18
3	—	7	—	21
3	—	8	—	24
3	—	9	—	27
4	—	4	—	16
4	—	5	—	20
4	—	6	—	24
4	—	7	—	28
4	—	8	—	32
4	—	9	—	36
5	—	5	—	25
5	—	6	—	30
5	—	7	—	35
5	—	8	—	40
5	—	9	—	45
6	—	6	—	36
6	—	7	—	42
6	—	8	—	48
6	—	9	—	54
7	—	7	—	49
7	—	8	—	56
7	—	9	—	63
8	—	8	—	64
8	—	9	—	72
9	—	9	—	81
12	—	2	—	24
12	—	3	—	36
12	—	4	—	48
12	—	5	—	60
12	—	6	—	72
12	—	7	—	84
12	—	8	—	96
12	—	9	—	108
12	—	10	—	120
12	—	11	—	132
12	—	12	—	144

Write down the ſame *Multiplicand,* but altered, and form ten, twenty, or thirty Sums of it, by multiplying by 851, 852, 853, *&c.* and then let 98761, 98762, 98763, *&c.* be the *Mutipliers* to the ſame *Multiplicand* altered, and of it form ten, twenty, or thirty Sums; and then let 876541, 876542, 876543, *&c.* be the *Multipliers* to the ſame *Multiplicand* altered, and of it form ten, twenty, or thirty Sums. Only remember when you multiply by the Figure in the third Place, to ſet the firſt Figure under the third Figure, or *Hundreds,* of the firſt Line or *Product*; when you multiply by the Figure in the fourth Place, to ſet the firſt Figure under the fourth Figure, or *Thouſands,* of the firſt Line or *Product*; and ſo on, when you multiply by the Figure in the *fifth* and *ſixth* Places, to ſet the firſt Figures under the *fifth* and *ſixth* Places of the firſt Line or *Product.*

Laſtly, if the *Multipliers* have Cyphers only in the firſt, ſecond, *&c.* Places, ſet the firſt ſignificant Figure under the *Units* of the *Multiplicand,* and proceed as before, only barely writing down the Cyphers on the Right Hand of the *Product.* Thus, 87 × 10, 20, 30, *&c.* to 100, 200, 300, *&c.* 1000, 2000, 3000, *&c.* The Manner of Operation is this, 87 × 10; 0 is 0, once 7 is 7, once 8 is 8; in 87 × 20, 0 is 0, twice 7 is 14, 4 and carry 1, twice 8 is 16, and 1 is 17; in 87 × 3000, 0 is 0, 0 is 0, 0 is 0, 3 Times 7 is 21, 1 and carry 2, 3 Times 8 is 24, and 2 is 26; all which the Learner will be pleaſed to work upon his Slate. But if there are likewiſe Cyphers only in the firſt, ſecond, *&c.* Places of the *Multiplicand,* ſet the firſt ſignificant Figure of the *Multiplier* under the firſt ſignificant Figure of the *Multiplicand,* only barely writing down all the Cyphers in both, placing them on the Right Hand of the *Product.* 58000 × 38000 (altered to differ from other Youths' Books); here, diſregard the Cyphers, and 58 × 38 (ſo altered) making a *Product,* ſet ſix Cyphers on the Right Hand of 4, which you will find in the *Product.* Let 98,760,000 (altered in the ſignificant Figures) be the *Multiplicand,* and 7,850,000, 7,860,000, 7,870,000, *&c.* be the *Multipliers,* and form ten, twenty, *&c.* Sums of them.

MULTIPLICATION *of* MONEY.

AS this will be an Affair of great Uſe, on many Occaſions, in the leſſer Concerns of the World, I hope the worthy Youth will give due Attention to it. But young Ladies, in particular, if they never travel any further in

Accounts than to the End of this *Multiplication*, will be hereby enabled to caſt up what the *Cambricks*, *Linen*, *Lawns*, *&c.* which they purchaſe in Shops amount to; and then, ſurely, it is well worth their while to take a little Pains in what ſo nearly concerns them, and for which they have ſufficient Abilities, were they but properly exerted.

Suppoſe a Lady buys 3 Yards of Cambrick, at 6s. 9¾d. a Yard, and ſhe would know how much ſhe is to pay for the ſame: Here ſhe muſt pay three Times as much as if ſhe bought one Yard only; and therefore ſhe muſt make the Price of one Yard three Times more; and this is done by making a little Sum in Addition of *6s. 9¾d.* by letting the Price be wrote three Times down, and then adding all up together, which will make 1*l.* 0*s.* 5¼*d.* or ſhorter by *Multiplication*, ſay 3 Times ¾ is 9, that is ¼, and carry 2 (to the Pence) 3 Times 9 is 27 and 2 is 29, that is 5 and carry 2 (to the Shillings) 3 Times 6 is 18 and 2 is 20: So that we have again 1 *l.*0*s.* 5¼*d.* the Money to be paid the Tradeſman.

Now, from hence, the Learner perceives what tedious Work it would be if the Lady bought a great many Yards, and was to find the Value of the whole by Addition; and therefore it is well worth while to be able to *find the Amount* of ſuch Purchaſes by *Multiplication*. *Suppoſe ſhe buys* 18 *Yards at 6s. 9¾d. a Yard, what will they coſt her?* Now, conſider what two Numbers multiplied together will make 18 Yards; and theſe, by the *Multiplication Table*, will be 3×6=18; therefore, as before, find what 3 Yards of Cambrick will come to; then, becauſe you have made the Price only 3 Times more, you muſt make that 3 Times 6 Times more, becauſe the Price muſt be made 18 Times more. Now, then, ſay 6 Times 1 is 6, that is ½ and carry 1 (to the Pence), 6 Times 5 is 30 and 1 31, that is 7 and carry 2 (to the Shillings), 6 Times 0 is 0, but 2 is 2, 6 Times 1 is 6; ſo that I find 18 Yards will coſt 6*l.* 2*s.* 7¼*d.* But ſuppoſe there are 19 Yards, I muſt then add the Price of one Yard to the Value of 18 Yards, becauſe the Price of one Yard muſt be made 19 Times more.

If the Half Yard, or Quarter, is to be paid for, I ſhall not make the young Lady ſo bad a Compliment, as to ſuppoſe ſhe cannot tell what the Half Yard or Quarter will come to, eſpecially as for ſuch Sort of Goods Tradeſmen ſeldom reckon lower than Pence per Yard, ſuppoſe 6*s.* 9*d.* Therefore, when you have made the Price 18, 19, 25, 30, *&c.* Times more, add (if there is this odd Meaſure) the three

Quarters of a Yard, by ſaying Half a Yard is 3*s.* 4½*d.* and a Quarter of a Yard is 1*s.* 8¼*d.* (becauſe ½ Yard is 40 Pence ½) and ſo you will have 19, &c. Yards and ¾ at 6*s.* 9*d.* per Yard.

Make ten, twenty, or thirty Sums in buying Cambrick; thus 20 Yards, 21, 22, 23, &c. and let a Sum every now and then have ¾ Yard, and let the Prices be 6*s.* 9*d.* per Yard, 6*s.* 10*d.* 6*s.* 11*d.* 3*s.* 0*d.* 3*s.* 1*d.* &c. that is, 20 Yards at 6*s.* 9*d.* 21 at 6*s.* 10*d.* 22 at 6*s.* 11*d.* &c.

Let the Price of ſome Sort of Goods be 1*l.* 17*s.* 6*d.* per Yard, and let the Learner form ten, twenty, or thirty Sums of it, beginning at 25 Yards, 26, 27, &c. Yards. The firſt Sum is the Price made 25 Times more, which anſwer thus, $5\times5=25$, then 5 Times 6 is 30, that is 6 and carry 2 (to the Units of Shillings), 5 Times 7 is 35, and 2 is 37, that is 7 and carry 3 (to the Tens of Shillings), 5 Times 1 is 5 and 3 is 8 and carry 4 (becauſe the Half of 8 is 4), 5 Times 1 is 5 and 4 is 9; that is 9*l.* 7*s.* 6*d.* for the Price 5 Times more; then becauſe I muſt make this laſt Sum ſtill 5 Times more ($5\times5=25$), I ſay 5 Times 6 is 30, that is 6 and carry 2 (to the Shillings), 5 Times 7 is 35 and 2 is 37, that is 7 and carry 3, the Half of 3 is 1 and 1 over, ſet down the 1 (10*s.*) 5 Times 9 is 45 and 1 is 46: And here we find 25 Yards at 1*l.* 17*s.* 6*d.* will coſt 46*l.* 17*s.* 6*d.* and thus proceed to find all the reſt.

The Learner may remember, that any Number of Yards, not exceeding the Bounds of the *Multiplication Table*, may be conveniently computed by the Price in the Manner above-mentioned. Thus $144=12\times12$, but in order to manage the Numbers between 9 and 12 (that is $9\times9=81$, $12\times12=144$) and occaſionally to multiply by eleven, there muſt be another Table got by Heart, as in the Margin. But if the Number is 12, or under, more than 144, we may manage it thus; $12\times12=144$ Times, and then, in the Mind, multiply the Price by 12, 11, 10, &c. and ſet it under what it comes to at 144 Times, and then we have 156 Times or Yards, which is a greater Number than is uſually wanted in theſe leſſer Concerns of buying Shop-Goods. If the Number of the Yards, &c. as in larger Concerns, ſhould run much higher, the Rule of Arithmetic called *Practice*, hereafter to be taught, will eaſily anſwer all ſuch Queſtions.

10 Times 10 is 100
11 — 2 — 22
11 — 3 — 33
11 — 4 — 44
11 — 5 — 55
11 — 6 — 66
11 — 7 — 77
11 — 8 — 88
11 — 9 — 99
11 — 10 — 110
11 — 11 — 121

Let there be ten, twenty, or thirty Sums formed from the Price 5 *l.* 17 *s.* 9 *d.* and increaſing 2 *d.* every Time, and let the Number of Yards, &c. be 156 and downwards, decreaſing by 5 Yards in each Example.——The Learner may be aſſured of receiving much Advantage, if he will concur with his Maſter's Endeavours, in underſtanding how to work theſe Sums.

DIVISION.

THIS Rule *undoes* that which *Multiplication does*; thus, $3 \times 3 = 9$, and $\frac{9}{3}$ or $9 \div 3 = 3$, which is juſt the Reverſe; for as 9 is 3 Times more than 3 by *Multiplication*, ſo 3 is 3 Times leſs than 9 by *Diviſion*. If 9 *l.* is to be equally divided among 3 Perſons, the Share of each will be 3 *l.* becauſe $3 \times 3 = 9$, and leaves no Remainder; but if 10 *l.* is to be divided among 3 Perſons, there will be 3 *l.* for each Perſon, and 1 *l.* will remain belonging to the 3 Perſons. The Learner is to remember, that the Number to be divided is called the *Dividend*, the Number of Perſons among whom it is to be divided, or the Number dividing the Dividend, is called the *Diviſor*, and the Anſwer, or Share of each Perſon, is called the *Quotient*; and when there is not an equal Diviſion, or when ſomething is over, it is called the *Remainder*. The Queſtion to be aſked in *Diviſion* is thus, the *threes* in 9? 3 *Times*, becauſe 3 Times 3 is 9; the *ſixes* in 54? 9 *Times*, becauſe 9 Times 6 is 54; the *eights* in 72? 9 *Times*, becauſe 9 Times 8 is 72. Now, from hence the Learner may ſee that if the *Diviſor* and *Quotient* are multiplied together, the *Product* muſt be the ſame as the *Dividend*; for if 3 Perſons have each 3 *l.* out of 9 *l.* their Shares, when *added* together, muſt make the 9 *l.* and *Multiplication* only does this *Addition* a ſhorter Way.

3	in	9	—	3 times
3	—	12	—	4
3	—	15	—	5
3	—	18	—	6
3	—	21	—	7
3	—	24	—	8
3	—	27	—	9
4	—	16	—	4
4	—	20	—	5
4	—	24	—	6
4	—	28	—	7
4	—	32	—	8
4	—	36	—	9
5	—	25	—	5
5	—	30	—	6
5	—	35	—	7
5	—	40	—	8
5	—	45	—	9
6	—	36	—	6
6	—	42	—	7
6	—	48	—	8
6	—	54	—	9
7	—	49	—	7
7	—	56	—	8
7	—	63	—	9
8	—	64	—	8
8	—	72	—	9
9	—	81	—	9

The Table in the Margin will prevent that Confuſion which Children diſcover, at their firſt ſetting out, when they want to know how often the Diviſor is contained in the Dividend; for they need only get the foregoing Table perfectly by Heart, backwards and forwards, and then they will go on with more Pleaſure. Thus, if the *Diviſor* is 8, and the *Dividend* is 72, then the *Quotient* will be 9, becauſe 9 *Times* 8 is 72; or if the *Diviſor* is 9 then the *Quotient* is 8, according to the Table.

Write down $\frac{78540,}{2,\ \&c.}$ that is, divide this Number by 2, 3, 4, &c. to 9. When 2 is the Diviſor, ſay the Half of 7 is 3, the Half of 18 (becauſe 1 is over, which call 10 and 8 is 18) is 9, the Half of 5 is 2, the Half of 14 (becauſe 1 is over, which call 10 and 4 is 14) is 7, the Half of 0 is 0: All which will ſtand thus, $\frac{78540}{2}=39270$ the Anſwer or Quotient; and this Form will not only be more elegant and convenient, but will uſe the Learner to the Expreſſing of Fractions, which will alſo accuſtom him to the ſame Form in the Solution of a few Arithmetical Queſtions by *Algebra*, which is, unfortunately for young People, but too often conſidered as a myſterious Affair, at leaſt extremely difficult; tho' it is quite the contrary, as the Learner, to his agreeable Diſappointment, ſhall ſee, if he is but willing to obſerve what ſhall be ſaid on that Occaſion.

We will work another of theſe Sums in Diviſion, and leave the reſt to the Ingenuity of the Learner. We will take 8 for the *Diviſor*, and 78540 for the *Dividend*, and then the Expreſſion will be $\frac{78540}{8}=9817\frac{4}{8}$ the Anſwer or *Quotient*; that is, if a Fortune, expreſſed by the *Dividend*, is divided equally among 8 Children, each will have 9817 *l.* and 4 *l.* over; to be work'd thus, the *eights* in 78, 9 Times; the *eights* in 65 (becauſe 6 over to be called ſix Tens, which is 60 and 5 is 65) 8 Times; the *eights* in 14 (becauſe 1 over to be called Ten, and 4 is 14) 1; the *eights* in 60 (becauſe 6 over to be called ſix Tens which is 60, and 0 is ſtill 60) 7 Times, and there remains 4, which expreſs $\frac{4}{8}$, to be read four Eights. Hence, the Learner ſees all depends upon his getting the laſt Table well by Heart; and when he ſays the *eights* in 78, he knows he muſt take $8\times9=72$ from 78, and 6 remains; the *eights* in 65, he muſt take $8\times8=64$ from 65, and there remains 1; the *eights* in 14, he muſt take $8\times1=8$ from 14, and there remains 6; the *eights* in 60, he muſt take $8\times7=56$ from 60, and there remains over 4. And to prove his Work, if he ſets the *Diviſor* underneath the *Quotient* and multiplies, he will agreeably ſee that the *Product* will be the ſame with the *Dividend*, provided he takes in the Remainder 4, which he knows belonged to the *Dividend*. Thus, $8\times7+4=60$, that is 0 and carry 6, $8\times1+6=14$, that is 4 and carry 1, $8\times8+1=65$, that is 5 and carry 6, $8\times9+6=78$ that is 8 and carry 7, which proves the Diviſion was rightly made.

We muſt requeſt the Learner will form ten, twenty, or thirty different *Dividends* of 78540, by multiplying it ſeparately by 5, then by 6, 7, 8, 9, 10, 11, 12, 13, *&c.* for ſo many new *Dividends*, and that he will let the *Diviſors* to each *Dividend* be 2, 3, 4, 5, 6, 7, 8, 9, in order to make himſelf expert at this Sort of *Diviſion*, which will leſſen his Toil and Perplexity as he proceeds forwards.

Let the ſame *Dividend* 78540 be divided by 10, 11, 12, 20, 100, 200, 1000, 2000. But in order to divide by 11 or 12, we muſt, for the Learner's Eaſe, form another Table to be got by Heart, both backwards and forwards, that is, the *elevens* in 33, three *Times*, or the *Threes* in 33, eleven *Times*, and ſo on for the reſt, as was directed in the laſt Table. Now, $\frac{7854,0}{1,0}=7854$; here is no Difficulty, for it is only cutting off the firſt Place in the Dividend with a Comma, and doing the ſame in the Diviſor 10, and the other Figures remain unaltered, as 1 neither multiplies nor divides; for if we ſay the *ones* in 7, 7 *Times*; the *ones* in 8, 8 *Times*; the *ones* in 5, 5 *Times*; the *ones* in 4, 4 *Times*, there is no Alteration.

11	in	22	— 2 times
11	—	33	— 3
11	—	44	— 4
11	—	55	— 5
11	—	66	— 6
11	—	77	— 7
11	—	88	— 8
11	—	99	— 9
12	—	24	— 2
12	—	36	— 3
12	—	48	— 4
12	—	60	— 5
12	—	72	— 6
12	—	84	— 7
12	—	96	— 8
12	—	108	— 9

So $\frac{785,40}{1,00}=785\frac{40}{100}$, becauſe there is a ſignificant Figure 4 with 0, when cut off by the Comma; in like Manner, $\frac{78,540}{1,000}=78\frac{540}{1,000}$, becauſe there are two ſignificant Figures 54 with 0, when cut off by the Comma,

Now divide $\frac{78540}{11}=7140$, which is performed thus; the *Elevens* in 78, 7 *Times*; the *Elevens* in 15 (becauſe 1 remains, which is 10 and 5 is 15) 1; the *Elevens* in 44 (becauſe

4 remain, which is 40 and 4 is 44) 4 *Times*; the *Elevens* in 0, no *Times*. And now the Learner ſees the Neceſſity of being perfect in the Table of *Elevens*, otherwiſe he muſt take a round-about Way to divide by *Eleven*. And now if the diligent Youth will take the Pains to multiply 7140 by 11, that is, the *Quotient* by the *Diviſor*, he will find the *Product* is the ſame as the *Dividend*. Thus again, $\frac{78540}{12} = 6545$ performed thus, the *Twelves* in 78, 6 *Times*; the *Twelves* in 65 (becauſe 6 remain, which is 60 and 5 is 65) 5 *Times*; the *Twelves* in 54 (becauſe 5 remain, which is 50 and 4 is 54) 4 *Times*; the *Twelves* in 60 (becauſe 6 remain, which is 60 and 0 is ſtill 60) 5 *Times* exactly: Then $6545 \times 12 = 78540$ the *Dividend*.

Alſo divide $\frac{7854{,}0}{2{,}0} = 3927$; here is no Difficulty, as it is only pointing off the Cypher in the *Diviſor*, and the firſt Place in the *Dividend*, and taking the Half of 7854. In like Manner, dividing by 200, 2000, *&c.* it is nothing more than cutting off two, three, *&c.* Places in the *Dividend*, and taking the Half of the Places on the Left of the Comma. So when the *Diviſors* are 300, *&c.* 400, *&c*, 500, *&c.* to 900, *&c.* you have nothing more to do than to point off in the *Dividend*, according to the Number (two, three, *&c.* of the Cyphers in the *Diviſors*, and divide the Places on the Left of the Comma, by 3, 5, 4, *&c.* to 9.

But if there are two ſignificant Figures, ſuppoſe 21, 22, 23, *&c.* for *Diviſors*, we muſt place them differently, and the Operation is more troubleſome, but a little Practice will ſoon remove the Difficulties. Thus, let 78540 be the *Dividend* and 21 the *Diviſor*, to ſtand thus, 21)78540. The firſt Thing is to ſay, the *twos* in 7 will go 3 Times, which place under the Left of the Line for the *Quotient*, and becauſe there are two Places in the *Diviſor*, you muſt multiply them by 3, and take it from the two laſt Places in the *Dividend*; thus, $3 \times 1 = 3$ from 8, 5, $3 \times 2 = 6$ from 7, 1; ſo that 15 is the Remainder, or $78 - 63 = 15$; to this Remainder 15 bring down the 5 from the *Dividend*, and then you have 155. Now ſay, the twos in 15 will go 7 Times, which place on the Right of the 3 under the Line, and multiply 21 by 7, and take it from 155, ſaying $7 \times 1 = 7$ from 5 I cannot, but 7 from 15, 8 and carry 1, $7 \times 2 + 1 = 15$ from 15 0; ſo that 8 is now the Remainder, to which bring down the 4 from the *Dividend*, and then you have 84. Now ſay, the twos in 8 will go 4 Times, which place on the Right of 7 under the Line, and multiply 21 by 4, and take it from 84, ſaying 4×1=4 from 4, 0; 2×4=8 from 8, 0; ſo that 0 0 is now the Remainder, to which bring down the 0 from the *Dividend*, and then you have 000. Now ſay, the twos in 0 will go 0 Times, which place on the Right of 4 under the Line, and remember that as 0 cannot multiply the *Diviſor* 21, and as all the Figures in the *Dividend* have been divided or uſed, ſo nothing remains, becauſe the *Quotient* 3740×21 the *Diviſor* =78540 the *Dividend*.

Again, let us divide 78540 by 28, which may ſtand thus, as before, $\underline{28}$) 78540; here, as in the laſt Sum, the Line under the *Diviſor* is the Place where the *Quotient* is to ſtand, for the Convenience of ſetting the *Diviſor* under the *Quotient*; and multipling them together, to prove the Sum. The firſt Thing is to ſay, the twos in 7 will go 3 Times, which place under the Line, as in the laſt Sum; and becauſe there are two Places in the *Diviſor*, we muſt multiply them by 3, and take the *Product* from the two laſt Figures in the *Dividend*; thus, 3×8 is 24 from 28, (8 in the *Dividend* being here to be called 28) 4 and carry 2; 2×3 is 6, and 2 that I carry is 8 from 7 I cannot; ſo here I am catched, for ſurely we cannot take 84 (28×3) from 78; the *Diviſor* being multiplied by the Quotient Figure muſt be either equal to the *Dividend* or leſs than it is. Hence, then, the Learner concludes the Quotient Figure muſt not be 3 but 2; and therefore I ſay thus, 2×8 is 16 from 18 (8 in the *Dividend* being now to be called 18) 2 and carry 1; 2×2 is 4, and 1 that I carry is 5 from 7, and there remain 2: So that the Remainder is 22, to which I bring down the 5 from the *Dividend*, and then we have 225. Now conſider a little, this I muſt divide by 28, but tho' the *twos* will go in *two*, yet 28 will not go in 22, and therefore try 9, which you will find too much, and that you are catched again, for 9×28 is 252, which, ſurely, cannot be taken from 225. So try 8; then 8×28 is 224, and you ſee this is leſs than 225; therefore put 8 in the *Quotient*, and ſay 8×8 is 64 from 65, (5 in the *Dividend* being now to be called 65) 1 and carry 6; 8×2 is 16, and 6 that I carry is 22 from 22, and there remains 0; then we have the Remainder 1, to which bring down the 4 from the *Dividend*, and then we have 14. Now conſider again, you cannot make the *Diviſor* 28 go in a leſſer Number 14, and therefore, in ſuch Caſes, put a 0 in the *Quotient*, and bring down the 0 from the *Dividend*, and then we have 140. Now ſay, the *twos* in 14 will go 7 Times; but if you try 7×28 on your Slate, you will have 196, which you know is too much; then try 6 Times, and then 6×28 is 168, and even this is too much; then try 5 Times, and then 5×28 you will find to be 140, which you know will do; ſo I ſay 5×8 is 40, from 40, (0 in the *Dividend* being now to be called 40) 0 and carry 4; 5×2 is 10, and 4 that I carry is 14 from 14 and there remains 0. Now take a little Breath, after this Toil, as, perhaps, you may call it.

Again, let us divide 78540 by 99, which will ſtand thus, 99) 78540; now ſay, the *nines* in 78 (for you know 9 will not go in 7) will go 8 Times; but try and you will find 99×8 is 792, which is more than the *Dividend* 785, and that it muſt not be ſo; for 785 divided among 99 Perſons cannot be 8 for each; but try 6 Times, and then ſay, 6×9 is 54 from 55 (5 in the *Dividend* being now to be called 55) 1 and carry 5; 6×9 is 54, and 5 that I carry is 59 from 58 (8 in the *Dividend* being now to be called 58) I cannot, but 59 from 78, that is 19; ſo that the Remainder is 191 which muſt not be; for ſurely that is not dividing 785 among 99 Perſons, when what remains is more than the Number of Perſons; it muſt be always leſs; and therefore you will conclude that each Perſon's Share will be more than 6. Now try 7 Times, and ſee what that will do; then 7×99 is 693, ſay 7×9 is 63 from 65 (5 in the *Dividend* being now to be called 65) 2 and carry 6, 7×9 is 63 and 6 I carry is 69 from 68 (8 in the *Dividend* being now 68) I cannot, but 69 from 78 gives 9, and ſo we have 92, which is leſs than 99; that is, 785 divided among 99 Perſons, each will have 7, and there will be 92 over, which is not one more for every Perſon. To this 92 I bring down the 4 from the *Dividend*, and then you have 924. Now you ſee the *nines* in 9 will go, but 99 in 92 will not; and therefore you muſt ſay, the *nines* in 92 will go 9 Times; but however we muſt try the Truth of it, by ſaying 9×99 is 891, which we ſee is leſs than 924, and therefore we may venture and ſay, 9×9 is 81 from 84 (4 being now to be called 84) 3 and carry 8; 9×9 is 81, and 8 I carry is 89 from 82 (2 being to be called 82) I cannot, but 89 from 92, and there remain 3, to which I bring down the 0, and we have 330; the *nines* in 33 will go 3 Times, and 33 over at laſt. And now reſt yourſelf again, and look about you, that you may, with the more Vigour, travel on with me.

Now, let us proceed on to the End of our intended Journey, where the young Traveller will forget the Fatigue he has met with in the Way. Let us divide 78540*l.* among 989 Sailors, who took the Prize which was worth a great deal more, but the reſt of its Value went to the Officers. Here, the Sum will ſtand thus, as before, 989) 78540. Now look at the *Di-viſor*; you ſee there are three Places, which are greater than the three laſt Places in the *Di-vidend*; therefore you will now uſe four Places of the *Di-vidend*, and ſay, the *nines* in 78 will go, you think, 8 Times; but try it, thus, upon your Slate, 9×989 is 8901, which you know cannot be taken from the four Figures 7854 of the *Di-vidend*, unleſs you can put your Hand into your Pocket and take out 9 any Thing, when you have only 7 there: It is impoſſible; and therefore you conclude inſtead of 8 you muſt try again with a leſs Figure, for the leſs the *Multiplier*, the leſs the *Product* is. Suppoſe 7 Times; then 7×989 is 6923, which, being leſs than the *Di-viſor*, you are encouraged to go on, ſaying, 7×9 is 63 from 64 (4 in the *Di-vidend* being now to be called 64) 1, and carry 6, 7×8 is 56, and 6 I carry is 62 from 65, (5 in the *Di-vidend* being now to be called 65) 3 and carry 6, 7×9 is 63, and 6 I carry is 69 from 68 I cannot, (8 in the *Di-vidend* being now to be called 68) but 69 from 78, that is 9: So that the Remainder is 931, to which bring down the 0 from the *Di-vidend*, and then we have 9310; and now ſay the *nines* in 9 will go, but the 98 in the *Di-viſor* is greater than 93, and therefore ſay, the *nines* in 93 will go 9 Times, 9×9 is 81 from 80 I cannot, (it would be ſtrange if you could! here 0 in the *Di-vidend* being now to be called 80) but 81 from 90, (10 being always to be added on theſe Occaſions) that is 9 and carry 9; 9×8 is 72, and 9 I carry is 81 from 81, (1 in the *Di-vidend* being to be called 81) 0 and carry 8; 9×9 is 81, and 8 I carry is 89 from 83 I cannot, (here 3 in the *Di-vidend* being to be called 83) but 89 from 93, that is 4. Hence then each Sailor will have 79*l.* and there will be 409*l.* over.

And if you are not quite tired down, let us divide the ſame Prize among 9895 Sailors. If this Buſineſs of dividing ſhould give you more Trouble than is agreeable to you, pray conſider the Advantages you may receive from it, and what Trouble you have in many Things which can bring you nothing but your Toil for your Pains. Now then let us proceed chearfully, as we are got to the laſt Stage of our Journey; 9895) 78540; here, you have four Places of Figures in the *Diviſor* ; ſay, the *nines* in 78 (as it will not go in 7) will go (try) 7 *Times* ; but now mind you will uſe all the Places in the *Dividend,* as there are four in the *Diviſor,* and only five in the *Dividend,* and as the *nines* will not go in 7. Now, then, 7×9895 is 69265, which, being leſs than the *Dividend,* may be right ; ſo then we proceed, ſaying 7×5 is 35 from 30 (here 0 in the *Dividend* is now to be called 30) I cannot, but 35 from 40 (10 being to be added on this Occaſion) 5 and carry 4 ; 7×9 is 63, and 4 I carry is 67 from 64 (here 4 in the *Dividend* is now to be called 40) I cannot, but 67 from 74 (here 10 being to be added) that is 7 and carry 7 ; 7×8 is 56, and 7 I carry is 63, from 65, (5 in the *Dividend* being now called 65) 2 and carry 6, 7×9 is 63 and 6 is 69 from 78 that is 9. And thus we find each Sailor will have 7 *l.* for his Share, and there will be 9275 *l.* over, which does not amount to one Pound over for every Man, tho' not much ſhort of it. So that the Anſwer is expreſſed 7 *l.* $\frac{9275}{9895}$. You know you may prove this by 7×9895+9275.

And, now, like two weary Travellers, that have gained an Aſcent, let us ſit down and view the Road we have paſſed over, that we may not be at a Loſs to know it again.

The following Obſervations will be of great Uſe : 1. If the firſt Figure in the *Diviſor* will not go, it will in the two left Figures of the *Dividend,* and then you uſe one more Figure in the *Dividend* than you have in the *Diviſor* : *Aſk the Queſtion,* by ſaying the *nines, eights, ſevens, &c.* in that Part of the *Dividend* you are to uſe. 2. Multiply all the *Diviſor* by the Figure you ſay will go, always taking the Product of each two ſingle Figures ſo multiplied from the reſpective Figure above it ; but when this is too little always add *ten* to it, that was called by the Name you give the Product of the two Figures multiplied together ; that is, if the Product is under ten, or between ten and twenty ; between twenty and thirty, *&c.* the Figure it is to be taken from muſt be conſidered as being under ten, or between ten and twenty ; between twenty and thirty, *&c.* 3. If the Remainder is greater than the Diviſor, this is a plain Demonſtration that the *Diviſor* would have gone oftener ; but if it is ſo much leſs, that, even when the next Figure is brought down, the *Diviſor* will not go, you muſt be ſure to put a Cypher in the *Quotient,* and bring down another Figure ; and then theſe Things begin again : 1. How often ; 2. Multiply and ſubtract at the ſame

Time; 3. Bring down; and then returns how often, or the twos, threes, fours, &c. in the *Dividend*.

If there are Cyphers in the *Diviſor* that have two or many ſignificant Figures, all the Cyphers are to be pointed off with a Comma, and the ſame Number of Places on the Right of the *Dividend*, as was mentioned before when we divided by the firſt Form. Thus, ſuppoſe we have this Sum to divide $\underline{98,00})785,40$; here are two Cyphers pointed off in the *Diviſor*, and two Places in the *Dividend*, ſo that we have $785 \div 98$; thus, the *nines* in 78 will go (try) 8 Times, then 9×98 is 882, which is too much, at it cannot be taken from 785; therefore I muſt have a leſs *Quotient* Figure that the Product may be leſs. Try 8 *Times*, then 8×98 is 784, which is under the Value of the *Dividend*; ſo then, I ſay, 8×8 is 64 from 65, that is 1 and carry 6; 8×9 is 72 and 6 I carry is 78 from 78, that is 0; and now the *Quotient* is 8 and 1 over; but becauſe I cut off two Figures, I muſt take away the Comma, and then the Anſwer is $8\frac{140}{9800}$.

It may be aſked, Why Youth are put upon a more difficult Way of dividing, when the old Method is eaſier for their Memories? I muſt beg Leave to anſwer, That the newer Form of dividing has its ſuperior Elegance, and alſo Advantages in ſaving much Time when Youth are uſed to it; even the old Way appears formidable enough to them, when they begin *Diviſion:* Beſides, the new Method, by ſubtracting as they multiply, obliges them to be careful, and fixes their Attention, while they have a Product on their Memory, till they diſcharge it by the Subtraction.

Now, by *Axiom* 6, dividing the Whole is the ſame as dividing its Parts; thus $\frac{125}{5} = 25$; $\frac{100+20+5}{5} = 20+4+1$; that is, the *fives* in 100 will go 20 *Times*, and the *fives* in 20 will go 4 *Times*, and the *fives* in 5 will go 1, and theſe *Quotients* added together make juſt 25. And $\frac{6370}{26} = 245$; and $\frac{5200+1040+130}{26} = 200+40+5$; that is, the *twenty-ſixes* in 5200 will go 200 *Times*, and the *twenty-ſixes* in 1040 will go 40 *Times*, and the *twenty-ſixes* in 130 will go 5 *Times*, and theſe *Quotients* added together make 245, which is the ſame with the whole 6370 divided by the ſame *Diviſor* 26. But this analyſing the Whole into Parts will be more diſtinctly ſeen as follows, which the Youth may alſo take down on his Slate, and may ſtand thus, for his greater Convenience, $26)6370(200+40+5$, ſaying the *twenty-ſixes* in 63 will go 2, but then this is 200, becauſe there muſt be ſuch a *Quotient* as, when multiplied by the *Diviſor*, will be leſs than the *Dividend*, and $26 \times 200 = 5200$; but had it been 3 Times (300) inſtead of 2 (200) the *Product* would be greater than the *Dividend*, and therefore 200 is the right Number. Now, $6370 - 5200$ is 1170, the *Remainder*, which we ſee can be divided by 26. Then the *twenty-ſixes* in 117 will go 4 *Times*, but this is 40, becauſe 26×40 is 1040, which is the neareſt Number, and $1170 - 1040$ is 130, the *Remainder*, which we ſee can alſo be divided by 26, and the *twenty-ſixes* in 130 will go 5 *Times*, becauſe 26×5 is 130. And thus the Learner ſees that, by knowing the Value of each Figure in the *Quotient*, he may ſubtract the whole Product from the whole *Dividend*, and the whole of each *Remainder* may be divided till the *Diviſor* will go no more. By this laſt Form of dividing the whole *Remainders*, the Learner will be enabled to do the like, which will greatly improve him in the Nature of analyſing a *Dividend*, and which will hereafter be of great Service to him on other Occaſions, if he intends to make himſelf Maſter of Arithmetic.

Let the *Diviſors* 345 to 390, increaſing 5 each Time, and let the *Dividends* to theſe *Diviſors* reſpectively be 78540, increaſing 20 each Time, but each *Dividend* ſo altered as not to be like any one in the School; then let each Sum be analyſed, as in the laſt Method of dividing the whole *Remainders*, till the *Diviſor* in each Sum will go no more.

And let me requeſt the Learner will go through the following Sums, in order to make himſelf *Maſter of Diviſion*, as the *analyſing* the *Dividends* laſt directed will give him an excellent Notion of the Value of the Places of Figures. It is but too frequently ſeen, that young People are always blundering quite through Arithmetic, and ſeldom arrive at what deſerves the Name of a Proficient in Accounts, chiefly owing to their not taking proper Care and Pains to be ready at dividing any Number that is propoſed to them. Now, if the Learner has a Deſire to conquer this impatient, heedleſs Humour, which is the Cauſe of Plague and Vexation to him too far through Life, he muſt reſolve to do many Sums by himſelf.

Therefore let the *Diviſors*, to ten, twenty, or thirty Sums, be 785476, but increaſing 5 each Time, and let the *Dividends* to theſe be 987687959000, but decreaſing by 100 from the firſt five Sums, and 95 afterwards; and above all to take Care, as he values his Honour, that his *Dividends* are ſo altered by him, that no two Cyphering Books in the School ſhall have the ſame Sums in them.

DIVISON *of* MONEY.

THIS will frequently be of great Uſe in the leſſer Concerns of the World, to Gentlemen and Ladies particularly, as there is a Kind of Elegance, Neatneſs, and Expedition in the Manner of Operation, when theſe marginal Tables are got off, or conſulted occaſionally. Thus, for Inſtance, in ſome of the Uſes of this ſhort Method of working theſe Sums; ſuppoſe a Fortune of 17,569*l.* 17*s.* 7¾*d.* is to be equally divided between *two* Children; here 2 is the *Diviſor*, and the Fortune is the *Dividend*; then ſay, the Half of 17 is 8 and carry 1, which is 10 (meaning 1000) and 5 is 15, the Half of 15 is 7 and carry 1, which is 10 (meaning 100) and 6 is 16, the Half of 16 is 8, the Half of 9 is 4 and carry 1*l.* which is 20*s.*+10*s.* in the 17 is 30*s.* + 7 is 37, the Half of 3 is 1 and carry 1, which is 10*s.* and 7 is 17*s.* the Half of 17*s.* is 8*s.* and carry 1*s.* which 12*d.* and 7*d.* is 19*d.* the Half of 19*d.* is 9*d.* and carry 1*d.* which is 4 Farthings and 3 *qrs.* is 7 *qrs.* the Half of 7 *qrs.* is ¾ and one Farthing over.

Now let the ſame Fortune be equally divided between *three* Children, then 3 is the *Diviſor* and the Fortune is the *Dividend*; ſay, the *threes* in 17 will go 5 *Times*, the *threes* in 25 (becauſe 2 over, or 2000) 8 *Times*, the *threes* in 16 (becauſe 1 over, or 100) 5 *Times*, the *threes* in 19 (becauſe 1 over, or 10) 6 *Times*, and 1*l.* over, which is 20*s.* and 17*s.* is 37*s.* the *threes* in 37*s.* will go 12 *Times*, and 1*s.* over, which is 12*d.* and 7*d.* is 19*d.* the *threes* in 19 will go 6 *Times*, and 1*d.* over, which is 4 *qrs.* and 3 is 7, the *threes* in 7 will go ½ and 1 *qr.* over.

We will now ſhew the Uſe of the three marginal Tables, in ſuppoſing 10823*l.* 19*s.* 11¼*d.* to be equally divided among *twelve* Relations, as they have all

£.		s.
2	is	40
3	—	60
4	—	80
5	—	100
6	—	120
7	—	140
8	—	160
9	—	180
10	—	200
11	—	220

s.		d.
2	is	24
3	—	36
4	—	48
5	—	60
6	—	72
7	—	84
8	—	96
9	—	108
10	—	120
11	—	132

d.		qrs.
11	—	44
10	—	40
9	—	36
8	—	32
7	—	28
6	—	24
5	—	20
4	—	16
3	—	12
2	—	8

behaved equally well. Here then the *Diviſor* is 12, and the Fortune left them is the *Dividend*; ſay, the *twelves* in 108 (for it will not go in 10) 9 *Times*, that is 900; the *twelves* in 2 will go 0 *Times*, but the *twelves* in 23 will go 1, and 11*l.* over; now this 11*l.* over by the Table is 220*s.* and 19*s.* is 239*s.* the *twelves* in 239 will (divide it on your Slate) go 19 *Times*, and 11*s.* over; now this 11*s.* over, by the Table, is 132*d.* and 11*d.* is 143*d.* the *twelves* in 143 will (divide it on your Slate) go 11 *Times*, and 11*d.* over; now this 11*d.* over, by the Table, is 44 *qrs.* and 3 is 47, the *twelves* in 47 will go 3 *qrs.* and 11 *qrs.* over, which is very near being ¼ more for each Relation. Were this Sum to be done in the common Way, we ſhould be obliged to bring the whole Fortune into Farthings, by multiplying by 20, 12, and 4, and divide by the Number of Relations to find how many Farthings each muſt have; and becauſe we muſt not reſt here, we ſhould be obliged to divide back again by 4, 12, and 20, to bring the Farthings into Pounds.

If you would be dextrous in dividing ſuch Sums as the laſt, which you may be, if you are very perfect in your Tables, ſay thus, the *twelves* in 108, 9; the *twelves* in 2, 0; the *twelves* in 23, 1; the *twelves* in 239, 19; the *twelves* in 143, 11; the *twelves* in 47, ¾. Now, in order to obtain this Dexterity, I muſt requeſt the Learner will form thirty-three Sums of the *Dividend* 97865876*l.* 19*s.* 11¾*d.* but to vary the *Pounds* only, in every Sum, in ſuch a Manner, that they ſhall not be any Thing like the *Dividends* to be found in any Cyphering Book in the School; and let the *Diviſors* be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, in all Eleven; ſo that when the ſame *Diviſor* returns, as there will be *three Courſes*, it will have a different *Dividend* to what it had before in the *firſt Courſe* of the *Diviſors*.

When theſe are worked, the worthy Youth is deſired to make Uſe of the *thirty-three* laſt *Dividends*, and the following *Diviſors*, 20, 30, 40, 50, 60, 70, 80, 90, and divide the above *thirty-three* Dividends by them in as many Courſes, by which each *Diviſor* will have *four Courſes*, and let 90 have *five Courſes*, by which he will have juſt *thirty-three* Sums. But leſt he ſhould meet with too much Difficulty, to overcome by himſelf, in dividing them, *at once*, by 20, 30, &c. and leſt his Maſter ſhould not be then at Leiſure to help him, I ſhall do two or three for the Youth, and then I am ſure he will trouble Nobody to do the reſt; but only to ſhew his Maſter what Diligence and Reſolution will do. In order to aſſiſt the

Youth, I ſhall take 10823*l.* 19*s.* 11$\frac{3}{4}$*d.* for a *Dividend*, and 20, 50, and 90, for the *Diviſors*; and then leave the Learner to form the above *thirty-three* Sums from his own *Dividends*.

Now, then, in 2,0)1082,3*l.* 19*s.* 11$\frac{3}{4}$*d.* you ſee the Cypher in the Diviſor, and the Figure 3 in the Dividend, are cut off; therefore ſay, the Half of 10 is 5, the Half of 8 is 4, the Half of 2 is 1; theſe, 541, as formerly, are to be placed under the *Dividend*, and not under the *Diviſor*. Now conſider there are 3*l.* over, and by the Table they make 60*s.* and 19 is 79*s.* now point off the 9, and ſay, the Half of 7 is 3*s.* and 1 over, which is 10 and 9 is 19*s.* over, which by the Table is 228*d.* (10*s.* is 120*d.* and 9*s.* is 108, and 120+108=228) and 11*d.* is 239*d.* now point off the 9, and ſay, the Half of 23*d.* is 11*d.* and 1 over, which is 10, and 9 is 19*d.* which by the Table is 76 *qrs.* (10*d.* is 40 *qrs.* and 9 is 36, and both 76) and 3 is 79; now point off the 9, and ſay, the Half of 7 is 3 *qrs.* and 1 over, which is 10 *qrs.* and 9 is 19 *qrs.* remaining over, which is very near $\frac{1}{4}$ more, as the Diviſor has been 20 all the Way. This Sum may be done by ſaying 20=5×4, for two Diviſors; and this to ſome young People may be as agreeable as doing it all at once, tho' not to thoſe who prefer the ſhorteſt Ways. In this Method of ſplitting the Diviſor into two Parts; for Inſtance, 20 by firſt 5 and then by 4, they are to be done in the ſame Manner, as when we divided the Fortune among three Children; that is, firſt dividing by 5, and then conſidering that Quotient as a Fortune, and dividing it by 4: So likewiſe, the Diviſor being 30, you may ſplit it into 6×5=30; 40, into 5×8; 50, into 10×5; 60, into 12×5; 70, into 10×7; 80, into 10×8; 90, into 10×9; and, in general, any two Numbers within 144, when multiplied together, making up the Diviſor. But, to proceed in doing it at once by 50 and 90.

That is, 5,0)1082,3*l.* 19*s.* 11$\frac{3}{4}$*d.* here the Cypher in the *Diviſor*, and the Figure 3 in the firſt Place of the *Dividend*, are cut off, as before: Then ſay, the *fives* in 10, *twice*, the *fives* in 8, *once*, and 3 over, which is 30, (meaning 300) and 2 is 32; the *fives* in 32 will go 6 *Times*, and 2 over, which is 20, and 3 is 23*l.* which is the Remainder, as it is leſs than the Diviſor 50. Now conſider, 10*l.* is 200*s.* and then 20*l.* is 400*s.* by the Table, and 3*l.* is 60*s.* ſo that 23*l.* is 460*s.* and 19*s.* is 479*s.* point off the 9, then the *fives* in 47 will go 9 *Times*, and 2 over, which is 20*s.* and 9 is 29*s.* now, by the Table, 10*s.* is 120*d.* and 20*s.* is 240*d.*

240*d.* and 9*s.* is 108*d.* and 11*d.* is 359*d.* then point off the 9, and ſay the *fives* in 35 will go 7 *Times*, and 9*d.* over, which is 36 *qrs.* and 3 is 39, in which the Diviſor will not go, ſo that 39 *qrs.* remain over.

Now, to do the Sum by 90. Here, when the Figures are pointed off as before, ſay, the *nines* in 10 will go *once*, and 1 over, which is 10 (meaning 1000) and 8 is 18, the *nines* in 18 will go *twice*, the *nines* in 2 will go 0 *Times*, and 2 over, which is 20 and 3 is 23*l.* Now conſider, 10*l.* is 200*s.* by the Table, and 20*l.* is, of Courſe, 400*s.* and 3*l.* is 60*s.* then 23*l.* is 460*s.* and 19*s.* is 479; then pointing off the 9, ſay, the *nines* in 47 will go 5 *Times*, and 2 over, which is 20 and 9 is 29*s.* over; now, by the Table, 10*s.* is 120*d.* and, of Courſe, 20*s.* is 240 and 9*s.* is 108*d.* and 11*d.* is 359*d.* then pointing off the 9, ſay, the *nines* in 35 will go 3 *Times*, and 89*d.* over, which multiplied by 4, and taking in the odd 3 *qrs.* is 359 *qrs.* then pointing off the 9, the *nines* in 35 will go 3 *qrs.* and there remain 89 Farthings over undividable.

PROPORTION: *Or, The* GOLDEN RULE,

Commonly called

The RULE *of* THREE,

Applied to INTEREST, *Simple and Compound, Multiplication and Diviſion, and to ſome Concerns of the Gentry.*

BEING willing to throw into one connected Syſtem thoſe Parts of Arithmetic, more immediately of Uſe to thoſe whoſe Stations in Life may not require a more general Knowledge in all the Branches of Accounts, I have taken the Liberty, in this Place, of inſtructing the Learner in the Method of computing *Simple* and *Compound Intereſt*, as it makes ſo conſiderable a Part of the Concerns of the affluent and genteel Part of the World: And as ſuch I ſhall endeavour to make the Learner Maſter of the Subject, by explaining the Foundation of this agreeable Part of Arithmetic, and ſhall introduce ſome other Concerns, more immediately relating to the Gentry.

When two Quotients are the ſame, the Diviſors and Dividends are in Proportion; that is, if one *Diviſor* is 2, 3, 4, &c. Times leſs than its own *Dividend*, the other *Diviſor* will be 2, 3, 4, &c. Times leſs than its own *Dividend*. Let $\frac{6}{2}=3$; here the Diviſor 2 is three Times leſs than the Dividend 6, and the Quotient is 3: Let the other Dividend and Diviſor be $\frac{12}{4} = 3$; here alſo the Diviſor 4 is three Times leſs than the Dividend 12, and the Quotient is 3 alſo; then it will be as *Diviſor* 2 is to its *Dividend* 6, ſo is *Diviſor* 4 to its *Dividend* 12; or more cloſely expreſſed thus, as 2 : 6 : : 4 : 12; then, you ſee, as 6 is three Times more than 2, ſo 12 is three Times more than 4. Now, to explain this *Proportion* by an eaſy Example; ſuppoſe 2 Yards of Linen coſt you 6*s.* and you afterwards order 4 Yards more of the ſame Price, you know the Price of the 4 Yards muſt be three Times more than thoſe 4 Yards, becauſe you agreed to give three Times more Shillings for the firſt two Yards. Now conſider what a noble Property or Nature there is in theſe four Numbers, which have equal Quotients; and that is, if you multiply the firſt and fourth together, you will find that Product the ſame as when you multiply the ſecond and third together; that is, $2 \times 12 = 4 \times 6$.

But as the *Rule of Three* always gives *three Numbers*, and expects you to be ſo ingenious as to find out the *fourth*, conſider now which Way you would do it; in theſe Numbers, as 2 : 6 : : 4 : 12; by the Rule of Three you will and muſt have three Numbers, ſuppoſe as 2 : 6 : : 4 given; but 12 is to be found, that is, ſuch a Number that ſhall as much exceed 4 as 6 does 2. What do you think of multiplying the *ſecond and third together and dividing by the firſt?* You will then find the fourth proportional Number you want; for 6×4 is 24, and 24 divided by 2 is 12, the very Number required; or in 2 : 4 : : 6 : 12, for 4×6 is 24, and this divided by 2 is 12, the Number you would want, if you had only 2 : 4 : : 6 given: Or take the Theory of finding the fourth Number thus, by Diviſion, 24 $(= 6 \times 4) = 24$ $(= 2 \times 12)$; then divide either 24 by 2, which only undoes what Multiplication does, and you have $\frac{6 \times 4}{2} = 12$.

Let us ſuppoſe 2 Yards coſt 6 Shillings, and that 4 Yards, at the ſame Price, will coſt 12 Shillings, to ſtand thus, 2 *yds.*——6*s.*——4 *yds.*——12*s.* Here the 1ſt and 3d are both of a Name *(yds)*, and the 2d and 4th are both of a Name *(s.)*, and this muſt ever be the Order; that is, the 4th Number required muſt be of the ſame Name with the 2d Number. Now, ſince equal Quotients ſhew the four Numbers are in Proportion, let me obtain the Favour of you to form thirty or forty Sums, different from any in the School, and throw each of them into Proportion, then find the Product of the 1ſt and 4th, and of the 2d and 3d, and by them the fourth Number wanted in the *Rule of Three*, and the Conſequence will be, that you will find nothing in Arithmetic, nor indeed much higher, to which the Rule of Three can be applied, too hard for you. The Way to form theſe Sums is to fix on two Numbers, and then conſider how much more or leſs one is than the other; thus 3, 12; then $\frac{12}{3}=4$, that is, 12, the ſecond Number, is four Times more than 3, or 12, 3; then $\frac{12}{3}=4$; that is, 3, the ſecond Number, is four Times leſs than 12; in one the Diviſor is firſt, in the other it ſtands ſecond, and preſerves the Proportion; for if the firſt and third are both either *Dividends* or *Diviſors*, that is, both of a Name, the Proportion is not deſtroyed. Now for a third Number, any will do, and then, when the Order is 3, 12, &c. if you multiply by the *Quotient*, you will have the *fourth*; thus, ſuppoſe the *third* is 5, then, multiplying 5 by 4, the *Quotient* of $\frac{12}{3}$, you have 20 the fourth Number; and now you have 3 : 12 : : 5 : 20, and 12 × 5 60=3×20=60; and if you divide 60 by 3, the firſt Number, you have 20 the fourth: Or if the Order is to be 12 : 3 : : 20 : 5; here the third Number 20 muſt be *divided* by the Quotient of the firſt and ſecond, which is juſt the contrary of 3 : 12 : : 5 : 20, when the Quotient was *multiplied* by the third Number ſo taken at Pleaſure.

Applied to Simple and Compound INTEREST.

THIS Rule, in Arithmetic, ſhews the Youth how much muſt be allowed for lending out any Sum for a Year, or more or leſs Time, but this Allowance, called Intereſt, is uſually paid either yearly or half yearly, and is eſtimated always as 100*l.* 3, 4, 5, that is, at 3*l.* 4*l.* 5*l.* for every 100*l.*

Rule to be got by Heart.

In the Rule of Three you have three Numbers, which ſo place, that the firſt and third may be both of one Name, and then the odd Number ſtands in the Middle, and the Number belonging to it muſt ſtand firſt; then multiply the ſecond and third Numbers together, and divide by the firſt, and this Quotient, or fourth Number, will be the Anſwer.

Suppoſe you lend a Perſon 267l. *for one Year, at* 5 *per Cent.* that is, to have 5*l.* for the Lent of every Hundred Pounds. Here you have only two Numbers given in the Figures, but *per Cent.* (100*l.*) is always one, which make three Numbers. Now, for what you have learned concerning Proportion: Here you have, as 100*l.* : 5*l.* : : 267*l.* to ſome Number to be found, and which ſhall bear the ſame Proportion to the Money lent, as 5*l.* does to 100*l.* Therefore, according to the Rule, multiply the ſecond and third Numbers together, and divide by the firſt, and what remains, multiply it by 20, and divide by the firſt Number for Shillings.

Here I ſhall ſhew the Theory of multiplying the Remainder by 20, which I do not recollect ever to have ſeen done before, eſpecially without the Aſſiſtance of the theoretical Part of Fractions. The Remainder in the laſt Sum is 35 Parts of one Pound divided into 100 Parts; here then, one Pound being 20 Shillings, we have this Proportion, 100 *parts* : 20*s.* : : 35 *parts* : 7*s.* that they are truly Proportionals, we diſcover by the Quotients being equal, becauſe $\frac{100}{20}=5$, and $\frac{35}{7}=5$, and by $100\times7=35\times20$, being the two Extremes; ſo that as the ſecond Number is 5 Times leſs than the firſt, and as the fourth is 5 Times leſs than the third, *the Rule is, multiply the Remainder by* 20, *and divide by* 100.

Again, *ſuppoſe I lend a Perſon* 1869*l.* 17*s.* 6*d. for one Year at* 5 *per Cent. What is the Intereſt thereof?* The Principal being multiplied by the Rule, and divided by 100, will ſhew the fourth Number in Pounds, and there will be a Remainder of 49 Parts of one Pound divided into 100 Parts. Here alſo we have 20, the Shillings in one Pound, by which three Numbers we find the fourth; that is 100 *parts* : 20*s.* : : 49 *parts* : 9*s.* and a Remainder of 87. Now this Remainder would be 80, but becauſe the 7*s.* muſt be taken in when we multiply by 20, (ſaying 0 is 0, but 7 is 7) it is 87 Parts of one Shilling divided into 100 Parts; here then we having one Shilling, making 12 Pence, we alſo have three Numbers to find a fourth; that is 100 *parts* : 12*d.* : : 87 *parts* : 10*d.* and a Remainder of 50, becauſe we take in the 6*d.* when we multiply 87 by 12, ſaying 12 Times 7 is 84 and 6 is 90, 0 and carry 9, *&c.* This Remainder is 50 Parts of one Penny, making 4 Farthings, divided into 100 Parts, and therefore we have 100 *parts* : 4 *qrs.* : : 50 *parts* : 2 *qrs.* or ½, and if there had been any Farthings, they muſt have been taken in when you multiplied the 50 by 4. Now then, you may be Maſter of the Reaſon of multiplying the Remainders by 20, 12, and 4, and of taking in the Shillings, (which is the ſame as adding them afterwards) Pence, and Farthings.

And now I muſt requeſt the Learner will form many Sums of this laſt, and make them differ each Time from any Cyphering Book in the School.

But let ſeveral of the many Examples be propoſed in Shillings, Pence, and Farthings, and then let all be multiplied by the given Rate of Intereſt, 5 *per Cent. per Annum*, as in the laſt, according to what was taught under the Head of *Multiplication of Money*, but be ſure you never fail to form the Proportions, and to give the Reaſon of multiplying by 20, 12, and 4. By this Means no Principal, put out to Intereſt for one Year, can be propoſed, but you will readily be enabled to give the true Reaſons of what you are about, and an exact Anſwer, even in Farthings, and in the Remainder or Parts of a Farthing. If the Intereſt ſhould be 3 or 4 *per Cent. per Annum*, the ſame Method will do; but if it ſhould be 3 ½, 3 ¾, 4 ¼, 4 ½, or 4 ¾ *per Cent per Annum*, there will be a Variation in the Performance.

Thus, ſuppoſe a National Debt at 129,786,521*l.* 19*s.* 6¾*d. at* 3 ¾ *per Cent. per Annum*; that is, 3*l.* 15*s.* 0*d.* for every 100*l.* for one Year: *Required the annual Intereſt?* Here, firſt multiply all by the Rate; but then conſider, to multiply by 1 *per Cent.* the Principal would not be altered, but to multiply by 2 *per Cent.* the Principal would be twice as much, and therefore to multiply by ½ *per Cent.* the Principal will be Half as much; and to make a Number Half as much is to divide it by 2; and to multiply by ¼ *per Cent.* the Principal will be a Quarter as much, but to make a Number a Quarter as much, is to divide it by 4, or to take the Half of what it comes to by multiplying by ½. Therefore, for the ¾ *per Cent.* take the Half of the Principal for ½ *per Cent.* or 10*s.* and then for ¼ or 5*s. per Cent.* take the Half of what it comes to at ½ *per Cent.* When this is done, add together what it comes to by multipling by 3, and by dividing for the ¾; then divide by the firſt Number, 100, as you have now multiplied the ſecond and third Numbers together, and you have the fourth Number in Pounds; then multiply the Remainders by 20, 12, and 4, to find the Shillings, Pence, and Farthings; remembering to take in the Shillings, Pence, and Farthings, in the Product of the ſecond and third Numbers, as you multiply the Remainders. You know dividing by 100 is only pointing off the two firſt Figures on the Right Hand in

Pounds, Shillings, Pence, and Farthings. Thus, you have a ſhort Method, and yet you keep the Theory, that is, the Proportion of your Numbers, in View, by which Means you will always have a Light with you, inſtead of groping in the Dark.

And now I again beg, as I have undertaken to make you perfect as far as you learn, that you will form a great many Sums of this National Debt, for which the Public Creditors receive Intereſt; and let the Rates always be an odd ¼ *per Cent. per Annum* from 3 to 5 *per Cent.* and let the Principal be alſo made quite different from your School-Fellows Sums; and conſider what a Delight you will be to your Maſter, to your Parents, and to yourſelf, if you can give the Proportions, that is, the Reaſon for what you do, and are never at a Loſs to work ſuch Sort of Sums, which you never will, if you are ſo diligent as to form a great many Queſtions, and their Proportions, from the given National Debt.

Now, if the Time be ¼, ½, ¾ of a Year, under or over a Year, there can be no Difficulty in this; for ſurely whatever the Intereſt comes to in one Year, for ¼ of a Year it is the fourth Part of the whole, or ½ of the whole for ½ a Year; and if the Time is ¾ of a Year, the Intereſt is the Sum of what ½ a Year and ¼ of a Year's Intereſt come to: So likewiſe, if the Intereſt is to be computed for 2, 3, 4, *&c.* Years and Quarters, it is only multiplying the Intereſt of one Year by 2, 3, 4, *&c.* Years, and taking the Parts for the Quarters accordingly; that is, dividing one Year's Intereſt by 2 for ½ a Year, by 4 for ¼ of a Year, and ſo on as before. And here again be pleaſed to form ſeveral Sums from the Principal 3763*l.* 19*s.* 11¼*d.* at all the above Rates and their Quarters *per Cent.* for all Times from ¼ of a Year to 7 or 14 Years, varying ¼ of a Year in each Sum, and let the Principal be ſcrupulouſly altered, that not one Sum ſhall knowingly agree with any one Sum in the School.——So much for Years and Quarters.

We come now to the Method of computing Intereſt by Days, which is abſolutely neceſſary to be done, as in great Concerns this is always practiſed; however, if you were never to have any Occaſion for this Sort of Calculation, it will be a Satisfaction that you can do it. And here I muſt beg Leave to obſerve, That it ſeems to be an Error in Education, to propoſe different Rules to Youth, when ſuch Methods make them entirely loſe Sight of the Reaſon of the Operations: For to what Purpoſe is it that they are taught the

Rationale of their Rules as they go on, if the Youth's Attention is diverted from it by Principles which lie not within his Reach?

What is the Intereſt of 50765*l.* 17*s.* $6\frac{3}{4}$*d.* (which call *p*) *for* 25 *Days*, at $3\frac{2}{3}$ *per Cent. per Annum?* In all ſuch Queſtions, where *Days* are concerned, firſt find the *Intereſt* (which call *r*) of *p* for *one Year*; then you have this plain Proportion or *Theory*, as 365 *Days* is to *r*, (which reduce into Shillings, Pence, and qrs. if neceſſary) ſo is 25 *Days* to a fourth Number, which will be the *Intereſt required*; becauſe the ſecond Number is Intereſt, and the firſt and third Days. And here again, the diligent Youth will be ſo much a Friend to himſelf, as to form ten, twenty, or thirty Sums of this Kind, of different Days, Rates, and Principals, from the above, *differing* from the *Principals* of his School-Fellows.

Applied to MULTIPLICATION.

SUppoſe I buy 8 Yards of Linen at 3 Shillings a Yard, you know you were taught to find out how much you are to pay the Tradeſman, by conſidering 2 Yards would be twice 3*s.* and 8 Yards would be eight Times 3*s.* and to make 3*s.* eight Times more, is to multiply the Price by the Yards. But the true Theory lies in Proportion, that is, 1 *yd* : 3*s.* : : 8 *yds* : 24*s.* here you ſee that thoſe practical Queſtions in *Multiplication*, before taught you, are only ſo many Sums in *the Rule of Three*; for 24 is 3 Times more than 8, and 3, the ſecond Number, is 3 Times more than 1, and the firſt and third are both of a Name (*yds*), and ſo are the ſecond and fourth (*Money*), and $\frac{3}{1}=3$, and $\frac{24}{8}=3$, and $1\times24=3\times8$, and therefore the Numbers are in Proportion. Here the Youth is deſired, in order to fix this Theory in his Mind, to form many ſuch Proportions, by making the Figure 1 the firſt, and any two Numbers he pleaſes the ſecond and third, and theſe, multiplied together, will give the fourth, to form the Proportion.

Applied to DIVISION.

SO if I buy 3 *Yards of Linen for* 1l. *What muſt I give for* 24 *Yards at that Rate?* By the practical Queſtions in Diviſion you were taught to find a Quotient or Anſwer, but now you ſee the Quotient was nothing leſs than the fourth

Number, from three Numbers in Proportion to find the fourth. For 3 *yds* : 1 *l.* : : 24 *yds* : 8 *l.* here the ſecond is three Times leſs than the firſt, and the fourth three Times leſs than the third; alſo the firſt is three Times more than the ſecond, and the third three Times more than the fourth, and $3\times 8=1\times 24$, likewiſe $\times\frac{3}{1}=3$, and $\frac{24}{8}=3$; and therefore the Numbers are in Proportion, and both Diviſion and Multiplication are nothing leſs than a ſhort Method of performing *the Rule of Three*. And here again the Learner will do well to form many ſuch Sums, by making any Number above 1 the firſt, and 1 always the ſecond, and then any Number he pleaſes the third, and this, divided by the firſt, will give the fourth, to form the Proportion.

Applied to REDUCTION *of Pounds, Shillings, and Pence.*

PRoportion gives the Theory alſo of this Rule, called *Reduction*; for Inſtance, if you want to reduce 245 *l.* into Shillings, Pence, and Farthings, we have, as 1 *l.* : 20 *s.* : : 245 *l.* : 4900 *s.* Here the firſt and third are both of one Name, and ſo are the ſecond and fourth, and as the ſecond is 20 Times more than the firſt, ſo is the fourth 20 Times more than the ſecond, and as $1\times 4900=245\times 20$, therefore the Numbers are in Proportion, and the Rule in *Reduction* is eſtabliſhed, which orders you to multiply Pounds by 20 to bring them into Shillings. Indeed you muſt needs ſee the Reaſon by common Multiplication; for as there muſt be 20 Times more Shillings than Pounds; ſo, to make them 20 Times more, you muſt multiply by 20. To bring Shillings into Pence, we have, as 1 *s.* : 12 *d.* : : 4900 *s.* : 58800 *d.* and therefore the Rule is, multiply the Shillings by 12 to bring them into Pence, not only becauſe this Proportion tells you that 4900 *s.* × 12 *d.* = 58800 *d.* but becauſe you muſt have 12 Times more Pence than Shillings: So to bring Pence into Farthings, we have, as 1 *d.* : 4 *qrs.* : : 58800 *d.* : 235200 *qrs.* that is, the Rule ſays multiply the Pence by 4 to bring them into Farthings, for the Reaſon before given.

And to bring all theſe back again into Pounds, we have, as 4 *qrs.* : 1 *d.* : : 235200 *qrs.* : 58800 *d.* and as 12 *d.* : 1 *s.* : : 58800 *d.* : 4900 *s.* and as 20 *s.* : 1 *l.* : : 4900 *s.* : 245 *l.* In all theſe you ſee the Proportions are preſerved; and that if you multiply the ſecond and third Numbers together, as *the Rule of Three* directs, and divide by the firſt, you have the Anſwer: And from the whole you conclude, that to bring Farthings into Pounds, you divide by 4, 12, and 20, becauſe, as another Reaſon, you muſt have leſs Pence, Shillings, and Pounds than Farthings; and to make a Number 4, 12, or 20 Times leſs, you muſt divide by theſe Numbers.

I muſt again requeſt, that the Learner will form many Sums of 3859*l.* 19*s.* 11¾*d.* but to alter the Pounds, and bring all into Farthings, taking in the 19*s.* when you multiply by 20, the 11*d.* when you multiply by 12, the ¾ when you multiply by 4; and when this is done, to divide all back again by 4, 12, and 20; but always to alter the Farthings, that they may not agree with any Boy's Book.

Applied to ſome Concerns of the Gentry.

ADMIT *a Gentleman's Eſtate is* 1760*l.* 19*s.* 6*d.* *a Year,* (which call *e*) *What is that a Day, or a Week, or a Month, or a Quarter?* The Proportions are as 365 Days is to *e*, ſo is 1 Day to the Income thereof; here the firſt and third Numbers are both of one Name *(Days)*, the ſecond the Eſtate, which reduce into Pence, and then find the fourth, by the Rule, by only dividing the Eſtate by 365: So as 52 Weeks is to *e*, ſo is 1 Week to the Income thereof; and when the ſecond Number *e* is reduced, and multiplied by the third, (1 which does not multiply, and is only dividing the Eſtate by 52) the *fourth* is found in the ſame *Name* you have the *ſecond*. Laſtly, as four Quarters of a Year is to *e*, ſo is 1 Quarter to the Income thereof; that is, you need only divide *e* by 4, according to the Method taught in the Diviſion of Pounds, Shillings, and Pence, to find the fourth Number required. Now, in order to be ready in theſe Calculations, let the above Eſtate be branched out into many Sums, under the Variations often mentioned.

The Owner of the laſt Eſtate appropriates 517*l.* 3*s.* 9*d.* *a Year for the younger Children's Fortunes, What has he then coming in a Day, a Week, a Month, a Quarter?* Here is no other Difference from the laſt, than ſubtracting the Money appropriated from the Eſtate, and proceeding as before. But at the End of ten Years, after thus ſaving and putting out the 517*l.* 3*s.* 9*d.* to Intereſt at 4 *per Cent. per Annum*, it is required to know how much the Gentleman will have ſaved for his three younger Children's Fortunes, and how much there will be then for each? Now, we are to ſuppoſe that the firſt Year's Saving is put out to Intereſt, and at the End of the Year the Intereſt thereof is added to the Saving, and conſidered in the ſecond Year as a Principal; and ſo on, for every Year after, adding the Intereſt to the Principal, and for every one of the other Year's Saving. You muſt ſee to how much amounts the firſt Year's Saving in 10 Years, at Intereſt upon Intereſt, or *Compound Intereſt*; how much the ſecond amounts to in 9 Years; how much the third amounts to in 8 Years; and ſo on, till the laſt Year, when the tenth Saving will have only one Year's Intereſt due upon it: And when you have found the Amount of each Saving, for its own Number of Years, your own Judgment will tell you, that theſe Amounts muſt be all added together, and then divided by 3, the Number of Children; and you will ſee what a *handſome, nay great Fortune* each will have, and what *Encouragement* there is for Parents to *appropriate* even a much leſs Sum out of their Fortunes, when their *Children deſerve* this Care. I muſt here requeſt to be indulged in my conſtant Humour of having the Saving, 517*l.* 3*s.* 9*d.* ſo altered, as not to be like the Saving in any other Youth's Cyphering Book. If my diligent and ingenious Pupil ſhould attend me to the End of this Treatiſe, he will meet with a much ſhorter Method of anſwering the laſt, and all ſuch *extremely uſeful* Queſtions; but it cannot, conſiſtently, be introduced here; yet you muſt take Care not to flatter yourſelf, that the Multiplicity of the Operations will deter your Maſter from examining them, as he, with very little Trouble to himſelf, by that ſhorter Method, will ſoon detect you, if you give up your Honour.

A young Gentleman, ten Years of Age, is left to the Care of Guardians with a clear Fortune, when the Steward's Salary, &c. *are deducted, of* 1760*l.* 19*s.* 6*d.* *a Year*; *but, by his Father's Will,* 100*l.* *a Year is to be allowed for the firſt ſix Years, and* 200*l.* *a Year till he is of Age, to bring him up, What ready Money will he have when he comes at Age?* This Queſtion is likewiſe to be altered, meaning the Principal 1760*l.* 19*s.* 6*d.* at firſt going off, that the Youth may not give into that *odious mean-ſpirited* Practice of *ſtealing* Anſwers from other Boy's Books: And, ſurely, it is very well worth the Pains (when he will have ſuch a great Fortune, what with his clear Eſtate, and what with the Amount of all the Savings) to learn, not only how to do it, but actually to go through all the Operations by himſelf, as the Theory, or Reaſon, lies within his own Reach. From what has been ſaid of the noble Truths of unerring Proportion, my Sentiments are, If he is unwilling to undertake the Taſk, which is quite entertaining, and ſo productive of Knowledge, he is not worthy of the Fortune, and that it ſhall be given to thoſe that are Maſters of more Diligence.

A Gentleman bought 5800 *of* 3 *per Cent. Annuities at* 75, *What muſt he pay for this Property in the Funds?* The Learner muſt be told that theſe Annuities, or Intereſt at 3 *per Cent. per Annum*, which is made of Money lent to the Government, may be bought or ſold; that is, the Perſons throughout the Nation who at firſt lent the Money upon the Terms agreed on, may ſell their Debts to others, at a lower or higher Price than was originally allowed to the Perſons who firſt lent the Money. The Proportions for finding the Value of 5800*l.* Stock, as it is uſually called, though more properly a public Debt, are as follow: As 100 : 75 : : 5800; that is, as 100*l.* which the Government acknowledges to owe a Perſon, and to pay 3 *per Cent. per Annum*, is to 75*l.* the Price which Buyers and Sellers have agreed to ſet at a particular Time, or rather as the Worth of Money then was, ſo is 5800*l.* the Money lent the Government, to the Money the Buyer muſt pay according to the Market-Price, or the Worth of Money *per Cent. per Annum*. Then $\frac{75 \times 5800}{100} = 4350$; that is, the ſecond and third multiplied together, and divided by the firſt, gives the fourth Number. When 100*l.* Share fell down to 75*l.* in the Sale, the Reaſon was, the Buyer, being able elſewhere to make 4 *per Cent.* of his Money, would give no more than 75*l.* for 100*l.* for as 75*l.* : 3 *Int.* : : 100*l.* : 4 *Int.* or as 4 : 100 : : 3 : 75; ſo that the Buyer made 4 *per Cent.* of his Money; whereas, had he given 100*l.* for 100*l.* he would have made only the 3 *per Cent.* the Government paid. But ſuppoſe at the late Peace, that Money was only 3 *per Cent.* in that Caſe the Price, inſtead of 75, ſhould be 100, and then the Purchaſer muſt have paid full 5800*l.* inſtead of 4350*l.* for what he bought, and he would have made only 3 *per Cent. per Annum* of his Money.

Hence, we perceive, that the Purchaſer conſiders the Worth of Money; that is, whether, from the Wants of the Government he can get 3 or 4 *per Cent.* &c. So that when Money was worth 4 *per Cent.* the 3 *per Cent. Annuities* could not be ſold for 100*l.* but 75*l.*

And now I muſt deſire the Learner to compute the Value of theſe Stocks, from 75 to 100, riſing 5 $\frac{1}{8}$ in every Queſtion, and let the Sum bought be 15760*l.* but ſo altered, in every Queſtion, as to differ from any Cyphering Book in the School. But here I muſt obſerve that $\frac{1}{8}$, *&c.* are computed as in the Margin; yet I ſhall ſhew how to multiply the ſecond and third Numbers together, and divide by the firſt, without being obliged to reduce the ſecond Number, which would be troubleſome. Thus, as 100 : 80$\frac{1}{8}$: : 15760 : the Purchaſe, performed as follows: Firſt, multiply the Stock 15760 by 80; then, $\frac{1}{8}$ becauſe it is $\frac{1}{8}$ of a Pound more, ſo to make the Stock $\frac{1}{8}$ more is to divide it by 8, which Quotient muſt be added to 15760 × 80. Then to divide by 100, the firſt Number, is only to point off two Figures on the Right, as a Remainder in Pounds, to be multiplied by 20, then by 12 and 4, if there are any Remainders in Shillings or Pence, and ſtill dividing on by the firſt Number. When you have $\frac{1}{4}$ or $\frac{1}{2}$, there can be no Difficulty, as, inſtead of 8, you divide the Stock by 4 or 2, and add either to the Product of the ſecond and third Numbers as above ſhewed: But when there are either $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{4}$, or $\frac{7}{8}$, proceed thus: For $\frac{3}{8}$, find for $\frac{2}{8}$ or $\frac{1}{4}$, then for $\frac{1}{8}$ or the $\frac{1}{2}$ of $\frac{1}{4}$, and adding them to the Product of the ſecond and third Numbers: For $\frac{5}{8}$, find for $\frac{4}{8}$ or $\frac{1}{2}$, then for $\frac{1}{8}$ or the $\frac{1}{4}$ of $\frac{4}{8}$, and add them to the Product: For $\frac{3}{4}$, find for $\frac{1}{2}$, then for $\frac{1}{4}$, and add them to the Product: Laſtly, for $\frac{7}{8}$, find for $\frac{4}{8}$ or $\frac{1}{2}$, then for $\frac{2}{8}$ or $\frac{1}{4}$, and for $\frac{1}{8}$ the $\frac{1}{2}$ of $\frac{1}{4}$, and add theſe to the Product of the ſecond and third Numbers.

		s.	*d.*
$\frac{1}{8}$	is	2	6
$\frac{1}{4}$	—	5	—
$\frac{3}{8}$	—	7	6
$\frac{1}{2}$	—	10	—
$\frac{5}{8}$	—	12	6
$\frac{3}{4}$	—	15	—
$\frac{7}{8}$	—	17	6

If the Quantity of Stock happens to be ſmaller than the Price *per Cent.* as ſometimes a common Tradeſman, nay, even a Servant, wants to buy into the Annuities, the Proportion is the ſame for caſting up 20*l.* 25*l.* *&c.* that is, as 100 : 75, *&c.* : : 25 : the Purchaſe or Value of 25*l.* *&c.* But many Times, in the Quantity of Stock, within 144*l.* a ſhorter Way may be taken, like what was taught in Multiplication of Money; thus, ſuppoſe the Quantity of Stock be 120*l.* and the Price be, ſuppoſe, 98 $\frac{7}{8}$, or 98*l.* 17*s.* 6*d.* then the Multipliers are 10 and 12, becauſe 10 × 12 = 120; the laſt Product muſt be divided by 100, the firſt Number, and the Remainders multiplied by 20, 12, and 4, and ſtill reſpectively divided by 100. And here again I plead for many ſmall Quantities of Stock being done in this laſt Method alſo, that the Learner may be very dextrous in this Buſineſs of employing ſmall Sums of Money upon Government Securities. I requeſt alſo, that the Learner will likewiſe vary his ſmall Quantities of Stock, that they differ from thoſe of his School-Fellows.

Thus much for the Price of Stock when it is *below Par*; that is, under 100*l.* as this will ever be the ſteady Caſe, when more than 3 *per Cent.* can be got upon Government (meaning Parliamentary) Security; but if you ſuppoſe the Intereſt of Money hereafter to be lent upon ſuch Security, ſhould ever be only 2 *per Cent.* then the abovementioned 3 *per Cent.* Annuities (or Intereſt) will be *above Par*, or ought to be worth 150; for as 2 : 100 : : 3 : 150. Now you perceive when Money is lent the Government (whoſe Wants chiefly raiſe or lower Intereſt) at 4 *per Cent.* the Stocks lent at 3 would fall, and are only worth 75; and if Money ſhould be lent at 2 *per Cent.* the 4 *per Cent.* Annuities would riſe and be worth 200*l.* tho' they would be all *at Par* when the Money is firſt lent. The Way to compute ſuch Stocks *above Par*, is by the ſame Proportion as before; that is, as 100*l.* : 150*l.* : : any Quantity of Stock to the Money which muſt be paid for that Quantity.

And thus we may compute *Bank* and *India Stock*, which are *above Par*; that is, the preſent Value is above 100*l.* which was the original Price; but the Value of Money now being leſs than at that Time, when the Money was lent to thoſe incorporate Bodies, the preſent Price is *higher than Par*, for the Reaſon mentioned before. So likewiſe *South-Sea Stock* is *under Par*, partly on Account of the Difference of Money, and partly becauſe Government Securities are ſomething more ſatisfactory, and People, in Conſequence, deal more in them. But where there is not an encouraging Difference in Point of more Intereſt, the Public, in general, will, at all Times, give the Preference to Government Security, becauſe in trading Companies, or where there is any diſtant Hazard to be run, ſome People are too timerous to venture their Money for a greater Intereſt.

Let me now prevail on the Learner to caſt up all the intermediate Prices of *Bank Stock*, from 100*l.* *(par)* to 123*l.* making each Price 5 $\frac{1}{8}$*l.* more. Then *India Stock*, from Par (100*l.*) to 148*l.* making each Price 10 $\frac{1}{8}$*l.* more. Then *South-Sea Stock* from 100*l.* *(par)* to 92*l.* making each Sum on your Slate, 2 $\frac{1}{8}$*l.* more. But in all theſe Examples he will take Care, for his own Sake, to let the Quantities of Stock differ from thoſe done before by his School-Fellows. In like Manner you may make diſtinct Sums, and caſt up 3$\frac{1}{2}$ *per Cent.* Bank Annuities at 86; 3$\frac{1}{2}$ *per Cent.* Annuities for

1758,

1758, at 88; 4 *per Cent.* Annuities for 1763, at 93, or any other Stock, as the Proportions to find the fourth Number, or Value of any particular Quantity of Stock, are the ſame; that is, as *Par* (meaning 100*l.*) *is to the Price, ſo is the Quantity of Stock to be ſold to the Value.*

The next entertaining, as well as extremely uſeful, Buſineſs will be to ſee which Fund affords you moſt Intereſt for your Money, ſuppoſing they are all equally ſafe. Firſt then, if you buy into the Government's 3 *per Cent.* Annuities, the Price now being about 82, you have this Proportion, as 82 : 3 : : 100; that is, as the preſent Price will give you 3*l.* Intereſt for your buying 82 Stock, what is that *per Cent. per Annum:* Here 3 × 100, and this Product divided by 82, gives the Anſwer.

So in $3\frac{1}{2}$ *per Cent.* Bank Annuities, as 86 : $3\frac{1}{2}$: : 100 to the Intereſt of 100*l.* here in multiplying the ſecond and third together, you know you muſt take the $\frac{1}{2}$ of the third Number, and add it to 3 × 100 before you divide by 86: Alſo in $3\frac{1}{2}$ *per Cent.* Annuities for 1758, as 88 : $3\frac{1}{2}$: : 100 to the Intereſt *per Cent.* And in the 4 *per Cent.* Annuities, 1763, as 93 : 4 : : 100 to the Intereſt of 100*l.* In all theſe Queſtions, if the firſt Number has $\frac{1}{8}$, $\frac{1}{4}$, &c. you have nothing more to do than to multiply by the whole Rate, and to take the Parts of the $\frac{1}{8}$, $\frac{1}{4}$, &c.

The Bank, as a Company of Bankers, have their Gains, Charges, and Loſſes to conſider when they eſtimate their Dividends, and therefore the Intereſt to be made by thoſe who buy into Bank-Stock cannot always be the ſame, tho' the Variation may be very ſmall: However, we will ſuppoſe they give $4\frac{1}{4}$ *per Cent.* on their Dividends as a Company of Bankers, and then we have as 123 : $4\frac{1}{4}$: : 100 to the Intereſt of 100*l.* In like Manner the *Eaſt-India* Company, as a trading Body, cannot always aſcertain the Intereſt or Profit a Perſon may make, by buying into their Stock; but we will ſuppoſe their neat Gains will leave $4\frac{3}{4}$ *per Cent.* on their Dividends, and then we have, as 148 : $4\frac{3}{4}$: : 100 to the Intereſt or Profit of 100*l.* Laſtly, as the *South-Sea* Company have their Charges and Loſſes to deduct from their Gains, let us ſuppoſe the Profits may afford $3\frac{3}{4}$ *per Cent.* on their Dividends-and then we have, as 92 : $3\frac{3}{4}$: : 100 to the Intereſt or Profit of 100*l.*

I ſhall conclude this *Firſt Syſtem*, in ſhewing the Learner how to find the Value in Pounds of any Sum of Money received in *Guineas, Moidores, Pieces* of 36*s.* and 3*l.* 12*s.*—Here then, *ſuppoſe I receive* 5000 *Guineas, how many Pounds do they make?* The Proportion is, as 1 Guinea : 21 Shillings : : 5000 Guineas to the Shillings in the Guineas; that is to ſay, $\frac{5000\times 21}{20} = 5250$*l.* or ſhorter, bring 5000*s.* into Pounds, by dividing by 20, and add theſe to the 5000*l.* ſo 9768 Guineas received, will be as 1 : 21 : : 9768 to the Pounds, or $\frac{9768}{20}+9768=10256$*l.* 8*s.* *Suppoſe I receive* 3295 *Moidores, How many Pounds do they make?* Here the Proportion is, as 1 Moidore : 27 Shillings : : 3295 to the fourth Number in Shillings, or $\frac{3295\times 27 \text{ (or } 9\times 3)}{20} =$ to the Pounds requir'd. *If I receive* 6789 *Portugal Pieces of* 36*s.* the Proportion is the ſame, and then $\frac{6789\times 36 \text{ (or } 6\times 6)}{20} =$ to the Pounds. *Suppoſe I receive* 5706 *Pieces of* 3*l.* 12*s.* *How many Pounds do they make?* Here as 1 : 72 : : 5706; that is, as in thoſe above, multiplying the ſecond and third together gives the Number of Shillings, and this divided by 20, the Pounds required. It muſt needs be very eaſy, from the Theory and Practice here given, to find the Value of any foreign Money, ſuch as *French* Crowns, Louis d'Ors, Florins, Ducats, Dollars, Pounds *Flemiſh*, Guilders, *&c.* they depending on the ſame Proportion (Theory) and Manner of Operation, as the Learner will ſee, when he comes to that Part of Arithmetic called *Exchange*, if his deſtined Situation ſhould oblige him to travel ſo far with me; if not, I here take my Leave of the worthy Youth.

SYSTEM II.

PRACTICE.

IF the Learner, who is deſigned for Buſineſs, has well digeſted the four firſt Rules, *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*, and what has been ſaid of the Grounds of the *Rule of Three*, I preſume he will find no Difficulty, in going on with me, to maſter what is daily tranſacting in the buſy Scenes of Trade, and what has hitherto been purpoſely omitted, that thoſe who are deſigned for other Stations might not be troubled with what little concerns them.

This Rule, in Arithmetic, called *Practice*, is nothing more than Proportion, or the *Rule of Three*, when the firſt Number is *one*; and therefore performed by conſidering the Price as a Diviſor, and is very properly taught after a Youth is well grounded in finding a fourth Number, by having three given. But jumping over all the uſual Forms of teaching *Addition, Subtraction, Multiplication, Diviſion*, and *Reduction of Weights and Meaſures*, &c. requires ſome Apology; yet if the Learner is well grounded as he proceeds, and nothing is omitted which he may have Occaſion for in real Buſineſs, I hope the worthy Teacher will readily excuſe the Novelty.

AVOIRDUPOIS-WEIGHT.

AVoirdupois-Weight conſiſts of 16 Ounces to the Pound, and of 112 Pounds to the Hundred Weight. By this moſt Merchandizes are weighed, ſuch as Indigo, Sugars, Tobacco, Rice, Tallow, Pitch, Tar, Hemp, Flax, &c.

C.	*qrs.*	*lb.*
9	3	21
8	2	17
9	1	19
10	3	12
8	2	21
7	1	8
9	3	23
8	2	17
7	1	18
10	3	19

The Table to be got by Heart.

Note that
16 *Drams* make 1 Ounce *(oz.)*
16 *oz.* ——— 1 Pound *(lb.)*
28 *lb.* ——— ¼ of a hundred *(qr.)*
4 *qrs.* ——— 1 Hundred *(C.* or 112 *lb.*
20 *C.* ——— 1 Ton.

1. *Bought* 10 *Hogſheads of Sugar, whoſe neat Weights are as per Margin, at* 3d. *per* lb. *What Sum of Money will it all come to?* Here you muſt, *by the Table*, firſt caſt up what the Hogſheads all weigh, which, as you are ſuppoſed well grounded in what you have been taught, you cannot complain of the leaſt Difficulty in doing.

The Manner of Operation.

BEgin with the Units of the *lbs.* and ſay, 9, 17, 24, 27, 35, 36, 38, 47, 54, 55, 5, and carry 5 to the Tens of the *lbs.* 6, 7, 8, 10, 12, 13, 14, 15, 17, 7, and carry 1; ſo that you have 175 *lbs.* Now look into the Table marked *A* (which you are to get by Heart) in the Margin of the following Page, to diſcover how many *qrs.* 175 *lbs.* make, which, if you will not, you muſt have the Trouble of

dividing

Table A.

lbs.		*qrs.*
28	make	1
56	——	2
84	——	3
112	——	4
140	——	5
168	——	6
196	——	7
224	——	8
252	——	9
280	——	10
308	——	11
336	——	12

Table B.

qrs.		C.
4	make	1
8	——	2
12	——	3
16	——	4
20	——	5
24	——	6
28	——	7
32	——	8
36	——	9
40	——	10

dividing by 28 *lbs.* becauſe as 28 *lbs.* : 1 *qr.* : : 175 *lbs.* that is, $\frac{1\times175}{28}$ being the ſecond and third Numbers multiplied together, and to be divided by the firſt. Conſider now with yourſelf; had the *lbs.* been juſt 168, they would make exactly 6 *qrs. by the Table*; but you have 175, which is 7 more, for 175—168=7; therefore ſay 7 and carry 6 (to the *qrs.)* 9, 10, 12, 15, 16, 18, 21, 22, 24, 27, *qrs.* Now look into *Table* B, which I hope you will get by Heart alſo, and you will find 27 *qrs.* make 6 *C.* 3 *qrs.* that is, 3 and carry 6 (to the *Cs.)* 13, 21, and ſo on, as formerly taught you, as it will be affronting you to repeat what you undoubtedly already know how to do by yourſelf: Therefore I ſhall ſuppoſe you have very correctly diſcovered how many *Cwts. qrs. lbs.* there are in the 10 Hogſheads of Sugar.

The next Thing to be done is to bring thoſe *Cwts. qrs. lbs.* into *lbs.* as the Goods are, by the Queſtion, bought by the *lb.* and not by the *Cwt.* In order to this, you muſt multiply by 4, becauſe 4 *qrs.*=1 *Cwt.* for as 1 *Cwt.* : 4 *qrs.* : : the *Cwts.* in the Goods to the *qrs.* but you muſt not forget to take in the odd *qrs.* when you multiply by 4. Then you are to multiply by 28, becauſe 28 *lbs.*=1 *qr.* for as 1 *qr.* : 28 *lb.* : : the *qrs.* in the Goods, to the *lbs.* and remember to take in the odd *lbs.* Now, pray what Difficulty have you found ſo far in this Queſtion? You can eaſily find the whole Weight in *Cwts. qrs.* and *lbs.* and there is no Myſtery in bringing them into *lbs.* as you have been taught the Nature of Addition and Multiplication, and you ſee they are only here again brought into Uſe.

We are now to conſider the Queſtion was deſigned, by finding the *lbs.* to diſcover how much muſt be paid for them at 3*d. per lb. The Rule of Three* tells you, as 1 *lb.* : 3*d.* : : *x lbs.* (here *x* ſtands for all the *lbs.)* to the Value in Pence, becauſe the ſecond Number is in Pence, and then dividing the fourth, which is the Anſwer in Pence, by 12 and 20, we have the Value in *£. s. d.* But as this takes up Time, People in Buſineſs uſe a ſhorter Way, *by making the Price a Diviſor*, as was mentioned before. Thus, 3 *d.* is a Diviſor, and the Pence in a Shilling the Dividend; for $\frac{12}{3} = 4$ the *Quotient*, that is, 3 *d.* is the 4th of 12 *d.* becauſe $3 \times 4 = 12$; and therefore uſing theſe *Quotients* as *Diviſors* to the Goods, we have the true Value in Shillings; for before we multiplied the *lbs.* by the Price 3 *d.* and we had the Value in Pence, but by dividing by 4, which makes the Pence 12 Times leſs than when the *lbs.* were × 3, we bring the Value into Shillings; for multiplying by 3 *d.* and then dividing by 12 to bring them into Shillings, is the ſame as dividing by 4; that is, both Ways make the Pence 12 Times leſs. Now then, draw a Line under the *lbs.* and ſay, the *fours* in the *lbs.* and this Quotient will give you the Shillings which all the Goods are worth; and theſe Shillings divided by 20 will give you the Pounds, &c.

Diviſ.		*Quot.*
¼ *qr.*	of a *d.* is the	4th
½ *qrs.*	———	2
1 *d.*	of a *s.* is the	12th
1½ *d.*	———	8th
2 *d.*	———	6
3 *d.*	———	4
4 *d.*	———	3
6 *d.*	———	2
1 *s.*	of a *l.* is the	20th
1 *s.* 8 *d.*	———	12th
2 *s.*	———	10th
2 *s.* 6 *d.*	———	8th
3 *s.* 4 *d.*	———	6th
4 *s.*	———	5th
5 *s.*	———	4th
6 *s.* 8 *d.*	———	3d
10 *s.*	———	2
¾ *qrs.*	— 4th of	3 *d.*
¾ *qrs.*	— 8th of	6 *d.*

From what has been ſaid, you will not be diſpleaſed to ſee theſe *Diviſors*, *Quotients*, and *Dividends*, or what Part the Price is of its own *Dividend*, thrown into a Table, as *per* Margin; thus, ſuppoſe the Prices were 1 *qr.* (or ¼) 3 *d.* or 5 *s.* &c. then we have $\frac{4\ qrs.}{1\ qr.}$ 4th, $\frac{12\,d.}{3\,d.} = 4$th, $\frac{20\,s.}{5\,s.} = 4$th; or if the Prices were 2 *qrs.* (or ½) 2 *d.* or 4 *s.* &c. then we have $\frac{4\ qrs.}{2\ qrs.} = 2$, $\frac{12\,d.}{2\,d.} = 6$th, $\frac{20\,s.}{4\,s.} = 5$th. The Table of *Quotients*, &c. muſt be got by Heart, otherwiſe you will be obliged to find your *Quotients*, as I have juſt ſhewn you, or look into the Table, which is not clever in a Youth, when he does a Sum, for he ſhould have all his Buſineſs in his Head.

Now, from what has been ſaid, the Learner is to let the ſame 10 Hogſheads be altered; that is, to vary the Weight of each Hogſhead, and then to caſt them up as before taught, and then find how many *lbs.* Weight there are; and for the Sake of the Rules in Practice, let the Price be a Farthing *per lb.* Then ¼ is the 4th, that is, divide the *lbs.* by 4 and you have the Pence, then by 12 for Shillings, and laſtly by 20, and you have the Pounds, *&c.* the Goods will coſt you.

Then alter the Weight of each of the 10 Hogſheads again, proceed as before, and let the Price be at ½*d. per lb.* Here ½ is 2, that is, divide the Goods by 2, and you have the Pence, then divide by 12 and 20, and you will ſee how many £. *s. d.* you are to pay for the Goods bought. When this is done, again alter the Weight of each of the 10 Hogſheads, and proceed as before to find the Amount of the whole in *C. qrs. lbs.* and then to reduce them into *qrs.* and *lbs.* and let the Price be ¾ or Farthings *per lb.* Here 3 Farthings is no *even Part* of a Penny, therefore the Parts it is to be ſplit into are 2 for ½, and then the Half (2) of that for ¼, that is, what it comes to at ½*d.* for ¼ it muſt ſurely come to the Half of that; and theſe being added together give you the Pence, and dividing by 12 and 20 give the Sum you are to pay.

So far, for the Parts of a Penny, in the two laſt Sums; we come now to the Parts of a Shilling: Begin with 1*d.* then 1¼*d.* 1½*d.* and ſo on, making the Price ¼*d.* more every Time, for ſome Sort of heavy Goods, till you come to 12*d.* and every now and then quite alter the Weight of each Hogſhead, and proceed as before. By thus frequently varying the Weight of the Sugar, *&c.* you will be perfect in that Sort of *Addition of Weights*, and by reducing them into *lbs.* you grow Maſter of *Reduction of Weights* in that Particular, and making each Sum a Farthing in the Price more, you will want no Precedent to go by, and be perfect in *Practice* within theſe Limits. But that you may go with a ſtill greater Light in your Hand, I ſhall, in the Margin, ſplit the Parts for you, which you may conſult and be ſet forwards, inſtead of being perplexed, when your Maſter is engaged with ſome of your School-Fellows, and cannot then help you.

What is meant by ſplitting the Parts, are as follow: Suppoſe the Price is 1¾*d.* then the Diviſors are 12, 2, 2, becauſe 1*d.* is the 12th of a Shilling, ½ is the ½ of 1*d.* and ¼*d.* is the ½ of ½*d.* So if the Price is 2¾*d.* here the Parts or Diviſors are 6, 4, 2, becauſe 2*d.* is the 6th of a Shilling, ½*d.* is 4th of 2*d.* and ¼ is the ½ of ½*d.* In like Manner let the Price be 3¾*d.* here the Diviſors are 4, 4; becauſe 3*d.* is the 4th of a Shilling, and ¾*d.* is the 4th of 3*d.*

Price		*pt.*	*pt.*	*pt.*
¾ *qrs.*		2	2	0
1¼ *d.*		12	4	0
1¾		8	6	0
2¼		6	8	0
2½		6	4	0
2¾		6	4	2
3¼		4	12	0
3½		4	6	0
3¾		4	4	0
4¼		4	3	4
4½		3	8	0
4¾		3	8	2
5		3	4	0
5¼		3	4	4
5½		3	8 *up.*	0
5¾		3	8 *up.*	6
6¼		3	2	8
6½		3	2	4
6¾		2	8	0
7		2	6	0
7¼		2	6	4
7½		2	4	0
7¾		2	4	6
8		2	3	0
8¼		2	3	8
8½		2	3	4
8¾		2	3	8 *at* 6 *d.*
9	2	2	0	0
9¼ *d.*	2	2	12	0
9½	2	2	6	0
9¾	2	2	4	0
10	2	3 *up.*	0	0
10¼	2	2	3	4
10½	2	3 *up.*	8	0
10¾	2	3 *up.*	8	of 6 *d.*
11	2	3 *up.*	4	0
11¼	2	3 *up.*	4	4
11½	2	3 *up.*	4	of 6 *d.*
11¾	2	3 *up.*	4	8 *at* 6 *d.*

The Uſe of the Table in the Margin is as follows: Suppoſe the Price is ¾ *qrs.* under the Word *Price* you will find ¾ *qrs.* and againſt it two Diviſors 2, 2; that is, you muſt take the Half of the Goods for ½ *qrs.* and the Half of this Quotient for ¼ *qr.* and when theſe two are added together you have the Pence your Goods come to, which you muſt divide by 12 and 20. Suppoſe your Price is 1¼ *d.* againſt it you have 12, 4; that is, you take the 12th of the Goods for 1 *d.* and you have the Shillings they come to; then you take the 4th of what 1 *d.* comes to for ¼ *qr.* and theſe two Quotients, being added together, give you the Shillings the Goods come to, and then you divide by 20 for *Ls.* In like Manner, ſuppoſe the Price is 2¾ *d.* againſt it you have 6, 4, 2; that is, you divide by 6 for 2 *d.* then you divide this laſt by 4 for ½ *qr.* and this laſt by 2 for the odd ¼ *qr.* and adding theſe three Quotients together you have the Shillings the Goods come to, and then you divide by 20. Likewiſe, ſuppoſe the Price is 5¾ *d.* againſt it you have 3, 8, 6; that is, you divide by 3 for the 4 *d.* the Goods (which are here and elſewhere mark'd *up* for upper Line) by 8 for 1½ *d.* and by 6 for ¼ *qr.* Again, ſuppoſe the Price is 8 *d.* ¾ *qrs.* againſt it ſtand 2, 3, 8; that is, you take the Half for 6, the 3d for 2 *d.* and becauſe ¾ *qrs.* is no equal Part of 2 *d.* you take the 8th of what the Goods come to at 6 *d.* or 24 *qrs.* and not of the laſt Line or Quotient. Now let us ſuppoſe the Price is 9 *d.* ¼ *qr.* againſt it you have 2, 2, 12; that is, you take the Half of the Goods for 6 *d.* the Half of what it comes to at 6 *d.* for 3 *d.* and the 12th of this laſt for ¼ *qr.* and adding theſe Quotients together, as you muſt remember always to do, you have the Value of the Goods in Shillings, then divide by 20. So likewiſe, when the Price is 10 *d.* ¼ *qr.* you have againſt it 2, 2, 3, 4; that is, you draw a Line under the Goods, (which you are in all theſe Caſes ſuppoſed to do) and take the Half of them for 6 *d.* and the Half of this for 3 *d.* and the 3d of this laſt for 1 *d.* and the 4th of this laſt for ¼ *qr.* and adding them (meaning all the Quotients) together, you have the Value of the Goods in Shillings; then divide by 20. Laſtly, ſuppoſe the Price is 11 *d.* ¾ *qrs.* againſt it you have 2, 3 *up.* 4, 8 of 6 *d.* that is, you take the Half for 6 *d.* the 3d of the Goods (or *up.* for *upper Line)* for 4 *d.* the 4th of what it comes to at 4 *d.* for 1 *d.* and the 8th of what it comes to at 6 *d.* and adding theſe four Quotients together, you have the Value of the Goods in Shillings; then divide by 20, to find how many Pounds, Shillings, and Pence the Goods amount to.

And now the worthy Learner is fully enabled to caſt up what the Goods will come to from a Farthing to a Shilling: Nothing remains, but to prevail on him to read the Table over and over again, and to ſhew his Ingenuity in diſcovering the Reaſon of the Diviſors or equal Parts: It will give him a ſincere Pleaſure to *maſter* the Table, which has not the leaſt Difficulty in it, but what ariſes from firſt looking at it: The Appearance is *frightful* enough, but when the courageous Youth examines one Price and the Diviſors againſt it, and ſees the Reaſon of uſing ſuch Diviſors, he deſpiſes the Bugbear, and is reſolved to puſh forward, and to conquer all he meets with.

But he may, on ſome Occaſions, make the Work of ſome of theſe Sums ſhorter, by taking the Advantage of the Price; thus, at 10 *d.* he may only affix a Cypher, and divide by 12, to bring the Goods into Shillings. This ſetting 0 on the Right of the Goods, is only multiplying them by the Price; for 1 *lb.* : 10 *d.* : : the Goods to the Value in Pence. Thus, again, ſuppoſe the Price is 11 *d.* ¾ *qrs.* let the Goods ſtand for Shillings, (the Reaſon is, 1 *lb.* : 1 *s.* : : the Goods) as the Price wants only ¼ *qr.* of a Shilling; draw a Line, and divide by 4 for the ¼ *qr.* and bring theſe Pence into Shillings, and take them from the Goods. If the Price is 11½ *d.* do as in the laſt Sum, only divide by 2 for the Pence. So likewiſe, 11¼*d.* which has four Diviſors, may be done by doing as above, and dividing by 2, 2, for the Pence, and adding them together, bring theſe Pence into Shillings, which muſt be taken from the Goods ſtanding for Shillings: Likewiſe the Price being 11*d.* divide by 12, and ſubtract theſe Shillings from the Goods. So when the Price is 10½*d.* divide by 8 for the 1½*d.* wanting to make the Price a Shilling, and you have the Shillings the Goods will come to at 1½*d.* which muſt be taken from the Goods, and the Remainder is what they come to at 10½*d.* Now, in order to have theſe Advantages ready at Hand, I have thrown them together in the Margin, that you may remember to look into what has been done above for your Direction. On viewing the Table of Quotients, I do not perceive that you can pick out any more to throw into this little Column; but if, on your trying them, you ſhould meet with any, you may place them here among the reſt.

10*d.*
10½
11
11¼
11½
11¾

2. *Bought* 10 *Caſks of Indigo, at* 4*s. per lb. Neat, the Groſs and Tare being as per Margin, What have I to pay for theſe Goods?* By *Groſs* you are to underſtand the Weight of the *Caſk* and *Indigo* together; by *Tare* the Weight of the *Caſk* only; and by *Neat*, the Weight of the *Indigo* only. Whenever you find Mention is made hereafter of theſe Things, you muſt fix in your Mind that the *Groſs* means alſo the Weight of any Sort of Goods with what it is put into, ſuch as Bags, Caſes, Hogſheads, Barrels, Canniſters, &c. and that the Tare is the ſeparate Weight of theſe Bags, Caſes, Canniſters, &c. and that the *Neat* is the ſeparate Weight of the Goods theſe Bags, Caſes, &c. contain.

Groſs.				*Tare.*
C.	qrs.	lbs.		lbs.
6	3	21	—	76
5	3	18	—	78
7	1	25	—	79
6	0	7	—	80
6	2	19	—	75
7	3	6	—	77
6	2	23	—	80
7	0	5	—	76
6	3	17	—	79
5	3	27	—	75

The firſt Thing to be done is to find the Amount of the Groſs Weight, as was taught you in the laſt Queſtion; then bring all into *qrs.* and *lbs.* by multiplying by 4, and taking in the odd *qrs.* and by 28, taking in the Units of the odd *lbs.* when you multiply by 8, and taking in the Tens of the odd *lbs.* when you multiply by 2 (meaning 20). When this is done, caſt up all the *Tares*, (or Weights of the Caſks) which you know are already in *lbs.* Then take the *lbs.* of the

Price	*pts.*	*pts.*	*pts.*
3 *s.*	10	2	0
6	5	2	0
7	4	10 *up.*	0
8	5	5 *up.*	0
9	4	5 *up.*	0
11	2	10	0
12	2	5	0
13	2	5	2
14	2	5 *up.*	0
15	2	2	0
16	2	5 *up.*	2
17	2	2	5 of 10 *s.*
18	2	5 *up.*	5 *up.*
19	2	2	5 *up.*

the whole *Tare* from the Weight of the whole *Groſs lbs.* and then you have the *Neat Weight* of the Indigo, which you have bought at 4 *s. per lb. neat*; for, ſurely, you would not give this Price for the Weight of the Caſks, and therefore it is you take away the whole Weight of the Caſks from the whole Groſs Weight. Now, for the Price you have, as 1 *lb.* : 4 *s.* : : the Neat Weight of the Indigo, to its Value in Shillings, which, dividing by 20, gives you the Pounds, &c. But by the Rule of Practice you reaſon thus; 4 *s.* is the 5th of a Pound, therefore dividing by 5 gives me the Value of the Indigo in Pounds, &c. Here again you have to do with *Quotients* which become your *Diviſors*; for $\frac{20}{4}$ = 5th, and if the Price was 5 *s.* $\frac{20}{5}$ =4th, &c. as were mentioned in the *Practice Table* before. But when there are Pence beſides the Shillings, there ariſe other Diviſors or Parts to be taken; thus, 4 *s.* and 6 *d.* is 5, 8, becauſe 4 *s.* is the 5th of a £. and 6 *d.* is the 8th of 4 *s.* ſo 4 *s.* and 9 *d.* is 5, 8, and 2; and 4 *s.* and 9 *d.* 3 *qrs.* is 5, 8, 2, 4; in like Manner 3 *s.* 6¾ *d.* is 10, 2, 2, 8; and if the Price is 9 *s.* 6½ *d.* the Parts are 4, 5 *up.* (upper Line or Goods) 8, 12; alſo when it is 19 *s.* 9¾ *d.* the Parts are 2, 2, 5 *up.* (Goods) 8, 2, 4; and ſo on, always conſidering what Part the Pence are of the Shillings, and the Farthings of the Pence. Thus, for Inſtance, by the *Table of Shillings* in the Margin, ſuppoſe the Price is 17 *s.* 6¾ *d.* by the Table againſt the 17 *s.* you have 2, 2, 5 of 10 *s.* that is, the 5th Part of what it comes to at 10 *s.* then 6 *d.* is the 4th of 2 *s.* and ¾ the 8th of what it comes to at 6 *d.* and by the Aſſiſtance of the Table nothing will be too hard for you.

And now again I make my Court to the worthy Youth, requeſting him to form for himſelf many Sums from this 2d Queſtion, which he will be pleaſed to do, by changing the Groſs Weight of every Caſk and its Tare, and let the Price, for ſome Sort of Goods or other in ſuch Caſks, be from 12 *d.* to 20 *s.* uſing frequently odd Pence and Farthings. Is there any Difference between your doing this for yourſelf, or your

Maſter doing it for you? All he need be troubled with, is to examine you, whether you underſtand what you are about, and expreſs yourſelf in the genteeleſt Manner, which is not always the Caſe with Youth in a School.

3. *Bought 20 Hogſheads of Sugar, mark'd and numbered as per Margin, at 2 l. 7 s. 6 d. per C. neat. What have I to pay for them?* Here, as before, you are to caſt up the Groſs Weight; that is, find what all the Sugar and all the Hogſheads weigh together; then caſt up all the Tares you find in the next Page, that is, what the empty Hogſheads weigh together in *lbs.* When this is done, divide the *lbs.* by 28 to bring them into *qrs.* and theſe by 4 for *Cwts.* and take the one from the other. Surely, you will not give 2 *l.* 7 *s.* 6 *d. per C.* for lumbering empty Hogſheads. To take one from the other, I ſhall ſuppoſe the *Groſs Weight* to be 235 *C.* 2 *qrs.* 18 *lbs.* and the *Tare* to be 18 *C.* 3 *qrs.* 19 *lbs.* then conſider, you cannot take 19 *lb.* from 18 *lb.* but borrow 28 *lb.* or 1 *qr.* from the *qrs.* and you have $18+28-19=27$ *lb.* (or ſay 19 from 28, $9+18=27$, which is eaſier) and carry 1 *qr.* (to the *qrs.*) and 3 *qrs.* is 4 from 2 *qrs.* I cannot, but borrow 1 *C.* or 4 *qrs.* then $2+4-4=2$ *qr.* and carry 1 *C.* (to the *Cs.*) and 8 is 9 from 5 I cannot, but 9 from 15, 6 and carry 1 and 1 is 2 from 3, and there remains 1, 0 from 2, 2. So that the *neat Wegbt* is eaſily found.

No.		*C.*	*qrs.*	*lbs.*
			Groſs.	
1	—	9	3	27
2	—	10	2	18
3	—	9	3	7
4	—	9	1	16
5	—	10	2	5
6	—	9	2	13
7	—	10	1	19
8	—	8	3	27
9	—	9	1	6
10	—	10	2	3
11	—	9	3	5
12	—	8	3	19
13	—	10	1	2
14	—	9	3	11
15	—	8	3	26
16	—	9	2	13
17	—	10	1	10
18	—	9	3	27
19	—	8	3	16
20	—	10	2	13
Groſs.				

Having thus found the *neat Weight*, we are to compute the Value of the whole at 2 *l.* 7 *s.* 6 *d. per Cwt.* and here, ſurely, can be no Difficulty; for the neat C. Weight, at 2 *l.* is only multiplying by 2, and then 5 *s.* is the 4th Part of the Goods, and 2 *s.* 6 *d.* is the Half of what it comes to at 5 *s.* And we will ſuppoſe, beſides the *neat Cwt.* that there are 3 *qrs.* and 27 *lbs.* over; and then 2 *qrs.* is the Half of what it comes to at 1 *Cwt.* and 1 *qr.* the Half of the 2 *qrs.* 14 *lb.* the Half of 1 *qr.* 7 *lbs.* the Half of 14 *lbs.* and 4 *lbs.* is the 7th of 28 *lbs.* ($\frac{28}{4}=$7th), and 2 *lb.* is the Half of 4 *lbs.* all which make the Parts of 3 *qrs.* and 27 *lbs.* I ſhall not make the Learner ſo bad a Compliment, as to think he ſtands in Need of any farther Aſſiſtance in Sums of this Sort; and therefore ſhall only requeſt him to form and alter many Sums from this Queſtion, that he may be perfect, not only in *Addition* and *Subtraction* of theſe Weights, but alſo in finding the Value of the Goods at any Price: As theſe Sums, ſo formed, will be at a Price between one Pound and any Number of Pounds, he will only have to multiply by the Pounds, and to take the Parts for the Shillings, *&c.*

4. *Bought 25 Caſks of Indigo, each, by Agreement, weighing 5 C. 3 qrs. 27 lbs. Groſs, Tare 10 lbs. per C. each Caſk, at* 4s. 6d. *per lb. Neat; To how much Money will the Goods amount?* Now, the firſt Thing you have to do, is to conſider how to make the Weight of one Caſk 25 Times more, in order to find the whole Groſs Weight of the 25 Caſks. This you can do by this Proportion, as 1 Caſk : 5 *C.* 3 *qrs.* 27 *lbs.* : : 25 Caſks to the Groſs Weight of 25 Caſks; here the ſecond and third multiplied together, and divided by the firſt, is the Anſwer; but work this Proportion or Stating two Ways; the firſt is to multiply the ſecond Number by 4, taking in the 3 *qrs.* and 28, taking in the 27 *lbs.* then multiply theſe *lbs.* by 25 the third Number, and you have the Groſs *lbs.* in all the Caſks; or by the ſecond Way, multiply the *Cwt. qrs. lbs.* in one Caſk, firſt by 5, and that Product again by 5, becauſe 5×5=25; thus, 5 Times 7 is 35, 5 and carry 3, 5 Times 2 (meaning 20) is 10 and 3 is 13; ſo that there are 135 *lbs.* which, by the *Table marked A.* you will find to be 23 *lbs.* and carry 4 *qrs.* 5 Times 3 *qrs.* is 15 and 4 is 19 *qrs.* which, by *Table B.* is 3 *qrs.* and carry 4 *C.* 5 Times 5 *C.* is 25 and 4 is 29 *C.* ſo that we have 29 *C.* 3 *qrs.* 23 *lbs.* which we muſt make 5 Times more. Then 5 Times 3 is 15, 5 and carry 1, 5 Times 2 (meaning 20) is 10 and 1 is 11; ſo that there are 115 *lbs.* which, by *Table A.* is 3 *lbs.* and carry 4 *qrs.* 5 Times 3 *qrs.* is 15 and 4 is 19 *qrs.* that is, by *Table B.* 3 *qrs.* and carry 4 *C.* 5 Times 9 is 45 and 4 is 49, 9 and carry 4, 5 Times 2 is 10 and 4 is 14; ſo that we have 149 *C.* 3 *qrs.* 3 *lbs.*

No.	*Tare.*
1 —	98 *lbs.*
2 —	103
3 —	97
4 —	96
5 —	105
6 —	93
7 —	102
8 —	90
9 —	95
10 —	103
11 —	99
12 —	87
13 —	105
14 —	96
15 —	89
16 —	98
17 —	103
18 —	97
19 —	86
20 —	105
Tare.	

3 *lbs.* Groſs Weight, which we muſt bring into Groſs *lbs.* by multiplying by 4, taking in the 3 *qrs.* and by 28, taking in the 3 *lbs.*

When we have the *Groſs lbs.* by either of theſe two Ways, we are to conſider the *Tare* is proportioned, by Agreement, at 10 *lbs. per Cwt.* that is, if 112 *lbs. Groſs allow* 10 *lbs. Tare, What will the whole lbs. Groſs allow?* That is, multiply the whole *Groſs lbs.* by 10, and divide by 112, for the Tare in *lbs.* and this is the conſtant Rule when the Tare is to be any Number of *lbs. per* 112 or C. and the Proportion here given is the Reaſon of your doing ſo. The *Tare* thus found muſt be taken from the *Groſs*, and there will remain the *neat* Indigo, which you are to pay for, at 4*s.* 6*d. per lb.* then 4*s.* is the 5th of a £. and 6*d.* is the 8th of what it comes to at 4*s.* and thus ends the Sum. And now again I requeſt ten, twenty, or thirty Queſtions of this Sort may be formed from the Weight of one Caſk, Bag, Cheſt, or whatever contains any Sort of Goods; but do not let the Groſs Weight of one Caſk, Cheſt, *&c.* be the ſame, that is, 5 *C.* 3 *qrs.* 27 *lbs.* and let the Number of Caſks be between 2 and 144; likewiſe vary the Tare between 8 and 16 *lbs. per* 112, and laſtly, let the Price be varied; ſo that upon the whole your Queſtions ſhall not be the ſame with any Cyphering Book in the School.

5. *The Weſt-India Fleet arriving has brought* 1276 C. 3 qrs. 27 lbs. *of Indigo, Groſs: The Tare of the whole, by Agreement, is to be* 10 *lbs. per Cwt. The ſeveral Merchants to whom the Indigo belonged ſold it all to* 25 *Salters, at* 33*l.* 12*s.* 6*d. per Cwt. neat: What Part of the* 1276 *C. qrs.* 27 *lb. Groſs Weight muſt each Salter have; and what muſt he pay for his neat Part, and what is the Value of all the Indigo bought by the* 25 *Salters?* The firſt Thing to be done is to divide the *Groſs Weight,* 1276 *C.* 3 *qrs.* 27 *lbs.* among the Salters; here then, as 25 Salters : the whole Groſs Weight : : 1 Salter; that is, multiply the ſecond and third Numbers together, and divide by the firſt (that is 5×5). Now then ſay, the *fives* in 12, 2 and carry 2; the *fives* in 27, 5 and carry 2; the *fives* in 26, 5 and carry 1, which is 4 *qrs.* and 3 is 7; the *fives* in 7, 1 *qr.* and carry 2 *qrs.* which is 56 *lbs.*+27 *lbs.*=83 *lbs.* the *fives* in 8, 1 and carry 3; the *fives* in 33, 6 and 3 *lbs.* over, which diſregard. So then we have 255 *C.* 1 *qr.* 16 *lbs.* which muſt again be divided by 5, by ſaying the *fives* in 25, 5; the *fives* in 5, 1; the *fives* in 1 *qr.* 0 Times, but 28 *lbs.*+16 *lbs.*=44 *lbs.* the *fives* in 44, 8 *lbs.* and 4 over,

which diſregard. Then we have 51 *C.* 0 *qrs.* 8 *lbs. Groſs* for each Salter; but we muſt now compute the *Tare,* which is agreed upon to be 10 *lbs. per Cwt.* and therefore you muſt bring the *Groſs* into *lbs.* by multiplying by 4, to bring the *Cs.* into *qrs.* and theſe into *lbs.* by multiplying by 28, taking in the 8 *lbs.* Now you are to multiply the *Groſs lbs.* by 10, and divide by 112, becauſe as 112 Groſs : 10 *lbs.* Tare : : the *Groſs lbs.* each Salter is to have, to his *Tare* in *lbs.* This *Tare* is to be taken from the *Groſs,* and we have the *Neat lbs.* which being brought into *Cs. qrs. lbs.* by dividing by 28, to bring them into *qrs.* and then by 4, to bring theſe into *Cwts.* So that we have 46 *C.* 2 *qrs.* 2 *lbs.* Neat for each Salter, at 33*l.* 12*s.* 6*d.* *per Cwt.* Here then, 46 × 33 = *£s.* and $\frac{46}{2}$ = what the *Cwt.* come to at 10*s.* and as 2*s.* 6*d.* is the 4th of 10*s.* ſo dividing what the *Cwts.* come to at 10*s.* by 4, gives us the *£s.* for 2*s.* 6*d.* but becauſe there is ſome odd Weight, (2 *qrs.* 2 *lbs.*) we reaſon thus; 2 *qrs.* is the Half of a *Cwt.* therefore $\frac{33l.\ 12s.\ 6d.}{2}$ = 16*l.* 16*s.* 3*d.* for 2 *qrs.* then as 2 *lbs.* is the 28th Part of 2 *qrs.* ſo 16*l.* 16*s.* 3*d.* taken apart on your Slate and divided by 4 × 7 = 12*s.* which you are to add to the reſt, you will have 1564*l.* 3*s.* 3*d.* which each Salter has to pay: And now to find the Amount of what all the Salters had, we have the laſt Sum multiplied by 5 × 5 = 39104*l.* 1*s.* 3*d.* the Value of all the Neat Weight.

And now give me Leave to hope that you will form ten, twenty, or thirty Sums of the laſt Queſtion, by altering the whole Quantity of Indigo belonging to the Merchants, the *Tare per Cwt.* and the Price, and alſo the Number of Salters who thus buy up all the Indigo brought from abroad:. The Conſequence will be, you will find yourſelf ſurprizingly improved, and ready at all ſuch Queſtions which daily occur in Buſineſs, and ſufficiently rewarded for all your Labour.

6. But when the Price is *per* Cwt. and the Tare is ſo many *lbs. per* Cwt. for divers Sorts of Goods, we may frequently ſhorten the Work; that is, we are not obliged to bring the Groſs *Cs. qrs. lbs.* into *lbs.* according to the Proportion before, as 112 *lbs. Groſs* : 10 *lbs. Tare* : : the Groſs Weight in *lbs.* to the Tare required. *Thus, ſuppoſe the Groſs Weight of any Thing I buy be* 75 C. 3 qrs. 24 lbs. *and the Tare,* per *Agreement, is to be* 14 lb. per C. *at* 28l. 13s. 6d. *per Cwt. Neat,* then, as in the Margin, the Diviſor

Tare.		*pts.*
7 *lb.*	is the	16th
8	———	14
14	———	8
16	———	7

Diviſor is 8, (becauſe $\frac{112}{14}=8$) that is, the 8th Part of the Groſs will be the Tare: Thus, the *eights* in 75, 9 and carry 3 *C.* which is 12 *qrs.* + 3 = 15 *qrs.* the *eights* in 15, 1 *qr.* and carry 7 *qrs.* which is 7 × 28 *lbs.* + 24 = 220 *lbs.* the *eights* in 22, 2 (meaning 20 *lbs.*) and carry 6; the *eights* in 60, 7 and 4 *lb.* over, which may be diſregarded, unleſs the Goods are of that valuable Nature as to require Ounces, which would oblige you to multiply the 4 *lb.* over by 16 Ounces in a *lb.* and divide on by the 8th Part, as before, and then we have *Tare* 9 *C.* 1 *qr.* 27 *lbs.* 8 *oz.* to be taken from the *Groſs*, by ſaying 8 *oz.* from 0 I cannot, but 8 from 16, 8 and carry 1 and 27 is 28 *lbs.* from 24 I cannot, but 28 from 24 + 28 (= 52 *lbs.*) 24 and carry 1 and 1 is 2 *qrs.* from 3, 1; 9 *C.* from 15, 6 and carry 1, from 7, 6. So that the *Neat Weight* will be 66 *C.* 1 *qr.* 24 *lbs.* 8 *oz.* at 28 *l.* 13 *s.* 6 *d.* *per Cwt.* It will be eaſier for you in ſubtracting the Tare from the Groſs, to ſay, 8 from 16, 8 and carry 1 and 27 is 28 from 28, 0 + 24; that is, to take the *lbs.* in the Subducend from the Quarter you borrow, and to the Remainder add the *lbs.* in the Minuend. And here again the Learner will do well to form ſeveral different Sums of this laſt, and to practiſe the foregoing *little Table* in the Margin.

7. *Bought* 57 C. 2 qrs. 9 lbs. *Groſs, of ſome Sort of Goods;* *Tare* 8 C. 3 qrs. 26 lbs. *Tret, at* 4 *lbs. per every* 104 lbs. at 2l. 18s. 6d. *per Cwt. Neat: What have I to pay for theſe Goods?* Here, to take the *Tare* from the *Groſs*, we ſay, 26 from 28, (by borrowing 1 *qr.*) as was ſaid before, 2 and carry 1, and 9 is 11 *lbs.* and carry 1 *qr.* (becauſe I borrowed 1 *qr.*) and 3 *qrs.* is 4 from 6, (becauſe I borrow 1 *C.* or 4 *qrs.*) 2 *qrs.* and carry 1 *C.* (becauſe I borrowed 1 *C.*) and 8 is 9 from 17 (as I borrow 1) 8, and carry 1 from 5, 4; hence we have a Remainder of 48 *C.* 2 *qrs.* 11 *lbs.* *Suttle*, as it is called, it not being yet the *Neat Weight*, as you have the Allowance of *Tret*, which is 4 *lbs.* for every 104 *lbs.* to take from the Remainder or *Suttle*. This *Tret*, by Cuſtom, is allowed the Buyer on ſome Sort of Goods, and you have this Method of finding the *Tret*, which is to be deducted from the *Suttle*, to come at the Neat; thus, as 104 : 4 : : the *Suttle lbs.* or $\frac{104}{4}=26$; that is, the 26th Part of the *Suttle* is the *Tret*. But as 26 is not a convenient

13	*in*	13	*is*	1
13	—	26	—	2
13	—	39	—	3
13	—	52	—	4
13	—	65	—	5
13	—	78	—	6
13	—	91	—	7
13	—	104	—	8
13	—	117	—	9

venient Number to divide by, let the Diviſors be 2, 13, for 2×13=26; and as this 13 is better managed, by getting off the foregoing Table of *Thirteens*, I hope the Learner will do it for his own Eaſe and Diſpatch in Buſineſs. Thus, then draw a Line, and ſay, the *thirteens* in 48, by the Table, is 3 *C.* and 9 over, which is 9 *C.* or 36 *qrs.*+2 *qrs.*=38; the *thirteens* in 38, 2 *qrs.* and 12 *qrs.* over, which is 12×28+11=147 *lbs.* the *thirteens* in 14, 1 and carry 1; the *thirteens* in 17, 1, and 4 *lbs.* over: So that we have 3 *C.* 2 *qrs.* 11 *lbs.* to be divided by 2; that is, the Half of 3 *C.* is 1, and carry 1, which is 4 *qrs.*+2=6 *qrs.* the Half of 6 is 3 *qrs.* the Half of 11 *lbs.* is 5 *lbs.* So the *Tret* is 1 *C.* 3 *qrs.* 5 *lbs.* and, therefore, 48 *C.* 2 *qrs.* 11 *lbs.* *Suttle*—1 *C.* 3 *qrs* 5 *lbs.* *Tret*= 46 *C.* 3 *qrs.* 6 *lbs.* *Neat*, at 2*l.* 18*s.* 6*d.* *per Cwt.* which I ſhall leave to the Learner to caſt up by himſelf, and requeſt he will form many Sums of this laſt Queſtion, in order to fix the Buſineſs of *Tret* on his Mind.

8. *Bought* 57 C. 2 qrs. 9 lbs. Groſs, *of ſome Sort of Goods*; Tare, 8 C. 3 qrs. 26 lbs. Tret, 4 lb. per 104 lbs. Clough, (*pronounced* Cluff) 2 lb. *for every* 3 C. *at* 12l. 18s. 6d. per Cwt. Neat: *What is the Value of the Goods?* This *Clough* is an Allowance on ſome Sort of Goods for the Turn of the Scale, and when you have deducted the *Tret*, as in the laſt Queſtion, that Remainder, 46 *C.* 3 *qrs.* 6 *lbs.* muſt ſtill be called *Suttle*, becauſe you have the *Clough* yet to be deducted from it. Here then we have, as 3 *C.* : 2 *lbs.* : : 46 *C.* to the *Clough* to be allowed 30 *lbs.* 10 *oz.* $\frac{(2 \times 16 \textit{ oz.})}{3}$; then 2 *qrs.* is the 6th $\frac{(\text{of } 2 \textit{ lbs.} \times 16 \textit{ oz.})}{6}$, and 1 *qr.* is the ½ of this, and 4 *lb.* is the 7th of the laſt, and 2 *lb.* the ½ of this, the *Clough* making together, when added up, 31 *lbs.* or 1 *qr.* 3 *lbs.* to be taken from the *Suttle*, and the Remainder will be 46 *C.* 2 *qrs* 3 *lbs.* *Neat*, at 2*l.* 18*s.* 6*d.* which, as before, I ſhall leave to the Learner to find the Value. I need not tell the worthy Youth that he will find himſelf the more ready, the more Sums he does of this Sort, by turning this Queſtion, at large, into ſeveral others; and that in taking the Parts of the odd Weight for *Clough*, 1 *qr.* is the 12th Part of the 2 *lbs.* for every 3 *C.* *Clough*; 2 *qrs.* is the 6th Part; 14 *lbs.* the ½ of 1 *qr.* and the 4th of 2 *qrs.*

TROY-WEIGHT.

THIS Weight conſiſts of 12 Ounces to the Pound, and was the original Standard; but when Cuſtom had introduced a Right to expect more than 12 Ounces to the Pound, in weighing ſeveral Sorts of Goods, either coarſe in their Nature or liable to waſte, *Avoirdupois*, at laſt, became the Standard for moſt Merchandizes, and *Troy-Weight* was chiefly confined to the leſs bulky and more coſtly, ſuch as Jewels, Gold, Silver, *&c.*

lb.	*oz.*	*dw.*	*gr.*
12	11	19	23
10	9	13	17
16	10	5	8
8	4	17	21
13	11	8	10
5	7	12	17
12	10	15	3
17	7	6	18
10	9	14	16
15	10	18	5

The Table to be got by Heart.

24 Grains *(gr.)* 1 Penny-Weight *(dw.)*
20 *dw.* ——— 1 Ounce *(oz.)*
12 *oz.* ——— 1 Pound *(lb.)*

9. *A Nobleman bought the Quantities of Plate of different Workmanſhip and Uſes,* (as in the Margin) *for which he was to give* 5s. 9d. *an Ounce, one Sort with another: What did he pay for the whole?*

The Manner of Operation.

Beginning with the *grs.* ſay, 5, 11, 19, 22, 29, 30, 38, 45, 48; that is, 8 and carry 4, (to the Tens of *grs.*) 5, 6, 7, 8, 10, 11, 13, making in all 138 *grs.* This you muſt divide by 24; for as 24 *gr.* : 1 *dw.* : : 138 *gr.* : 5 *dw.* 18 *gr.* which is 18 *gr.* and carry 5 *dw.* 13, 17, 23, 28, 30, 38, 45, 50, 53, 62; that is, 2 and carry 6 (to the Tens of *dw.* 7, 8, 9, 10, 11, 12, 13, making 132 *dw.* which is 12 *dw.*) and carry 6 *oz.* (for $\frac{132}{20}=6$ *oz.* 12 *dw.*) 15, 22, 29, 30, 34, 43, 44; that is. 4 *oz.* (and carry 4 *oz.* to the Tens of *oz.*) 5, 6, 7, 8, 9, making in all 94 *oz.* which is 10 *oz.* and carry 7 *lb.* (for $\frac{94}{12}=7$ *lb.* 10 *oz.*) to the *lbs.* and caſt them up as you did in Addition of whole Numbers, by ſtopping at 10, and you will find they make 125 *lb.* 10 *oz.* 12 *dw.* 18 *grs.*

Now you know the Silver Plate was ſold by the *oz.* therefore we muſt bring the 125 *lb.* into *oz.* which is done by this Rule, 1 *lb.* : 12 *oz.* : : 125 *lb.* that is, 125×12=1500 *oz.*+ 10 *oz.*=1510 *oz.* (becauſe you take in the odd 10 *oz.* when you multiply by 12) ſo that we have 1510 *oz.* 12 *dw.* 18 *gr.* to know how much it comes to at 5*s.* 9*d.* *per oz.* This I ſhall leave to your own Ingenuity, and ſhall only point out the Parts you are to take, as in the Margin, for the odd 12 *dw.* 18 *gr.* And now let me recommend it to you to vary this Queſtion ſo often till you find yourſelf extremely ready at caſting up *Troy-Weight*, and reducing the *lbs.* into *ozs.* and then finding what the Value of each Quantity of Plate amounts to, which, when you can do with Eaſe and Expedition, you will be extremely pleaſed with yourſelf, and will forget the Pains the Facility coſt you.

Of 5*s.* 9*d.*		
10 *dw.*	is	2
2 *dw.*	—	5
12 *gr.*	—	4
6 *gr.*	—	2

16. *The Nobleman, in the laſt Queſtion, having old Plate weighing* 69 lb. 11 oz. 18 dw. 20 gr. *for which he was to receive of the Silverſmith at the Rate of* 5s. 3d. per oz. *How much more Plate has he now than he had before, and what Money did he pay in Exchange?* The firſt Thing you have to do is to find the Value of the old Plate; in order to which, as the Price is *per oz.* you know the *lb.* muſt be reduced into *oz.* taking in the odd 11 *oz.* that is, $69\times12+11=839$ *oz.* 18 *dw.* 20 *gr.* at 5*s.* 3*d.* *per oz.* Here, becauſe 3*d.* is ſo long a Part of 5*s.* it is better to ſay 839×5*s.* and then 3*d.* is the 4th of the Goods: Then the Parts for the odd Weight 18 *dw.* 20 *gr.* will be as in the Margin, where *up.* ſignifies of 5*s.* 3*d.* When you have found the Value of the old Plate, take this from the Value of the new Plate, bought in the laſt Queſtion, and the Remainder will ſhew you what the Nobleman paid in Exchange. Then taking the Weight of the old Plate, which is 69 *lb.* 11 *oz.* 18 *dw.* 20 *gr.* from the new Plate, which was 125 *lb.* 10 *oz.* 12 *dw.* 18 *gr.* you have the Quantity of the Plate he has now more than he had before. But here, as the Youth has not done any Thing of this King hitherto, I ſhall do one Sum for him, and then leave him to form many other Queſtions of this Sort by himſelf, in order that he may be very ready in *Subtraction of Troy-Weight.* Thus then, 20 *gr.* from 24 *gr.* (having borrowed 1 *dw.* or 24 *gr.*) 4 + 18 *gr.* 22 and carry 1 and 18 *dw.* is 19 from 20 *dw.* (having borrowed 1 *oz.* or 20 *dw.*) 1 + 12, 13 and carry 1 and 11 *oz.* is 12 from 12 *oz.* (having borrowed 1 *lb.* or 12 *oz.*) 0 + 10, and carry 1 *lb.* and 9 *lb.* is 10 from 15, 5 and carry 1 and 6 is 7 from 12, 5; ſo that the

Of 5*s.* 3*d.*		
10 *dw.*	is	2
4 *dw.*	—	5 *up.*
4 *dw.*	—	5 *up.*
12 *gr.*	—	8
6 *gr.*	—	2
2 *gr.*	—	3

the Nobleman has now 55 *lb.* 10 *oz.* 13 *dw.* 22 *gr.* of Plate more than he had before.

11. *A foreign Ambaſſador, for his King, beſpoke of the ſame Silverſmith* 25 *Times the Quantity of Plate the Nobleman had bought of him, and was to pay* 6s. 7½d. per oz. *becauſe of its extraordinary Workmanſhip: Required the Quantity of Plate, and the Value in Money the Ambaſſador was to pay for it?* Here, 125 *lb.* 10 *oz.* 12 *dw.* 18 *gr.* $\times 5 \times 5 =$ to the Quantity of Plate the Ambaſſador bought; for 5×18 *gr.* is 90 *gr.* and $\frac{90}{24} = 3$ *dw.* 18 *gr.* that is, 18 and carry 3; $5 \times 12 + 3 = 63$ *dw.* and $\frac{63}{20} = 3$ *oz.* and 3 *dw.* that is, 3 *dw.* and carry 3 *oz.* $5 \times 10 + 3 = 53$ *oz.* and $\frac{53}{12}$ is 4 *lb.* and 5 *oz.* that is, 5 and carry 4 *lb.* $5 \times 125 + 4 = 629$ *lb.* hence we have 125 *lb.* 10 *oz.* 12 *dw.* 18 *gr.* $\times 5 = 629$ *lb.* 5 *oz.* 3 *dw.* 18 *gr.* then 18 *gr.* $\times 5 = 90$, and $\frac{90}{24} = 18$ *gr.* and carry 3 *dw.* 5×3 *dw.* $+ 3$ carried is 18 *dw.* 5×5 *oz.* $= 1$ *oz.* and 2 *lb.* 5×9 *lb.* $+ 2$ *lb.* $= 47$, 7 and carry 4, $5 \times 2 + 4 = 14$, and carry 1, $5 \times 6 + 1 = 31$: So that the whole Weight is 3147 *lb.* 1 *oz.* 18 *dw.* 18 *gr.* at 6*s.* 7½*d.* *per oz.* and therefore by bringing the Weight into *oz.* we have 37,765 *oz.* 18 *dw.* 18 *gr.* Hence, the *oz.* × 6*s.* give the *Shillings* the Plate in *oz.* comes to; then 6*d.* is the Half of the *oz.* in Shillings, and 1½*d.* the 4th of 6*d.* So far for the *oz.* at 6*s.* 7½*d.* and for the odd 18 *dw.* 18 *gr.* the Parts of the Price, 6*s.* 7½*d.* are as in the Margin. And now, I preſume, the Learner will require no Entreaty to cut himſelf out ſeveral Queſtions of this Nature from the preſent, in order to be ready at multiplying the Parts of *Troy-Weight*, inſtead of being obliged to reduce them to *gr.* by multiplying by 12, 20, 24, and then bringing them back again into *oz.* by dividing by 24, 20.

Of 6*s.* 7½*d.*		
10 *dw.*	is	2
4 *dw.*	*up.*	5
4 *dw.*	*up.*	5
12 *gr.*	—	8
6 *gr.*	—	2

12. *The King, for whom the laſt Plate was bought, gave the whole to* 81 *Captains of his Army, to be equally divided among them for their good Behaviour in an Attack upon a Place: What Quantity of Plate will each Captain have, and what is the Value of each Share?* Here, 3147 *lb.* 1 *oz.* 18 *dw.* 18 *gr.* is to be divided by $9 \times 9 = 81$ Captains; thus, the *nines* in 31, 3, and 4 over; the *nines* in 44, 4, and 8 over; the *nines* in 87, 9, and 6 over, which is 6 *lb.* $\times 12 + 1$ *oz.* $= 73$ *oz.* the

nines in 73, 8 *oz.* and 1 over, which is 1 *oz.* × 20 + 18 *dw.* = 38 *dw.* the *nines* in 38, 4 and 2 over, which is 2 *dw.* × 24 + 18 *gr.* = 66 *gr.* the *nines* in 66, 7 and 3 *gr.* over: So that we have 349 *lb.* 8 *oz.* 4 *dw.* 7 *gr.* which muſt ſtill be divided by 9. The *nines* in 34, 3 and 7 over; the *nines* in 79, 8 and 7 over, which is 7 × 12 + 8 = 92 *oz.* the *nines* in 92, 10 and 2 over, which is 2 × 20 + 4 = 44 *dw.* the *nines* in 44, 4 and 8 over, which is 8 × 24 + 7 = 199 *gr.* the *nines* in 199, 22; ſo that each Officer will have for his Share of the Plate 38 *lb.* 10 *oz.* 4 *dw.* 22 *gr.* at 6 *s.* 7½ *d.* *per oz.* Now, therefore, bring the Weight into *oz.* by multiplying as before, and you have 466 *oz.* 4 *dw.* 22 *gr.* to find the Value of the King's Preſent to each Captain. Then 466 × 6 *s.* = 2796 *s.* and 6 *d.* is the Half of the Ounces, and 1½ *d.* is the ¼ of what it comes to at 6 *d.* and the Parts for the odd 4 *dw.* 22 *gr.* may be found as in the Margin; and all added together, and brought into *£s.* you have the Anſwer to the Queſtion.

Of 6 *s.* 7½ *d.*	
4 *dw.* ———	5
12 *gr.* ———	8
6 *gr.* ———	2
4 *gr.* of 12 *gr.*	3

APOTHECARIES WEIGHT.

THOUGH Apothecaries buy and ſell their Goods by *Avoirdupois-Weight*, yet they compound their Medicines by particular Diviſions of the *Pound Troy*; but in order to make a Youth ready in the Apothecary's particular Diviſion of 12 *oz.* I ſhall eſtimate the Value of their Drugs, *&c.* by Apothecaries Weight.

The Table to be got by Heart.

20 Grains *(gr.)* 1 Scruple (℈)
3 ℈ ——— 1 Dram (ʒ)
8 ʒ ——— 1 Ounce (℥)
12 ℥ ——— 1 Pound (℔)

℔	℥	ʒ	℈	*gr.*
7	11	7	2	19
5	9	6	1	17
6	10	3	0	5
2	5	5	1	13
7	8	3	2	4
3	6	2	0	16
5	7	7	2	8
0	9	6	1	17
4	3	4	2	10
3	8	5	1	11
7	6	3	0	8
6	2	4	2	9

13. *An Apothecary bought the different Parcels of Muſk,* as in the Margin, *for* 1 *l.* 12 *s.* 6 *d.* per oz. Troy, *and to be eſtimated by the Apothecary's Diviſion of* 12 oz. *What is the Value of all the Muſk?* Here you are to caſt up the *gr.* and divide them all by 20, and ſet down the odd *gr.* and carry the Scruples forwards; add them up and divide them by 3,

and ſet down the odd Scruples, and carry the Drams forwards; add theſe up and divide by 8, and ſet down the odd Drams, and carry the *oz.* forwards; add theſe up and divide by 12, and ſet down the odd Ounces, and carry the Pounds to the Pounds. When you have done all this, and have found the whole Weight, you are to value the whole at 1 *l.* 12 *s.* 6 *d.* *per oz.* Therefore you muſt bring the *lb.* into *oz.* and when you have done this, let the *oz.* ſtand for £. for, ſurely, there muſt be as many £. as *oz.* Then 10 *s.* is the Half, and 2 *s.* 6 *d.* is the 4th of what 10 *s.* come to. Thus much for the Price; the odd Weight muſt be eſtimated as Parts of the Value of one Ounce, and make a Margin yourſelf in your own Book with thoſe Parts. I need not tell the Youth, that, if he is deſigned for the Profeſſion of an Apothecary, he will be obliged to form many Queſtions from this, and vary all the Circumſtances of it, in order to be ready at theſe Diviſions of the *Pound Troy*, and then he can never be at a Loſs to find the Value of the Drugs, *&c.* bought by *Avoirdupois-Weight.*

14. *The Apothecary, who bought the laſt rich Quantity of Muſk, was requeſted to let one of the ſame Profeſſion have* 17 ℔. 11 ℥. 7 ʒ. 2 ℈. 19 gr. *of it at the ſame Price: What is the Quantity of Muſk remaining in the Hands of the former Apothecary, and the Value of what each had in Poſſeſſion?* Firſt, you are to ſubtract the leſſer Quantity of Muſk from the greater; to do which you cannot want any Help, as you are ſuppoſed perfect in the Table of Apothecary's Weight, and in the Practice of adding up this Sort of Sums; for, in taking a leſſer Number from a greater, you are only to conſider what Denomination each has, and proceed accordingly: Thus, if I want to take 19 *gr.* from 18 *gr.* I take 19 from 1 Scruple (=20 *gr.*) and add the Remainder to 18 *gr.* So if I would take 7 Drams from 6 Drams, I take the 7 from the borrowed Ounce (=8 Drams), and as one remains I add it to the 6 Drams. When you find the Remainder, this ſhews what Quantity of Muſk the former Apothecary has in Poſſeſſion; and therefore proceed now to eſtimate the Quantity, each Apothecary has, by the Price, which is 1 *l.* 12 *s.* 6 *d.* *per oz.* and then you will have anſwered the Queſtion. If you will form many Queſtions of this, you will be the more ready in *Subtraction of Apothecaries Weight.*

15. *Several Apothecaries agreed to join and buy a rich Quantity of Muſk; by Computation, it is 64 Times more than the firſt Parcel, for which they were to pay* 1 l. 15 s. 9 d. per

oz. and the whole was equally divided among 96 *of theſe Apothecaries ſo joining to purchaſe this great Stock of rich Goods: What was the whole Quantity of Muſk, and each Man's Share of it, and what Money did each Perſon pay for his Proportion?* The Learner will not be pleaſed, if I ſhould do this Queſtion for him, as he is ſuppoſed by this Time to be able to do without Help, as he has had ſuch a Variety of Queſtions to anſwer, and which depend upon the ſame Principles of Reaſon: However, I ſhall give him general Directions, by which Means he will not be perplexed by the Varieties of the Queſtions

General Directions.

1. Multiply the Goods by 8×8=64 Times; but when you multiply the *Grains*, divide that Product by 20, to find how many *Scruples*; when you multiply the *Scruples*, divide by 3, to find how many *Drams*; when you multiply the *Drams*, divide by 8, to find how many *Ounces*; when you multiply the *Ounces*, divide by 12, to find how many *Pounds*, always remembering to ſet down the Remainder in each Denomination, and carry the Quotient forwards to that of its own Name.

2. Then, when you have found the whole Quantity of Muſk, you are to bring the *lb.* into *oz.* and calculate the Value of the whole, by having the Price given.

3. Then divide the whole Quantity of Muſk in Pounds, Ounces, *&c.* by 12×8=96, remembering to multiply the Remainder of the Pounds by 12, (to bring it into *oz.*) the Remainder of the *oz.* by 8, (to bring them into Drams) the Remainder of the Drams by 3, (to bring them into Scruples) the Remainder of the Scruples by 20, (to bring them into Grains) in all Caſes dividing the Remainder of every Denomination by 12×8=96. By this Means you will have the Quantity each Apothecary had.

4. In order to find what Money each Apothecary had to pay, it will be a ſhorter Method to divide the *Value* of the *whole Quantity* of Muſk by 12×8=96, than to eſtimate the *Quantity each* Apothecary had by the Price of one Ounce.

And now be pleaſed to form ſeveral Queſtions of this laſt, by varying the Circumſtances of it, and you will be richly rewarded for your Pains.

As I have been ſo full hitherto, I am now obliged to begin to be more brief, otherwiſe there will not be Room in this ſmall Treatiſe to introduce what the Title-Page promiſes;

but the Brevity, in the remaining Part of *Practice*, ſhall be no Loſs to the worthy Youth, as I hope he will find nothing omitted notwithſtanding, which may be wanted in any Station of Life.

CLOTH MEASURE.

The Table to be got by Heart.

2 Inches and a Quarter		1 Nail *(na.)*
4 Na.	———	1 Quarter of a Yard *(qr.)*
4 Qr.	———	1 Yard *(yds.)*
3 Qr.	———	1 Ell Flemiſh *(E. F.)*
5 Qr.	———	1 Ell Engliſh *(Ells.)*

All Sorts of Woollen-Cloths, Silks, Linens, &c. are meaſured by the Yard, Tapeſtry by the Ell *Flemiſh*, and Hollands by the Ell *Engliſh*.

16. Form *three* Sums (and ten of each) in Addition, one containing *Yds. qrs. na.* the ſecond, containing *Ells, qrs. na.* the third, containing *E. F. qrs. na.* and let the Price *per Yard*, or *Ell*, be any where between 1 *s.* and 20 *s.* Then take the three firſt Articles or Lines in each Sum from that Total, and find the Value of the Remainder by the Price. Then multiply each Total by any Number between 2 and 144, and find the Value of this Product alſo. Then divide this laſt Product by any Number that did not produce this Product, and find the Value of this Quotient likewiſe.

WINCHESTER MEASURE.

The Table to be got by Heart.

London,	8 Gallons *(gals.)*	1 Firkin of Ale *(Fir.)*	of Ale.
	2 Fir.	1 Kilderkin *(Kil.)*	
	2 Kil.	1 Barrel *(Bar.)*	
	32 Gals.	1 Barrel	
	9 Gals.	1 Firkin	of Beer.
	36 Gals.	1 Barrel	
	54 Gals. or 1½ Bar.	1 Hogſhead *(hhds.)*	
Moſt Parts of *England.*	8½ Gals. ——	1 Firkin	of Ale, Strong Beer, or Small Beer.
	2 Fir. ——	1 Kilderkin	
	2 Kil. 34 Gals.	1 Barrel	

17. Form three Sums (and ten of each) in Addition, one containing *Bar. Kil. Fir. Gal.* of Ale *(London)*; the ſecond, *Hhds. Bar. Fir. Gals.* of Beer *(London)*; the third, containing *Bar. Kil. Fir. Gals.* of Ale of the Country; and let the Price *per Hhd.* of Beer be any where between 36*s.* and 3*l.* *per Hhd.* and the Price *per Bar.* be any where between 22*s.* and 2*l.* then take the three or four firſt Articles or Lines in each Sum from that Total, and find the Value of the Remainder by the Price. Then multiply each Total by any Number between 2 and 144, and find the Value of this Product alſo. Then divide this laſt Product by any Number that did not produce this Product, and find the Value of this Quotient likewiſe.

WINE, BRANDY, &c. MEASURE.

The Table to be got by Heart.

10 Gallons ——— 1 Anchor of Brandy or Rum.
18 Gallons ——— 1 Runlet *(R.)*
42 Gallons ——— 1 Tierce *(T.)*
63 Gallons ——— 1 Hogſhead *(Hhds.)*
84 Gallons ——— 1 Puncheon *(P.)*
2 Hogſheads ——— 1 Pipe or Butt *(Pp.)*
2 Pipes ——— 1 Tun.

This Meaſure is 51 Parts of 282 leſs than the *Wincheſter* Gallon *or* 4 *Quarts*; that is, the Wine Gallon is 231 Parts, and the *Wincheſter* 282; ſo that the former is about the 5th Part and $\frac{1}{2}$ leſs than the latter Meaſure.

18. Form three Sums (and ten of each) in Addition, one containing *Pps. Hhds. Gals.* at any Price between 29 and 40*l.* *per Pp.* the ſecond, containing *Hhds. Gals. Qts.* at any Price between 12*l.* 10*s.* and 20*l.* *per Hhd.* the third, containing *T. Gals. Qts.* at any Price between 8*l.* 6*s.* 8*d.* and 13*l.* 6*l.* 8*d.* *per* Tierce. Then take the three or four firſt Articles or Lines in each Sum from that Total, and find the Value of the Remainder by the Price. Then multiply each Total by any Number between 2 and 144, and find the Value of this Product alſo. Then divide the laſt Product by any Number that did not produce this Product, and find the Value of this Quotient likewiſe.

DRY MEASURE.

The Table to be got by Heart.

2 Gallons —— ——	1 Peck *(P.)*	of Corn.
4 Pecks or 8 Gallons	1 Buſhel *(B.)*	
4 Buſhels —— ——	1 Sack *(S.)* or Comb	
8 Buſhels —— ——	1 Quarter *(qrs.)*	
40 Buſhels —— ——	1 Cart-Load *(L.)*	
3 Buſhels —— ——	1 Load *(S.)* in *York-ſhire*	
36 Buſhels in *London* ——	1 Chaldron	of Coals.
32 Buſhels in the Country	1 Chaldron	

A Gallon of this Dry Meaſure contains $268\frac{4}{5}$ Parts, whereas the *Wincheſter* Ale Gallon is 282 Parts; ſo that the Dry Meaſure is leſs than the Ale Gallon by 13 Parts and $\frac{1}{5}$; that is, leſs by about the one-and-twentieth Part.

19. Form three Sums (and ten of each) in Addition; one containing *Qrs. S. B.* at any Price between 24*s.* and 2*l.* 16*s.* *per Quar.* of Corn; the ſecond *L. Qrs. B.* at any Price between 6*l.* and 14*l.* *per* Cart-Load of Corn; the third containing *C. B. P.* at any Price between 20*s.* and 40*s.* *per London* Chaldron of Coals. Then take the three or four firſt Articles or Lines in each Sum from that Total, and find the Value of the Remainder by the Price. Then multiply each Total by any Number between 2 and 144, and find the Value of this Product alſo. Then divide this laſt Product by any Number that did not produce this Product, and find the Value of this Quotient likewiſe.

LAND MEASURE.

The Table to be got by Heart.

3 Feet in Length and 3 in Breadth	1 Square Yard *(yd.)*
$30\frac{1}{4}$ Yards in Length and $1\frac{1}{2}$ in Breadth	1 Square Pole or Perch *(P.)*
40 Perches in Length and 1 in Breadth	1 Rood of Land *(R.)*
4 Roods in Length and 1 in Breadth	1 Square Acre *(A.)*

Becauſe it may be of Service to ſome Learners, I ſhall here alſo ſhew them how to eſtimate an Eſtate, when the Price of an Acre is agreed upon between the Buyer and Seller.

20. Form ten Sums in Addition, containing *A. R. P. Yds.* by Survey, at any Price between 8*l.* 5*s.* and 33*l.* *per* Acre. You muſt remember to bring the Yards into Quarters by ×4, and then dividing by 121 Quarters gives the Perches, becauſe 30¼ Yards make one Perch, and when you have caſt up the Perches, divide them by 40, and the Roods by 4. Then take the three or four firſt Articles or Lines in each Sum from the Total, and find the Value of the Remainder by the Price. Then multiply each Total by any Number between 2 and 144, and find the Value of this Product alſo. Then divide this laſt Product by any Number that did not produce this Product, and find the Value of this Quotient likewiſe.

LONG MEASURE.

In *Land-Meaſure* we conſider the Breadth of each Denomination, as 40 Perches in *Length* and one in *Breadth*; but in *Long Meaſure* we conſider only a ſtraight Line in *Length*.

The Table to be got by Heart.

4 Inches ——— ——— ———	1 Hand Horſe Meaſure.
12 Inches ——— ——— ———	1 Foot *(F.)*
3 Feet ——— ——— ———	1 Yard *(Yd.)*
5½ Yards (on ſome Occaſions more)	1 Pole *(P.)* or Rod.
40 Poles, or 220 Yards ———	1 Furlong *(Fur.)*
8 Furlongs, or 1760 Yards ——	1 Meaſured Mile *(M.)*

21. Form two Sums (and ten of each) in Addition; one containing *Yds. F. I.* at any Price (for Flooring meaſured) between 2*s.* 6*d.* and 5*s.* *per Yd.* the ſecond *M. Fur. P. Yds.* at any Price between 131*l.* 10*s.* and 185*l.* *per Mile*, for a new Road, at a Breadth agreed on. Here the Yards, when caſt up, muſt be turned into ½ Yards, and divided by 11, becauſe 5½ Yards, (or 11 Halves) make a Pole or Rod. Then take the three or four firſt Articles or Lines in each Sum from the Total, and find the Value of the Remainder by the Price. Then multiply each Total by any Number between 2 and 144, and find the Value of this Product alſo. Then divide this laſt Product by any Number that did not produce this Product, and find the Value of this Quotient likewiſe.

TIME.

The Table to be got by Heart.

60 Seconds ———— 1 Minute.
60 Minutes ———— 1 Hour.
24 Hours ———— 1 Day.
7 Days ———— 1 Week.
4 Weeks ———— 1 ſhort Month.
13 ſhort Months * }
12 Calendar Months } 1 Year.

April, *June*, *September*, and *November*, have 30 Days each; the reſt have 31, except *Febr.* which has 28, and in Leap Year 29 Days.

* There are generally reckoned 13 ſhort Months 1 Day and 6 Hours to the Year; that is, from the Sun's apparently leaving one particular Point in the Heavens to his returning to it, after his annual Journey to bring about the Seaſons of Summer and Winter; but this is too much, for he only takes up 365 Days, (28 Days in a ſhort Month × 13*m* + 1*d*) 5 Hours, 48 Minutes, and 55 Seconds; ſo that 6 Hours is too much by 11 Minutes and 5 Seconds every Year, or 44 Minutes and 20 Seconds in four Years, when we call it *Leap Year*, becauſe the 6 odd Days in 4 Years make a Day more in *February*; by this Means we reckon wrong, and ſhall be obliged, in Ages to come, to drop 11 Days, as was done in 1752.

22. Form ten Sums in Addition, containing *M.* (ſhort Months) *W. D.* ſuppoſing ten Men of War in each Sum, the Number of Months' Pay multiplied by the Crew; for Inſtance, 17 Months Pay × 500 Men = 8500 Months' Pay due to the whole Crew of one Ship, at any Wages due to each Man, between 18*s.* and 36*s.* *per* Month. Then take the three or four firſt Articles or Lines in each Sum from that Total, and find the Value of the Remainder by the Price. Then multiply each Total by any Number between 2 and 144, and find the Value of this Product alſo. Then divide the laſt Product by any Number that did not produce this Product, and find the Value of this Quotient likewiſe.

COMMISSION, *or* BROKERAGE.

23. *Sold Goods to the Amount of* 5765 l. 17 s. 6 d. *for which I am to pay the Perſon who tranſacted the Buſineſs for me at* $\frac{1}{2}$ per Cent. *that is,* 10 s. *for every* 100 l. *How much does the Commiſſion, Brokerage, or Proviſion amount to?* The Proportion is, as 100*l.* : 1*l.* : : any Sum : the Commiſſion; that

is $\frac{5765l.\ 17s.\ 6d.}{100}$ and then for ½ or 10*s.* the Half, is the Anſwer. I ſhall not make the Youth ſo bad a Compliment to do this Sum for him, as he is ſuppoſed perfect in what was ſhewn him in Intereſt. Now, if the Commiſſion is ⅛ (2*s.* 6*s.*) ¼, ⅜, ⅝, *&c.* or any Sum under 20*s.* the Learner knows it is only taking the Parts accordingly; and if the Commiſſion is above 20*s.* he may conſider it as Intereſt, and take his Meaſures accordingly. I requeſt he will alter this Sum, and find the Commiſſion from ⅛ to 2½ *per Cent.*

DISCOUNT.

24. *Bought Goods to the Amount of* 765l. 17s. 6d. *to pay in* 12 *Months; but Money coming in the ſame Day, the Seller diſcounts the Debt with me at* 5 per Cent. per Annum: *What is the preſent Money I muſt pay him?* Diſcount is often confounded with Intereſt, from which it differs, in that the Seller has only a Right to ſuch a preſent Sum as will, with its Intereſt, make up his Money at the End of the Time; and this the Buyer cannot think any ways unreaſonable; but if he deducts the Intereſt from the Debt, he does not allow the Seller a ſufficient Sum: Thus, ſuppoſe the Debt only 200*l.* here the Buyer deducting 10*l.* as the Intereſt, pays down 190*l.* which is too little by 9*s.* 6¼*d.* and this Difference, in very large Sums of Money, would amount to ſomething conſiderable. Again, let the Debt be only 105*l.* if the Buyer pays downs 100*l.* he could only make 105*l.* of it at the End of the Time, and the Seller, by being paid 100*l.* preſent Money, and putting it to Intereſt, will have his full Demand of the Buyer at the Expiration of the Year. Hence then, there is Juſtice done on both Sides, and we have two Numbers 105*l.* : 100*l.* : : that is, if 105*l.* twelve Months hence at 5 *per Cent.* is worth 100*l.* now, what will any Debt, twelve Months hence, be worth?

And therefore the Youth ſees the Reaſon of the Rule; *Find the Intereſt of* 100l. *for the Time, and add Principal and Intereſt together, for the firſt Number in the Stating, then* 100l. *will be the ſecond, and the Debt the third, to find the fourth: If the Diſcount is required, it is only taking the preſent Money or Worth from the Debt, and you have this Anſwer alſo.* And now let there be eight Sums formed of 765*l.* 17*s.* 6*d.* when properly altered, and let the Intereſt be from 3 to 5 *per Cent. per Annum,* making the Intereſt ¼ *per Cent.* more in every Sum, and let the Time be between 1 and 18 Months, and always find the Diſcount by having the preſent Money or Worth.

25. *Bought Goods amounting to* 765l. 17s. 6d. *to pay in two four Months, (that is* ½ *in* 4 *Months, and* ½ *in* 8 *Months) but diſcounted at* 5 per Cent. per Annum: *What is the preſent Money and Diſcount?* All the Difference between this and the laſt Queſtion is, that the Youth muſt divide the Debt into two equal Sums, and then find the Anſwers by the Rule, as two different Debts due at four and eight Months. If the Times of Payment were *two three Months*, the Meaning is ½ the Debt in 3 Months, and the other ½ in 6 Months; ſo if the Times were *three two Months*, the Meaning is ⅓ of the Debt is to be paid in 2 Months, ⅓ in 4 Months, and ⅓ in 6 Months; ſo if the Times were *three four Months*, this is ⅓ in 4 Months, ⅓ in 8 Months, and ⅓ in 12 Months. In all theſe Varieties, or others that may ariſe, it is only conſidering each Time when a Debt is to be paid, and finding the preſent Money by the Rule, and adding the particular preſent Worth together, and this Sum diſcharges the whole Debt. And here again, of 765*l.* 17*s.* 6*d.* due at different Times, and properly altered, form ten, twenty, or thirty Sums, at 3, *&c.* Intereſt.

EQUATION *of* PAYMENTS.

The Meaning of this Rule is to find out a fixed Time for the Payment of ſeveral Debts, for Goods bought, and due by Agreement at different Times, as mentioned before in Diſcount. The uſual Way of doing this is near enough the Truth for commercial Concerns; but the Principles of the true Calculation, when very large Sums of Money are to be equated, lie not at preſent within the Learner's Reach. *Let the Debt be* 150*l.* *to be paid in three four Months; required to reduce them to one Time of Payment.* The Rule is, multiply each Debt by its Time, and add them together, and divide the Total by the whole Debt: Or, ſhorter for Money, call the *Sum* of each Debt, multiplied into its Time *S*. then as 150*l.* : 1 Month : : S. to the equated Time; that is to ſay, $\frac{50\times4+50\times8+50\times12}{150}$ = 8 Months, the Anſwer.

26. *Bought different Goods to the Value of* 976*l.* 18*s.* 6*d.* *of which I am to pay* 200*l.* *in* 3 *Months,* 200*l.* *in five Months,* 300*l.* *in* 7 *Months, and the Remainder in* 9 *Months; but by Agreement the whole is to be paid at one equated Time: Required*

this Time of Payment. The Learner will do well to vary this Queſtion in all its Circumſtances, and form of it ſeveral Sums, and try the Iſſue of the Anſwers as follows: It was ſaid before that 8 Months is the equated Time for the 150*l.* which was to be paid in three equal Sums at three different Times; but you may reaſon thus, the Intereſt of the firſt 50*l.* for 4 Months is 16*s.* 8*d.* the ſecond 50*l.* for 8 Months is 1*l.* 13*s.* 4*d.* and the Intereſt of the third 50*l.* for 12 Months is 2*l.* 10*s.* 0*d.* Now theſe Principals and their Intereſts, being all added together, make 155*l.* and the 150*l.* Principal and its Intereſt during the equated Time 8 Months, make alſo 155*l.* And thus you may procced in all the Queſtions formed from the 26th.

LOSS *and* GAIN.

It is very requiſite the Youth ſhould be well acquainted with the Method of knowing what his Gains are *per Cent.* on the Goods dealt in, otherwiſe it will be difficult for a Perſon to form his Judgment, whether he ſhould not do better to employ his Fortune at Intereſt, than to be at high Charges in Trade for little Profit.

27. *Bought Goods to the Amount of* 695*l. for which I paid ready Money*; *and* 12 *Months after ſold the ſame for* 745*l. with which I gave* 6 *Months Credit: Whether do I gain or loſe, when compared with putting my Mouey out at* 4 per Cent. *Intereſt?* Firſt find the Intereſt of 695*l.* at 4 *per Cent.* for 18 Months: Then $745-695=50l.$ Gain; and as $695 : 50 :: 100 : x$ the Gain *per Cent.* in 18 Months, and 18 Months : x :: 12 Months : the Gain *per Cent. per Annum.* When all this is done you will find that I have made 4*l.* 16*s.* *per Cent. per Annum* of my Money; and though I gained 50*l.* by the Bargain, that the bare Intereſt of my Money amounts to 41*l.* 14*s.* ſo in Fact I have but 8*l.* 6*s.* 0*d.* for all my Trouble, Charges, and Hazard in truſting a Perſon with my Money. Here vary the Circumſtances of this Queſtion, and form ten other Sums of it.

28. *But having the Command of the Price, and conſidering all Contingencies, and that I have had the Goods by me* 12 *Months, and that I muſt give* 6 *Months Credit with the Goods, I reſolve to make* 20 per Cent. per Annum *Profit: What muſt I ſell them for?* Here 20 *per Cent. per Annum* is 30*l.* in 18 Months, (for 12 *m* : 20*l.* :: 18 *m* : 30*l.*) and then we have 100*l.* : 30*l.* :: 695*l.* : 208*l.* 10*s.* Gain; and therefore I ſell the Goods for 695*l.*+208*l.* 10*s.*=903*l.* 10*s.*——And here

again form ſeveral Sums of this Queſtion, and vary every Circumſtance of it.

29. *Bought Goods, amounting to* 185 l. *at* 4s. *per Yard,* &c. *for* 6 *Months Credit, and ſold them at* 4s. 3d. *for* 12 *Months Credit: What is the Gain* per Cent. per Annum? Here you are to conſider you were 6 Months out of your Money, at 4 *per Cent. per Annum,* and therefore it was not all Gain, which it would have been, if you had had 12 Months Credit for the Goods. Now the Gain *per Cent.* is as 4*s.* : 3*d.* : : 100*l.* or $\frac{48}{3}$ (that is, 3*d.* Gain) $=$ 16th Part, and $\frac{100l.}{16\,pt.} = 6\frac{1}{4}\,l.$ *per Annum*; if you had paid ready Money for the Goods, the Advantage by Trade would be only 2¼ *per Cent. per Annum,* as you could have made 4 *per Cent.* Intereſt of the Money. But as you were only 6 Months out of your Money, the Gain in Trade was 6¼ *per Cent.* leſs 2 *per Cent.* for the Intereſt of your Money for 6 Months, leaving you the Advantage of 4¼ *per Cent. per Annum* by Trade. Hence in all ſuch Queſtions, in order to know the Gain on the Goods, *divide the prime Coſt by the Gain* per *Yard,* &c. *and then divide* 100*l.* by that Quotient, for the Gain *per Cent.* but to ſave the Reader ſome Trouble in this Particular, I have here inſerted a Table in the Margin, by which you may ſee the Gain *per Cent.* when you know what Part the Gain *per* Yard, *&c.* is of the prime Coſt. Thus, 4*s.* prime Coſt is 48*d.* and the Gain 3*a.* is the 16th Part; therefore look for the 16th Part in the Table, and you have 6¼ (or 6*l.* 5*s.*) *per Cent.* Gain. So 25*d.* prime Coſt and 1*d.* Gain is the 25th Part, againſt which you have 4 *per Cent.* Gain; and ſo on for any Gain higher or lower. It will be of great Service to the Learner, if he will form at leaſt ten Sums of this Queſtion; that is, mention what Credit he had with the Goods, (or whether bought for ready Money) and what Time he gives the Buyer; for unleſs he does this and computes the Intereſt

Table of Gain per Cent.

	£.	*s.*	*d.*
½ *Part is*	50	0	0
3d ——	33	3	4
4th ——	25	0	0
5th ——	20	0	0
6th ——	16	13	4
7th ——	14	5	8
8th ——	12	10	0
9th ——	11	2	2
10th ——	10	0	0
11th ——	9	1	9
12th ——	8	6	8
13th ——	7	13	10
14th ——	7	2	10
15th ——	6	13	4
16th ——	6	5	0
17th ——	5	17	7
18th ——	5	11	1
19th ——	5	5	3
20th ——	5	0	0
21ſt ——	4	15	2
22d ——	4	10	1
23d ——	4	6	11
24th ——	4	3	4
25th ——	4	0	0

tereſt of his own Money, he cannot be ſaid to know the real Advantages in Trade. I muſt likewiſe requeſt he will do them three Ways; that is, firſt by the Proportion, as (for Inſtance) 4*s.* : 3*d.* : : 100*l.* and, ſecondly, $\frac{48\,d.}{3\,d.}$=16th, and $\frac{100}{16}$=6*l.* 5*s.* *per Cent.* and, thirdly, $\frac{48\,d.}{3\,d.}$=16th, and then uſing the Table.

But if the Part of the prime Coſt ſhould be higher than the 25th, the Learner may ſupply that Defect by making Uſe only of either the firſt or ſecond Way. Yet if it is an even Parts, ſuppoſe 26, it will be Half the *per Cent.* of the 13th Part; if 28, it is Half the *per Cent.* of the 14th Part; if 30, it is Half of the 15th Part; and ſo on, for any other even Part above theſe. What has been ſaid of Gain, is equally true in the Manner of computing the Loſs, with this Difference, that if you loſe, ſuppoſe 8 *per Cent. per Annum* by the Goods, this is not all, as you have loſt 4 *per Cent. per Annum* in the Intereſt of your Money; ſo that as I ſhould at Intereſt have 104 back again, you will then, by Trade, loſe 12 *per Cent. per Annum* in that particular Article, for 100—8=92, and 104—92=12. But we will illuſtrate this by an Example.

30. *Bought Goods, for ready Money, amounting to* 100l. *at* 4s. per *Yard,* &c. *and* 12 *Months after ſold the ſame for ready Money at* 3s. 6d. per *Yard,* &c. *What have I loſt* per Cent. per Annum? Here, in the firſt Place, I have loſt 4*l.* in the Intereſt, and as 4*s.* : 6*d.* : : 100*l.* : 12*l.* 10*s.* or 8 Sixpences divided by 1, the Loſs *per* Yard, *&c.* is the 8th Part, and then $\frac{100}{8}$=12*l.* 10*s.* or, by the Table, againſt the 8th Part we have 12*l.* 10*s.* Therefore I have loſt 16*l.* 10*s.* *per Cent. per Annum*; for 104*l.*—87*l.* 10*s.*=16*l.* 10*s.* my 100*l.* laid out is reduced to 87*l.* 10*s.* which, with the ſame Gain, would have been 112*l.* 10*s.* and makes a Difference to me of 25*l.* And now, I wiſh the worthy Youth may never experience ſuch Loſſes in Trade, if he will form ten, twenty, or thirty Sums from this diſmal Queſtion, and conſider what Time he has had the Goods by him, ſince he paid for them, and will vary, as before, every Circumſtance, making the Value of the Goods 1875*l.* 17*s.* 6*d.* but ſo altered as to differ from any Cyphering Book in the School.

BARTER.

In the Courſe of Trade a Perſon has frequent Opportunities of buying Goods with Goods; that is, *A* has, or ſhould have, Occaſion for Goods which *B* has, and *B* alſo wants Goods which *A* has; and therefore both are willing to part with what they have at a Market-Price, or ſhould do ſo, and then there is nothing but Hononr in *Barter*, *Truck*, or *Exchange* of Goods.

31. A *has* 48 *C.* 3 *qrs.* 27 *lbs. neat Weight of Indigo, which is worth* 4s. 6d. per *lb.* B *has a Demand for it, and agrees to give* A, *who wants to export Medley Cloths at* 14s. 6d. per *Yard: How many Yards of Cloth muſt be given in* Barter, Truck, *or* Exchange? Common Senſe tells the Youth that he muſt find the Value of *A*'s Indigo; call this Amount x; then we have 14*s.* 6*d.* : 1 *yd.* : : x to the Number of Yards the whole Money will buy. If *B* expects half Money and half Indigo for his Cloth, which is not uncommon when Woollen Goods are trucked for any other Sort of Merchandize, then the Proportion is 14*s.* 6*d.* : 1 *yd.* : : $\frac{x}{2}$ to the Number of Yards of Cloth: Here $\frac{x}{2}$ is Half *A*'s Goods. Form ten or twenty Sums of this Queſtion.

32. A *in* London *ſends* B *in* Portugal, *Bays, Long-Ells, Cloth,* &c. *to the Amount of* 1895l. 17s. 6d. *including all Charges when* B *receives them. By Agreement* B *was to anſwer for Half the Value in his own Bill on* London, *and was to ſhip on board, for* A's *Account, Pipes of Wine for the other Half of the Value, at* 12l. 10s. *per Pipe: How many Pipes of Wine ought* A *to expect?* Here we ſhall ſuppoſe both *A* and *B* are to make the Barter on the prime Coſt of the reſpective Goods, being content with their bare Commiſſion only. In order to find the Quantity of Wine *A* muſt have, we have 12*l.* 10*s.* : 1 *Pipe* : : $\frac{x}{2}$ to the Quantity of Wine. Here x ſtands for the Value of *A*'s Goods, and $\frac{x}{2}$ is the Half of it. The Youth will not repent of his Labour, if he, of this uſeful Queſtion, will form ten or twenty others, in order to make the Manner of working them quite familiar to himſelf.

SINGLE FELLOWSHIP, *or* COMPANY.

This Rule of Arithmetic is of Uſe, when two or more Merchants put in their reſpective Sums of Money or Goods, previouſly aſcertained, to be employed in Trade for their mutual Advantage.

33. A *in* London *with* 5765l. B *in* Portugal *with* 6878l. *and* C *in* Cadiz *with* 3985l. *enter into Partnerſhip to employ this Capital in Trade, and after* 18 *Months the Accounts are ſettled, and they find they have gained* 1739l. 17s. 6d. *and now intend to withdraw their reſpective Capitals: What is each Merchant's Share of the Gain, and what is his Advantage in Trade, when compared with employing his Money at* 4 per Cent. per Annum *Intereſt?* You know all their Money acquired all the Gain, and that *C* cannot expect ſo much as either *A* or *B*; therefore your Proportions are (call the Gain *x*) as $A+B+C : x :: A$'s Capital to his Gain; and likewiſe as $A+B+C : x :: B$'s Capital to his Gain; and alſo as $A+B+C : x :: C$'s Capital to his Gain; and therefore all their Gains added together muſt make the whole Gain, provided there is no Error in any of the Operations, otherwiſe the Nature of Proportion muſt be deſtroyed; which it never can be.

34. *The ſame Merchants being concerned as before, only* D *puts in* 7850l. *and* A *in* London *has all the Trouble in tranſacting the Buſineſs, for which he is to have* $\frac{1}{2}$ per Cent. *for Commiſſion or Management: What is each Perſon to draw out of the whole Stock and Gain upon the Diſſolution of the Partnerſhip, and what is his Gain when compared with Intereſt at* 4 per Cent. per Annum? Here is another Partner now concerned in this Trade, and therefore there will be four Statings, as *D*'s Gain muſt be now conſidered. But the Youth muſt be aware the total Gain will not be 1739*l.* 17*s.* 6*d.* becauſe, by Agreement, *A* is to have $\frac{1}{2}$ *per Cent.* Commiſſion for managing the Capitals of the other Partners, which Commiſſion is to be firſt deducted from the total Gain. And in like Manner, whether there be a total Loſs or total Gain, you may work theſe Sums in Fellowſhip, were there never ſo many Partners; but as this would be very tedious, the ſhorteſt Way * here would be to compute what the Gain (ſuppoſe) to each, would be in the Pound, and then each Stating is turned into *a Sum in Practice*; that is, as the whole Capital is to the whole Gain, ſo is 1*l.* to the Gain of 1*l.* and as $1l. : g :: A$'s, *&c.* Capital to his Gain; (here *g* is the Gain

of 1 *l.*) that is, *in Practice*, *A*'s, *&c.* Capital at *g*; therefore multiply the Pounds in each Perſon's Capital by the Shillings in *g*, and take the Parts accordingly.

The diligent Youth is deſired to form ten Sums of Queſtion 33, then of Queſtion 34, and ten of this Method.*

COMPOSITION.

The Meaning of this is to proportion what any Creditor may have in the Pound, when a Debtor is ſo unfortunate as not to be able to pay his Debts.

35. *A Merchant failing owes* 35,683l. 18s. 9d. *to divers Perſons, and, on making up his Accounts, it appears he has* 16,578l. 19s. 6d. *to go among his Creditors: What is this in the Pound?* That is, as he cannot pay every Perſon 20*s.* in the Pound, which is the full Demand, What between Nothing and 20*s.* can he pay? Here, as in Single Fellowſhip, all the Money he owes expects all he has; and therefore as 35,683*l.* 18*s.* 9*d.* all the Debts, is to 16,578*l.* 19*s.* 6*d.* all he has to pay with, ſo is 20*s.* to what it will be in the Pound. Or, in another Expreſſion, If all the Debts decreaſe to all his Effects, what will 20*s.* decreaſe to. Here the firſt and third Numbers are of the ſame Nature (Debts), and the ſecond and fourth will be the Effects to pay with. Now ſuppoſe one Creditor's Demand upon this unfortunate Merchant is 159*l.* 17*s.* 6*d.* and there will be *x* in the Pound, the Proportion is 20*s.* : *x* : : 159*l.* 17*s.* 6*d.* to this Creditor's Dividend; or, *by Practice*, 159*l.* 17*s.* 6*d.* at *x* (ſuppoſe 9*s.* 9*d.* in the Pound); then 159*l.*×9*s.* then 6*d.* is the $\frac{1}{2}$ of 159*l.* 3*d.* is $\frac{1}{2}$ of what 6*d.* comes to; and for the odd 17*s.* 6*d.* 10*s.* is the $\frac{1}{2}$ of 9*s.* 9*d.* 5*s.* is the $\frac{1}{2}$ of what 10*s.* comes to, and 2*s.* 6*d.* is the $\frac{1}{2}$ of what 5*s.* comes to. In the very ſame Manner, when there are Legacies, and the Effects do not reach to pay 20*s.* in the Pound, each Demand is to be computed. And here again I muſt beg the Youth will form ten or twenty Sums of this Queſtion, in order to make himſelf perfect in theſe neceſſary, tho' unhappy, Calculations.

COMPOUND FELLOWSHIP.

The Meaning of this Rule in Arithmetic is, that ſeveral Merchants, or others, engage to try the Event of Trade for a certain Time for the Benefit of all; but each for a Time he chuſes, or thinks moſt convenient, to employ his Money with a Proſpect of more Gain than he can make of his Capital at Intereſt.

36. Three Merchants, *A*, *B*, and *C*, engage, ſuppoſe, in the Courſe of Exchange, to diſcount Bills, or any other Buſineſs wherein the Accounts can be finally cloſed in 12 Months certain. But tho' *A* immediately employs his 30,000*l.* yet it does not ſuit *B* to pay in his 30,000*l.* till 3 Months after the Commencement of this mutual Engagement, nor can *C* ſpare his 30,000 till 6 Months after. On adjuſting the Accounts there is a neat Gain of 3600*l.* and the Queſtion is, How muſt the Gain be proportioned among the Partners, as each has a Right during the whole 12 Months Tranſaction? And what is the Advantage in Trade more than employing their Money at 4 *per Cent. per Annum* Intereſt? The Rule of Compound Fellowſhip is, *Multiply every Partner's Money by the Time during which it is employed, and then ſay, as the Sum of theſe Products* (call it *P*) *is to the whole Gain or Loſs,* (call it *G*) *ſo is each Product* (call it *p*) *to the Share of each Partner* (call it *g*); that is, as $P : G :: p : g$. Here the firſt and third Numbers are both of one Name; (that is, Shares multiplied into Time), and ſo are the ſecond and fourth of one Name. In theſe Caſes alſo, the Work will be ſhorter, if you find what the Gain is *per* Pound for each Partner's Capital, when multiplied by its own Time, and take the Parts accordingly. The ſame Rule and Method hold good for any Number of Merchants, let the reſpective Sums or Times be never ſo different; only it muſt be obſerved, all the Partners are bound to run the Riſk of the whole Time agreed upon, however late ſome of them may pay in their Money. Whoever engages in the World as a Merchant, may probably have Occaſion to put this Part of Arithmetic in Practice; and therefore we recommend it to the Youth as ſuch, and hope he will form many Queſtions from this, that he may ſo effectually fix the Manner of Operation on his Mind, as not to be obliged to learn Compound Fellowſhip, or *Company with Time,* when he is a Partner.

EXCHANGE.

THIS Part of Arithmetic teaches us how to caſt up Bills of Exchange; that is, the mutual Intercourſe of Payments by written Orders; for if you deſire your Correſpondent abroad to buy Goods for you, he either draws a Bill on you, or you ſend him a Bill to receive the Money himſelf: In all this Buſineſs of computing the Value, *the Rule of Three* is eminently concerned, or a ſhorter Method by *Practice.* It muſt be obſerved, that there is always underſtood to be an

Equality in Point of Value between foreign Money and our own, called *Par*; but, from divers Circumſtances, the Courſe of Exchange, or the preſent Price of thoſe foreign Coins, real or imaginary, riſe and fall; but, for the Learner's Inſtruction, or Introduction to the Nature of Exchange, we ſhall inſert a Table.

		Value.		
Spain, &c.	Dollars	0	4	8
Portugal	Mill-Rees	0	6	8
France	Crowns	0	4	6
Venice	Ducats	0	4	4
Florence	Ducatoons	0	4	5
Frankfort	Florins	0	4	11
Holland, &c.	*Flem.* £.	1	14	6
Vienna	Rix-Dollars	0	4	10
Ruſſia	Ruble	0	4	8
Dublin, 100*l.*		108	6	8
Weſt-Indies, 100*l.*		144	9	7

37. *My Correſpondent in* Cadiz *has drawn on me for* 2785 *Dollars, at* 4s. 8d. per *Dollar: What Sterling Money muſt I pay?* This Buſineſs needs no Direction, and therefore I ſhall not affront the Learner with doing it for him. He will be pleaſed to remember it will be neceſſary for him to form a Sum from each Article in the Table, by ſuppoſing a Bill drawn upon him from each Place, in the Manner of the Example of this 37th Queſtion; and then draw on each Place a Sum of our Money, to be anſwered there in their own Courſe of Exchange; that is, to find how much will be paid abroad for the Money you receive here for your Bill on your Correſpondent. In the two laſt Articles of the Table, the Meaning is, that 100*l.* here is worth 108*l.* 6*s.* 8*d.* in *Dublin*; and 100*l.* here is worth 144*l.* 9*s.* 7*d.* in the *Weſt-Indies*. As to the Riſe and Fall of the Exchange between one Country and another, whereby there may be either Loſs or Gain in negotiating Bills, that ſhall be ſhewn in a more proper Place.

INVERSE PROPORTION,

AND THE

The DOUBLE RULE *of* THREE.

ALL Queſtions which come under the Denomination of the *Double Rule of Three*, may be anſwered by two ſingle Statings: However, after premiſing what is meant by *Inverſe Proportion*, I ſhall propoſe an eaſy Method of placing the five Numbers, and anſwering the Queſtions by two ſingle Statings,

Statings; then in a ſhorter Way, which will oblige the Learner to exerciſe his Ingenuity, in diſcovering whether the Proportion is a Deviation *(Inverſe)* from what he has learned, and afterwards exhibit the mechanical Form of placing the Numbers.

1. *Of Inverſe Proportion.*

The Proportions hitherto in this Treatiſe are called *Direct*; but if the third Number is *greater* than the firſt, and requires, from the Meaning of the Queſtion, that the fourth ſhould be *leſs* than the ſecond; or if the third Number is *leſs* than the firſt, and requires, from the Nature of the Queſtion, that the fourth ſhould be *greater* than the ſecond; in either of theſe Caſes the Proportion is called *Indirect*, or *Inverſe*, and then the Rule is, *Multiply the firſt and ſecond Numbers together, and divide by the third:* Thus, as 4 : 3 : : 1 : 12, if the third Number requires more, for $\frac{4\times3}{1}=12$; that is, if 4 Men are 3 Days in finiſhing a Wall, 1 Man will be 12 Days about it. The Proportion here, from the Senſe, is, that the *fourth* muſt exceed (or be leſs than) the *ſecond*, as much as the *firſt* exceeds (or is leſs than) the *third*; for 12 is 4 Times more than 3, as 4 is 4 Times more than 1: For, ſurely, 1 Man will require 4 Times the Days 4 Men require. Now, if we want a *greater* Quotient, or fourth Number (12 Days), we muſt have a leſs Diviſor than the firſt Number is; that is, $\frac{4\times3}{1}=12$; but if we ſhould aſk, *How long 6 Men will be about it?* Here we want a *leſs* Quotient (for, ſurely, 6 Men will do it in leſs Time) and a greater Diviſor than the firſt Number, for 4 *m* : 3 *d* : : 6 *m* : 2 *d*. or $\frac{4\times3}{6}=2$ Days; and therefore the Youth muſt conſider, whether he wants a *leſs* or *greater* Quotient than he can have by dividing by the *firſt* Number. And that they are in exact Proportion, he may be convinced thus; as 2 *d* : 3 *d* : : 4 *m* : 6 *m*; for $\frac{3\times4}{2}=$ 6 Men, and $2\times6=3\times4$; that is, as before, the *fourth* Number is as much leſs than the *ſecond*, as the *firſt* is leſs than the *third*.

38. *It is computed that* 1175 *Workmen of all Sorts will build a magnificent Palace in* 7 *Years, (*365 *Days to the Year): but it is more agreeable to have the Building finiſhed in* 3 *Years and* 25 *Days: What Alteration muſt there be in the Number of Artificers?*

Artificers? Bring both Times into Days, (365×7, and $365\times3+25$) and making the Men the ſecond Number thus, (2555 *d.*——1175 *m.*——1120 *d.*); conſider whether you want a *leſs* or *greater Quotient* (that is, a fourth Number or Anſwer) than you can have by dividing by the firſt. Form ſeveral Sums of this Queſtion, and let Half of them conſiſt of a leſs Time to finiſh the Work, and Half of the Sums to conſiſt of a longer Time to perform the ſame, and let the four Numbers of each Sum be thrown into the *Inverſe* and *Direct* Proportion, as above ſhewn, in order to enlarge the Conception, and ground the Youth in this Part of Arithmetic. By this Means it will be extremely eaſy for him to anſwer any Queſtion in *Inverſe* Proportion, as they all depend upon conſidering whether *more* requires *leſs*, or *leſs* requires *more*, as is explained above.

2. *Of the Double Rule of Three.*

There are five Numbers always given in this Rule to find a ſixth, and, by two ſingle Statings, the Judgment of the Learner is no otherwiſe ſtrained than to diſcover, from the Nature of the Queſtion, whether Inverſe or Direct Proportion muſt take Place to bring out a proper Anſwer. I ſhall here give the firſt Queſtion in Simple Intereſt, that the Learner may more clearly ſee the Nature of the *Double Rule of Three*, when the Proportions are *Direct*.

39. *What is the Intereſt of* 250l. *for* 3 *Months, at* 4 per Cent. per Annum? Here you have five Numbers, 100*l.* 12 Months, 4 *per Cent.* 250*l.* 3 Months, to find *Intereſt*; and therefore in this, and in all other Caſes, ſet the Number of the ſame Name with that you want to know in the Middle, and the two Numbers belonging to it one under another in the firſt Place, and let the other two correſpond to them in the third Place; thus, 100*l.* —— 4 *Intereſt.* —— 250*l.*
12 *m.* — ——— 3 *m.*
Here you perceive the firſt and third, in both Lines, are of the ſame Name. Now, conſider, the third Number 250*l.* is greater than its firſt (100) and requires the Intereſt in 12 Months ſhould be greater than the Intereſt of 100*l.* in the ſame Time, and therefore, as the Proportion is *Direct*, $\frac{250\times4}{100}=10$ Intereſt in 12 Months. Then if in 12 Months 250*l.* will require 10*l.* Intereſt, what will 3 Months require? Surely, leſs Intereſt, becauſe the Time is leſs; and therefore as

this Proportion is *Direct* alſo, $\frac{10\times 3}{12}=$ 2*l.* 10*s.* the Anſwer. Now, as *another Method*, multiply each Principal by its own Time, and you have a ſingle Stating and *Direct*, becauſe the other two are both *Direct*; that is, as 100 × 12 : 4*l.* : : 250 × 3 to the Intereſt required: For in 1200 : 4 : : 750, the firſt and third Numbers are both of one Name; that is, Principal multiplied into its own Time: Then the Anſwer will be $\frac{750\times 4}{1200}=$ 2*l.* 10*s.* as before, becauſe 1200 × 2*l.* 10*s.* = 4 × 750, and therefore Proportionals.

Now, from this Stating 1200 : 4 : : 750, we have the Reaſon of the Rule, of the uſual mechanical Method of placing the five Numbers; that is, the Cauſe (100) of Gain or Loſs muſt ſtand firſt; the Time (12 Months) in which the Cauſe produces the Gain, muſt ſtand in the ſecond Place, and the other Number (4*l.*) which belongs to the other two (100 and 12) ſtand in the third Place; then ſet the other two (250*l.* and 3 Months) under thoſe of the ſame Name, and you will have 100*l.* —— 12 *Months* —— 4*l.* Here
250*l.* —— 3 *Months*
when the Blank falls under the third Place, the Rule is, multiply the three laſt Numbers together (4 × 250 × 3) for a *Dividend*, and the two firſt together (100 × 12) for a *Diviſor*; that is, $\frac{4\times 250\times 3}{100\times 12}=$ 2*l.* 10*s.* the Anſwer, the ſame Proportionals as in the other Method, with this Difference, that young Heads are too often puzzled to diſtinguiſh the *Cauſe*, *&c.* in order to know how to ſtate the Numbers in the laſt Form.

You may obſerve there is the ſame Reaſon runs through all the three Forms; the *firſt*, with great Eaſe, enables you to place the five Numbers, in order for you to give the Anſwer, by uſing two ſingle Statings; the *ſecond* ſhews you that you muſt, when the two Statings are found to be *Direct*, multiply the two firſt Numbers in both Statings for a *Diviſor*, and the two third Numbers in both Statings together, which again is multiplied into the ſecond for a *Dividend*, and thereby you have the Anſwer; the *third Form* you plainly perceive is only the Theory of the *ſecond*, with the Difference of placing the Numbers, but in ſuch a Manner, that the Principal and Time of the *ſecond Method* become the *Diviſor* in the third, and the other three Numbers in the *ſecond* are called the *Dividend* in the *third*.

39. *If 90 Buſhels of Oats will ſerve 80 Horſes 120 Days, How long will 240 Buſhels ſerve 160 Horſes?* Here the Number ſought is to conſiſt of Days; therefore place the Number of the ſame Name (120 Days) in the Middle, and the two Numbers belonging to it in the firſt Place, but one under another, and the other two, belonging to the Number wanted, in the third Place, but one under another, ſo as to be of the ſame Name with the two Numbers ſtanding in the firſt Place.

Thus, 80 *Horſes* —— 120 *Days* —— 160 *Horſes*,
90 *Buſhels* —— —— 240 *Buſhels*,

to be read thus, If 80 Horſes are maintained 120 Days (with 90 Buſhels of Oats) how many Days will 160 Horſes be kept with the ſame Quantity of Oats? Surely they will eat up the Corn in leſs Time; and therefore the Proportion is *Inverſe*; that is, $\frac{80\times120}{160}=60$ Days, the Anſwer to the firſt Stating. Then ſay, If 90 Buſhels are conſumed (by 160 Horſes) in 60 Days, in how many Days will 240 Buſhels be eat up? Here, ſurely, 160 Horſes will require more than the ſecond Number (60 Days), and therefore the Proportion is *Direct*; that is, $\frac{240\times60}{90}=160$ Days. Thus much for the *firſt* Method by reaſoning on the two ſingle Statings, without any Help from the Double Rule of Three.

Now, for the *ſecond Method*, we muſt reaſon thus; if both the Statings were *Direct*, we muſt multiply the two firſt Numbers in each Line together for a *Diviſor*, and the two laſt in each Line and the middle Term for a Dividend: But becauſe the Proportion in the firſt Stating is *Inverſe*, and we want a leſs Quotient than the middle Term, therefore, to get a greater Diviſor and leſs Dividend, we muſt multiply the firſt and third Terms in each Line croſs-ways; that is, 90 × 160 for a *Diviſor*, and 240 × 80 × 120 for a Dividend, and then we have 160 Days: For as 14,400 : 120 : : 19,200 : 160; becauſe the firſt and third are both of one Name, (Buſhels multiplied into Horſes) and the ſecond and fourth are of one Name, that is, Days; and becauſe the Product of the firſt and fourth is equal to the Product of the ſecond and third; and conſequently the Numbers are Proportionals. But leſt the Learner ſhould loſe Sight of theſe noble Properties of Numbers, I ſhall here ſtate the ſecond Method, to make the better Impreſſion on his Mind.

80 Horſes —— 120 Days —— 160 Horſes
90 Buſhels —— —— 240 Buſhels

14,400 : 120 : : 19,200
120

144,00) 23040,00
160 864
0

If the Learner, in ſtating the five Numbers, ſhould happen to find the *Inverſe* Proportion in the lower Line, and the *Direct* in the upper, he may change the Order, that the *Inverſe* Rule may be in the upper, as in this laſt Example before him. But ſuppoſe he had ſtated the Queſtion as follows, and chuſes to work it and not to change the Order, to make the Numbers ſtand as in the laſt: Then

90 *Buſhels* —— 120 *Days* —— 240 *Buſhels*
80 *Horſes* —— —— 160 *Horſes*

Here the *Inverſe* Proportion is in the loweſt Line; and now conſider, when you have multiplied acroſs, as before, you are to multiply the firſt and ſecond Numbers together, and divide by the third, for the Solution of the Queſtion; that is, $\frac{19,200\times 120}{14,400}=$ 160 Days, the Anſwer required, which you may be ſure is true, as the Numbers are in Proportion; for as the fourth is to the ſecond, ſo is the firſt to the third; that is, as 160 Days : 120 Days : : 19,200 : 14,400, and then $14,400\times 160=19,200\times 120$.

Hence then, by this ſecond Method, there are three Caſes: 1. When both the Lines are *Direct*, the firſt and third Terms in both are multiplied together as they ſtand, and the third Line is a *Direct* Proportion. 2. And if there is an *Inverſe* Proportion in the upper Line, the firſt and third Terms are multiplied acroſs, and the third Line is ſtill a *Direct* Proportion; but if the *Inverſe* Proportion is found in the lower Line, and not in the upper, you multiply acroſs as in the ſecond Caſe, and the third Line is an *Inverſe* Proportion.

And now for the *third Method*. The Learner will conſider the *Cauſe* to be the Horſes, therefore 80 muſt ſtand in the firſt Place; the *Time* (120 Days) they conſume 90 Buſhels of Oats is to be ſet in the ſecond, and what they conſume in the third Place; then the other two Numbers are to ſtand under thoſe of the ſame Name, thus:

80 *Horſes* ——— 120 *Days* ——— 90 *Buſhels*
160 *Horſes* ——— ——— 240 *Buſhels*

Here the Blank does not fall under the third Place, and therefore another Rule takes Place, which is, multiply the third and fourth together for a *Diviſor*, and the other three for a *Dividend*; that is, $\frac{80\times120\times240}{90\times160}=160$ Days, as before.

But for the greater Convenience in working Queſtions by this third Method, I ſhall ſhew the other Form of ranging the five Numbers in one Line. Thus, in the third Method of Queſtion 39, we have 100*l.*—12 *m.*—4*l.*—250*l.*—3 *m.* Here the Number wanted is Intereſt, and therefore the *Blank* falls under the 3d Number; and now they all ſtand in Order, to be multiplied as the Rule directs.

So, in the above Example, when the Numbers are properly ranged, we have 80 *H*—120 *D*—90 *B*—160 *H*—240 *B*. Here the Number wanted is Days, and therefore the *Blank* falls under the ſecond Place, and conſequently, by the Rule, the third and fourth Terms become the *Diviſor*, and the other three the *Dividend*. You muſt take Care to let the *firſt* and *fourth* be of the ſame Name, becauſe you will, when the Proportion is *Inverſe*, have Occaſion for the fourth of the ſame Name with the firſt; and if you take Care of the ſecond, to let it be always that Number which expreſſes the Time or Diſtance, it will throw the third in its proper Place, to be either multiplied alone by the fourth for a *Diviſor*; or when the Blank falls under the third Place, by the fifth alſo, for a *Dividend*.

This is not a proper Place in which to ſhew the *Theory* of the *Double Rule of Three*, when the Queſtion is not worked by *two ſingle Statings*. The worthy Youth cannot but have obſerved that he was working in the Dark, when he was uſing the *ſecond* and *third* Methods, particularly in the Stating, where the *Inverſe* Rule was found in the lower Line, as he had been before obliged to multiply acroſs, and yet afterwards told to multiply the firſt and ſecond Numbers together, and divide by the third; that is, 80 Horſes × 240 Buſhels; and this × 120 Days, and then the Product was divided by 160 Horſes × 90 Buſhels, to produce 160 Days. But he may ſee the Theory of theſe Operations, if he has Courage enough to travel on with me to the Algebraic Part.

ALLIGATION MEDIAL.

THIS Rule teaches us, between ſeveral given Quantities and their reſpective Prices, to find a Mean, by which to ſell the whole that ſhall amount to the ſame Sum of Money. *A Brandy Merchant has* 120 *Gallons of* French *Brandy, which he can ſell at* 11s. per *Gallon,* 60 *Gallons of an inferior Sort at* 9s. *and* 80 *Gallons of good* Engliſh *Brandy at* 8s. per *Gallon,* 90 *Gallons of an inferior Sort at* 6s. *and* 120 *Gallons of Spirits at* 4s. per *Gallon: For what Price* per *Gallon muſt he ſell the whole Mixture to bring him in the ſame Money?* The Rule, in all ſuch Caſes, is to find the Value of each given Quantity; then by having the Sum of the given Quantities, and the Sum of the Values, we have, as all the Gallons are to the Value of the whole, ſo is one Gallon to the Price required. Thus, 120 Gallons at 11*s.*=1320*s.* 60 at 9*s*=540*s.* 80 at 8*s.*=640*s.* 90 at 6*s.*=540*s.* 120 at 4*s.*=480*s.* then 120+60+80+90+120=470 Gallons in all; and 1320+540+640+540+480=3520*s.* therefore, as 470 Gallons : 3520*s.* : : 1 Gall : the Price of a Gallon. The Youth will do well to form ſeveral Sums in *Alligation Medial,* mixing Wine as above, then Wheat, Tobacco, Tea, *&c.*

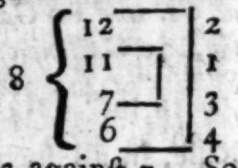
ALLIGATION ALTERNATE.

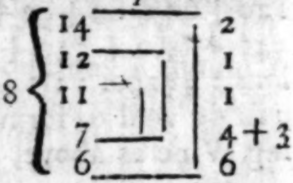
THE Meaning of this Diſtinction is, that, by having the Price only of each Sort of Goods, and the ſelling Price of the Mixture, the Quantity of each is required.

A Tea-Man has Bohea Tea worth 11s. per *lb. and an inferior Sort at* 7s. *What Quantity of each Sort will make a Mixture of* 8s. per *lb?* To anſwer this, place the Figures one under another thus, $8\left\{\begin{matrix}11\\7\end{matrix}\right.$ and put the Difference between the given and ſelling Price againſt the contrary Figure, and we have $8\left\{\begin{matrix}11-1,\\7-3,\end{matrix}\right.$ ſo that 1 *lb.* of 11*s.* and 3 *lb.* of 7*s.* give the Anſwer; for 11*s.*+21*s.*=32*s.* and 4 *lb.* at 8*s.*=32*s.* But ſuppoſe there was another Sort at 12*s.* then we have the Form, as in the Margin, where ſay, the Difference between 8 and 7=1, placed againſt 12, the Price yoked with 7; and the Difference between 8 and 12=4, placed againſt 7 the other Yoke-Fellow: But becauſe 11 and 7 are yoked together, we muſt inſert their Differences alſo, by ſaying, the Difference between 8 and 7=1, which place againſt its Yoke-

$$8\left\{\begin{array}{l|l}12 & 1\\ 11 & 1\\ 7 & 4+3\end{array}\right.$$

Fellow 11, and the Difference between 8 and 11=3, which is placed againſt 7, and we have 1 *lb.* of 12*s.* 1 *lb.* of 11*s.* and 7 *lb.* of 7*s. per lb.*=72*s.* in all: So that the Anſwer is, we may ſell 9 *lb.* of this Mixture at 8*s. per lb.*=72*s.* But let us ſuppoſe another Sort at 6*s. per lb.* and we want the Number of *lbs.* of each Sort to make up the Mixture that may be afforded at 8*s. per lb.* and then the Figures will ſtand as in the Margin. Now, obſerve always to link two Prices, one greater and the other leſs than 8 (the ſelling Price) together, and then ſay, 8—6=2 againſt 12; 12—8=4 againſt 6; 8—7=1 againſt 11; 11—8=3 againſt 7. So then we have 2 *lb.* at 12*s.*=24*s.* 1 *lb.* at 11*s.* 3 *lb.* at 7*s.*=21*s.* 4 *lb.* at 6*s.*=24*s.* then 24+11+21+24=80*s.* and 10 *lb.* the Quantity of the Mixture to be ſold at 8*s.*=80*s.* Again, let us ſuppoſe there is another Sort at 14*s. per lb.* to be mixt with the above, and all to be ſold at 8*s. per lb.* and they will all ſtand as in the Margin, when you link a Price greater or leſſer than 8; but as there is an odd Number (11) it muſt be yoked with one leſs than 8. Now, ſay, 14—8=6 againſt 6; 8—6=2 againſt 14; 12—8=4 againſt 7; 8—7=1 againſt 12; and becauſe 11 and 7 are yoked together, we compare them alſo; then 11—8=3 againſt 7; 8—7=1 againſt 11: So that we have 2 *lb.* at 14*s.* 1 *lb.* at 12*s.* 1 *lb.* at 11*s.* 7 *lb.* at 7*s.* 6 *lb.* at 6*s.*=136*s.* and the whole Quantity 17 *lb.* at 8*s.*=136*s.*





You may eaſily ſee the Reaſon of thus linking two Prices together, and that if there are many more than two the Theory is the ſame. Thus, as before, 11*s.* 6*s.* are yoked together, for this Reaſon, becauſe what one loſes in the Price the other exactly gains as much; for 11*s.*—9*s.*×3 *Gal.*=6*s.* Loſs; and 9*s.*—6*s.*×2 Gal.=6*s.* Gain. So if you yoke 11*s.* and 8*s.* together, we have 11*s.*—9*s.*×1 Gal.=2*s.* Loſs, and 9*s.*—8*s.*×2 Gal.=2*s.* Gain: In like Manner 9*s.*—6*s.*×1 *Gal.*=3*s.* Gain, and 10*s.*—9*s.*×3 Gal.=3*s.* Loſs; and ſo on for any two Prices, let their Number be never ſo many, the one always balancing and checking its Yoke-Fellow, and therefore the Price, at which the Mixture is to be ſold, is a true Medium between the Prices yoked together, and multiplied into the Quantities of each Price to be mixed. And hence you ſee the Neceſſity of yoking together one Price higher and one lower than the deſigned mean Price, otherwiſe it cannot be a Medium between two Prices higher or two lower.

But if the Quantity of one of the Sorts is given to find the ſeveral Quantities of the reſt, we muſt proceed as before to find the Differences in this Queſtion: Suppoſe a Perſon mixes 21 Gallons of *French* Brandy, worth 11*s.* *per* Gallon, with other Sorts worth 10*s.* 8*s.* and 6*s.* *per* Gallon, how much of each Sort ought to be added to the 21 Gallons, that the Mixture may be worth 9*s.* *per* Gallon. Firſt order the Terms, as in the Margin. Now, we muſt ſay, as 3 : 1 : : 21 : 7; and as 3 : 1 : : 21 : 7; and as 3 : 2 : : 21 : 14; that is, 21 Gallons at 11*s.* 7 at 10*s.* 7 at 8*s.* 14 at 6*s.*=441*s.* and 49 Gallons at 9*s.* the propoſed Price=441*s.* You canot fail of knowing the Reaſon of this Operation; for as there were 21 Gallons at 11*s.* and you want other proportional Quantities, we have, as 21 : 7 : : 3 : 1; for 3 : 1 : : 21 : 7; and you know the Column *a* contains the Proportionals, or Quantities of each Price, and theſe all are only multiplied by 7, as in Column *b*, which you remember do not alter the Proportion; for as 3 Gal. the Quantity at 11*s.* is to 1 Gal. the Quantity at 10*s.* ſo is 21 Gal. the given Quantity at 11*s.* (you ſee the firſt and third are both of a Name, and ever muſt) to 7, the Number of Gallons at 10*s.* of the ſame with the ſecond Number.

		a		*b*
9	11	3	—	21
	10	1	—	7
	8	1	—	7
	6	2	—	14
		7		49

Now ſuppoſe the Queſtion required 21 Gallons at 10*s.* then all Things would ſtand as above, only Column *b* would alter: For as 1 : 3 : : 21 : 63 againſt the Price 11*s.* and as 1 : 2 : : 21 : 42 againſt the Price 6*s.* and as 1 : 1 : : 21 : 21 againſt the Price 8*s.* and then 21, the given Quantity, will alſo ſtand againſt 10*s.* the Price belonging to it, by the Nature of the Queſtion. Now prove all this, and afterwards ſuppoſe the Queſtion required 21 Gallons at 8*s.* then at 6*s.* to find the proper Quantities at the other Prices to make up a Mixture at 9*s.* When you have done theſe, form many other Queſtions of the like Nature.

But ſuppoſe we would make up a Mixture from the foregoing Prices, to ſend away 49 Gallons at 9*s.* *per* Gallon, and it is required to find the proper Quantity at each Price, to make up this Parcel: If you obſerve Column *a*, you will find the Total of the Proportionals to be 7, and the Total of Column *b*, the other Proportionals, to be 49; then we have as 7 : 3 : : 49 : 21 ; and as 7 : 1 : : 49 : 7 ; and likewiſe as 7 : 1 : : 49 : 7 ; and 7 : 2 : : 49 : 14. Here you ſee the firſt and third in all are of the ſame Name ; that is, Totals of Numbers in Proportion ; and the ſecond and fourth in all are proportional Parts of each Column. Hence it appears the Number of Gallons, at each Price, is 21, 7, 7, 14= 49, which you ſee is only a Proof of the laſt Operation. And now you may underſtand what Authors mean when they talk of *Alligation Partial*, which is, when you had (a Part) 21 Gallons at any Price, to find the ſeveral Quantities of the reſt to mix with them ; and when they mention *Alligation Total*, they only mean the whole Quantity you intend to mix of all the Prices, ſo as to ſell the whole at a particular Price. And now let me requeſt the diligent Youth will form many Queſtions in *Alligation Total*, as its great Uſe, on many Occaſions, will amply reward him for any Pains he may take in this ingenious Part of Arithmetic. Notwithſtanding what has been ſaid of *Alligation*, tho' it gives true, yet they are not always the moſt ſuitable Anſwers, on ſome Occaſions, neither does it afford us all the poſſible Solutions to Queſtions of this Kind ; but the Learner will not find this Subject reſumed in the following Part of this ſmall Treatiſe.

I ſhall paſs over *Poſition*, or *Rule of Falſe*, as the leaſt Knowledge in Algebra enables a Youth to anſwer all ſuch Queſtions with great Eaſe and Delight, which I ſhall undertake to ſhew when we come to that ingenious and uſeful Part of Knowledge.

SYSTEM III.

VULGAR FRACTIONS.

THE Learner may have obſerved, on many Oocaſions, that there was a *Remainder* after the *Diviſion* was ended : Thus, ſuppoſe the Remainder was a Part of a Pound, Shilling, Penny, or Farthing, or of any other Denomination, the Subject we are now entered upon, ſhews us how to perform the four firſt Rules of Arithmetic and Reduction, alſo the Rules of Proportion, by the *fractional Parts :* And here the Remainder, after Diviſion, is called the *Numerator*, and the Diviſor is called the *Denominator :* Thus, ſuppoſe I divide 13*l.* by 4, then the *Remainder* is 1, to be placed in this Manner $\frac{1}{4}$ *l.* if the *Dividend* is 13*s.* then the *Remainder* or Fraction is $\frac{1}{4}$ *s.* &c. or when 13 *oz.* it will be $\frac{1}{4}$ *oz.* $\frac{3}{4}$ *Yd.* &c. and ſo on for any* other *Remainder* or *Diviſor*, how great ſoever.

Addition.] Now, if after working two Sums in Diviſion, there ſhould be two Remainders, $\frac{3}{4}$ *l.* and $\frac{4}{5}$ *l.* and it ſhould be required to add theſe two *Fractions* or *Remainders* together, the Learner cannot ſee, from any Thing he has hitherto learned in this Treatiſe, what Way he ſhould take to do this Buſineſs: But the Rule is, *multiply the Numerator of each Fraction into the Denominator of the other, for a new Numerator to that particular Fraction, and then multiply both the Denominators together, for a new Denominator to each Fraction,* and the Value of each Fraction is not altered. Then add both the new Numerators together, and put the Product of the Denominators underneath, thus, $\frac{31}{20}$ is the Sum of the two Fractions. This is commonly call'd reducing Fractions to a common Denominator, in order to prepare for Addition; but there is little to do here, you ſee, beſides finding a new Denominator. The Learner perceives, that if the Denominators are alike, there is nothing more required than adding the Numerators together, and ſetting one of the Denominators, as common already, under the Sum. The Youth will remember the Rule is the ſame, if there were never ſo many Fractions; that is, each particular Numerator muſt be multiplied into all the Denominators except its own, for a new Numerator inſtead of the old one, and then all the Denominators into one another for a new and common Denominator to each new Numerator. Now, to ſhew ſomething of the Uſe of Addition of Fractions, *ſuppoſe I had bought* $\frac{3}{8}$ *of a Ship, and another Partner concerned ſells me* $\frac{2}{7}$ *Shares which he had purchaſed, What Part of the Ship belongs to me?* Here, $21+16=37$ the Sum of the new Numerators, and $8\times7=56$ the new or common Denominator; therefore my Property in the Ship is $\frac{37}{56}$; that is, of the whole Value, conſidered in 56 Parts, I have 37 of thoſe Parts; but if I had bought $\frac{1}{8}$, &c. more I ſhould have $\frac{4}{8}$, &c. of the Ship, the Denominator being common, we have only to add together the Numerators.

Theory.] The Terms of a Fraction being multiplied by the ſame Number, the Value is not altered thereby: Thus, $\frac{3\times2}{4\times2}=\frac{6}{8}$; for as 4 : 3 : : 8 to a fourth Number 6; or as 3 : 4 : : 6 : 8. Hence, if there are never ſo many Denominators to be multiplied into a particular Numerator, the Denominator of that Fraction is alſo multiplied by the ſame Denominator. Thus, if it is required to reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, &c.

to a common Denominator, it will be $\frac{2\times4\times5\times6}{5\times4\times5\times6}=\frac{240}{360}$; for, as 2 : 3 : : 240 : 360, becauſe 2 × 360 = 3 × 240; that is, $\frac{2}{3}$ of any Thing = $\frac{240}{360}$ of that very Thing. And ſo on for all the other Fractions.

Subtraction.] If I want to take $\frac{3}{8}$ of any Thing from $\frac{5}{8}$, this is ſoon done, as I have only to take one Numerator from the other, and ſet the Remainder over one of the Denominators, as they are common, or the ſame, in both Fractions; thus $\frac{5}{8}-\frac{3}{8}=\frac{2}{8}$. But if the Denominators are unlike, they muſt be prepared; that is, reduced to a common Denominator, thus, $\frac{9}{11}-\frac{3}{7}$ will be $\frac{63}{77}-\frac{33}{77}=\frac{30}{77}$. And now I requeſt the Learner will form ten, twenty, or thirty Sums in Addition and Subtraction of Fractions, according to the Examples and Proportions, or *Theory*, given him, in order to ground himſelf in theſe valuable Parts of Arithmetic.

Multiplication.] If you have Occaſion to multiply two Fractions together, *the eaſy Rule is, multiply the Numerators together for a new Numerator to the Product, and the Denominators together for its new Denominator*: Thus, $\frac{3}{5}\times\frac{5}{7}=\frac{15}{35}$; $\frac{3}{8}\times\frac{7}{8}=\frac{21}{64}$, &c.

Theory.] Were $\frac{3}{5}$ multiplied by 1, it would be the ſame as it is, but as $\frac{5}{7}$ is not ſo much as 1, therefore the Product $\frac{15}{35}$ of two Remainders or Fractions $\frac{3}{5}$ and $\frac{5}{7}$ will be leſs than the Multiplicand $\frac{3}{5}$: For as one is to any Multiplier, ſo is the Multiplicand to the Product; that is, as $\frac{1}{1}$: $\frac{5}{7}$: : $\frac{3}{5}$: $\frac{15}{35}$; and this Product taken from $\frac{3}{5}$ will evidently prove that it is leſs than the Multiplicand; for theſe Fractions $\frac{3}{5}$ and $\frac{15}{35}$ being reduced to a common Denominator, will ſtand $\frac{105}{175}-\frac{75}{175}=\frac{30}{175}$ the Remainder; that is, $\frac{15}{35}$ is leſs than $\frac{3}{5}$ by $\frac{30}{175}$. Again, to multiply $\frac{3}{5}$ by $\frac{5}{5}$ is to keep the Multiplicand in the ſame Value, as we ſhewed before; but to multiply it by $\frac{5}{7}$ is to diminiſh the Value; becauſe when we increaſe the Denominator more than the Numerator, we leſſen the Fraction; but when the Numerator is increaſed more than the Denominator, the Value of the Fraction is greater.

Diviſion.] So if $\frac{3}{5}$ is to be divided by $\frac{5}{7}$, we have alſo *this eaſy Rule, change the Terms of the Diviſor, and proceed as in Multiplication*: Thus, $\frac{7}{5}$) $\frac{3}{5}$ (= $\frac{21}{25}$ the Quotient of $\frac{3}{5}$ divided by $\frac{5}{7}$. Now, as a Proof, the Quotient $\frac{21}{25}\times\frac{5}{7}$ the Diviſor = $\frac{105}{175}$ the Dividend, but in higher Terms; for if we divide them thus, $\begin{array}{l}35)\ \underline{105}\\35)\ 175\end{array}$ we have $\frac{3}{5}$ the Dividend, as the Terms, being divided by the ſame Number, continue in ſame Value, as was proved above, Diviſion being only the Reverſe of Multiplication, one undoing what the other does.

Theory.] Now, to divide $\frac{3}{5}$ by $\frac{1}{1}$ the Dividend continues unaltered; but $\frac{5}{7}$ is not ſo much as $\frac{1}{1}$, and a leſſer Diviſor gives a greater Quotient; that is, the Quotient will be greater than the Dividend, which is the very Reverſe of multiplying $\frac{3}{5}$ by $\frac{5}{7}$. To have a greater Quotient, therefore, cannot be obtained, unleſs we increaſe the Numerator, and diminiſh the Denominator of the Quotient, in juſt the contrary Direction to what was done in Multiplication; that is, as $\frac{1 : 7 :: 3 : 21.}{1 : 5 :: 5 : 25.}$ Again, as a farther Proof, to divide $\frac{5}{5}$) $\frac{3}{5}$ (the Quotient $=\frac{15}{25}$; but we have not altered the Dividend, becauſe $\frac{15}{25}$ is of the ſame Value as $\frac{3}{5}$, as was proved before: But we muſt have a Quotient greater than the Dividend, therefore $\frac{7}{5}$ will do this and not $\frac{5}{7}$; and hence we muſt change the Terms according to the Rule.

Reduction.] The Uſe of this Rule is to change Fractions out of one Form, or Denomination, into another, either for the more Eaſe in working or eſtimating their Value. But Fractions have different Names; if the Numerator is equal to, or greater than, the Denominator, they are called *improper Fractions*, as $\frac{4}{4}$, $\frac{5}{4}$; but if the Numerator is leſs than the Denominator, the Fraction is called *a proper Fraction*, as $\frac{3}{4}$; but a whole Number and a proper Fraction going with it, both together is called *a mixed Fraction*, as 5 $\frac{2}{3}$; and when the Word *of* comes between Fractions, they are called *compound Fractions*, as $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{7}$, &c. *The general Rule to find the Value of a Fraction of any Sort, is, multiply the Numerator by its known Value, and divide by the Denominator.*

1. *To find the Value of a Fraction when the Numerator is equal to the Denominator.* This is the full of the Denomination, for $\frac{4}{4}$ *l. s. d. qr. yd. oz.* &c. means one of theſe Denominations divided into 4 Parts, and the Numerator expreſſes the whole of thoſe Parts; thus, by the Rule, $\frac{4\times 20s.}{4}=20\,s.$ $\frac{4\times 12d.}{4}=12\,d.$ &c. for, as 4 *pts* : 20*s* :: 4 *pts* : 20*s*. &c.

2. *To reduce an improper Fraction to a mixed Number, or find the Value of a Fraction whoſe Numerator is greater than the Denominator:* Thus, $\frac{5}{4}$ *l.* is the mixed Number $1\frac{1}{4}$*l.* that is, dividing the Numerator by the Denominator for 1 *l.* and finding the Value of the Remainder, as the Fraction of a Pound; thus, $\frac{1\quad\times 20s.}{4}=5\,s.$ for, as 4 *pts* : 20*s* :: 1 *pt* : 5 *s*.

3. *To find the Value of a proper Fraction, ſuppoſe of a Cwt. &c.* Thus, $\frac{3}{7}$ *Cwt.* is $\frac{3\times4\,qrs.}{7}=1\,qr.$ and $\frac{5}{7}$, and $\frac{5\times28\,lbs.}{7}=$ 20 *lbs.* for, as 7 *pts* : 112 *lbs.* : : 3 *pts* : 48 *lbs.* or 1 *qr* : 20 *lbs.* or as 7 *pts* : 4 *qrs* : : 3 *pts* : 1 *qr.* $\frac{5}{7}$ *qr.* and as 7 *pts* : 28 *lbs* : 5 *pts* : 20 *lbs.*

4. To reduce Compound Fractions to ſingle ones, and to find their Value. Thus, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{5}$ of a Pound, &c. are reduced to a ſingle Fraction by this eaſy Rule, *multiply all the Numerators together for a new Numerator, and all the Denominators together for a new Denominator*; that is, $\frac{12}{60}$ *l.* &c. and to find the Value, it is $\frac{12\times20}{60}=4s.$ For $\frac{2}{5}$ *l.* $=8s.$ $\frac{3}{4}$ of 8*s.* is 6*s.* $\frac{2}{3}$ of 6*s.* is 4*s.* as before.

Theory of Compound Fractions.] As 5 : 20*s.* : : 2 : 8*s.* and as 4 : 8*s.* : : 3 : 6*s.* alſo as 3 : 6*s* : : 2 : 4*s.* Then in ſeveral Ranks of Proportionals the Products of the firſt and third Numbers are ſtill proportional; that is, as $5\times4\times3$: $2\times3\times2$: : 20*s* : $\frac{240}{60}=4s.$ which eſtabliſhes the Rule for multiplying all the Numerators for a new Numerator, and all the Denominators for a new Denominator; that is, $\frac{12}{60}$. In like Manner, the Products of the ſecond and fourth Numbers preſerve the ſame Proportion; for as $20\times8\times6$: $8\times6\times4$: : 20*s* : $\frac{3840}{960}=4s.$ as before.

5. *To reduce any Fraction to its Equivalent, by having one of the Terms of a deſigned Equivalent given*; thus let the Fraction be $\frac{2}{5}$, and the Numerator of a ſecond Fraction 6, to find the Denominator; then, as 2 : 5 : : 6 : 15; that is, $\frac{6}{15}=\frac{2}{5}$; for $\begin{matrix}3)\,6\,(2\\ \overline{3)\,15\,(5}\end{matrix}$ by equal Diviſion. Or let the Denominator of the required equal Fraction be 15 to find the Numerator; then as 5 : 2 : : 15 : 6. In like Manner, all Fractions, reduced to a common Denominator, are reduced to their Equivalents, or ſame Value.

6. *When two Fractions are given to find whether they are equal*; that is, proportional, as in the laſt; thus, multiply the Terms acroſs in theſe Fractions $\frac{2}{5}$, $\frac{6}{15}$, $2\times15=6\times5$, which are only the Products of the two Extremes multiplied by the two Means, as in the Proportionals above, 2 : 5 : : 6 : 15. Hence, the two Fractions $\frac{3}{7}$, $\frac{5}{8}$ are not equal, becauſe 3×8 is not the ſame with 5×7, and therefore not in Proportion.

7. *To reduce then a Fraction into other Values by a whole Number.* This has been partly treated of before, but may, with Propriety, be placed here: Let the Fraction be $\frac{5}{8}$, and the Numerator × 2; and we have $\frac{10}{8}$; here multiplying the Numerator is increaſing the Fraction; for ſurely 10 Parts of any Thing divided into 8 Parts, are more than 5 of thoſe Parts. But the ſame Increaſe may be made to the Fraction, by only dividing the Denominator of any Fraction by that whole Number, when that can be done without a Remainder. Thus, $2)\frac{5}{8}=\frac{5}{4}$, equal $\frac{10}{8}$, becauſe $5\times8=4\times10$, by *Article* 6 above.

8. *A Dividend is multiplied by its Diviſor, by taking away that Diviſor:* Thus, $\frac{15}{3}$ ſignifies the Diviſor and its Dividend, and to take the *Diviſor* means that Number is not to be divided: Thus, if 15 is to be divided by 3, and if we remove the *Diviſor*, the *Dividend* 15 will ſtand alone undivided, or not to be divided by any Number. Again, $\frac{15}{3}=5$; that is, it is really divided, and the Quotient is 5; and if we multiply the *Diviſor* into the Quotient, we have $15=5\times3$; and here again the *Diviſor* is removed, and the *Equality* on both Sides is preſerved; that is, $\frac{15}{3}=$ the Quotient 5, and $3\times5=15$ the Dividend. This will be of remarkable Uſe, and therefore the Learner is requeſted to take eſpecial Notice of what is here ſaid.

9. *To abbreviate a Fraction*, that is, reduce it to lower Terms: Thus, *find or try any Number that will divide both Terms without leaving a Remainder.* In this Fraction $\frac{1152}{1536}$, the Diviſors will be 8, 12, and 4; for the Eights in both Terms will bring them to $\frac{144}{192}$, and both divided by 12 will bring them to $\frac{12}{16}$; and again divided by 4 will reduce them to $\frac{3}{4}$ the loweſt Terms, for as 4 : 3 : : 1536 : 1152; that is, $4\times1152=3\times1536$, and conſequently the Value of the Fraction is ſtill preſerved. When both Terms have Cyphers adjoining, cut off an equal Number of Cyphers; thus, $\frac{3000}{4000}$ will be $\frac{3{,}000}{4{,}000}$; for, as 3 : 4 : : 3000 : 4000; that is, $3\times4000=4\times3000$. When both Terms end in 5, or one in 5, and the other in 0, the Diviſor may be 5: Thus, in $\frac{45}{90}$, both Terms divided by 5, we have $\frac{9}{18}$, and theſe by 9, we have $\frac{1}{2}=\frac{45}{90}$, as $2\times45=1\times90$: So when both Terms end in an even Number, they may be even continually divided by 2.

10. To reduce a whole Number (Integer) to an improper Fraction; thus, 4 is $\frac{4}{1}$, for dividing by Unit (1) does not alter the Value, as the ones in 4 is ſtill 4.

The Rule of Three.] Like whole Numbers (Integers), in the ſame Manner Fractions are often concerned in finding a *fourth* Number, by having *three* given: But as there is great Variety in the Fractions, we have in the laſt Articles ſhewn how to reduce or prepare them for the immediate Solution of the Queſtion. *Bought* $\frac{3}{4}$ lb. *of Silk for* $\frac{1}{12}$ l. *What muſt I pay for* $3\frac{7}{9}$ lb? Here the firſt and ſecond Numbers are *proper* Fractions, but the third is a *mixed* Fraction, which muſt be prepared, as in *Article* 10, and then have, as $\frac{3}{4} : \frac{1}{12} :: \frac{34}{9}$ to the Anſwer; that is, $\frac{1}{12} \times \frac{34}{9} \div \frac{3}{4} =$ to the fourth Number, or Price required, in *Direct* Proportion, becauſe more Silk requires more Money. *If* $\frac{3}{4}$ *of* $\frac{2}{5}$ *of a Ship coſt* 3000l. *What is the whole Ship valued at?* Here, the Compound Fractions reduced will be $\frac{3}{10}$; then, as 3 *pts* : 3000*l.* : : 10 *pts.* to a fourth Number, being the Value of the whole Ship. *If* 3 lb. $\frac{7}{9}$ *of* $\frac{3}{4}$ *coſt* $\frac{2}{3}$ *of* $\frac{7}{8}$ l. *What coſt* 24 lb. $\frac{7}{9}+\frac{4}{5}$—*of* $\frac{1}{2}$? Do not be frightened at the rugged Appearance of the Queſtion. The firſt Number is a whole Number, and has compound Fractions, which muſt be reduced to a proper (ſimple) Fraction, and then the firſt Number will be in a mixed Fraction, which muſt be reduced to an improper Fraction: The ſecond Number is a compound Fraction, and muſt be reduced to a proper Fraction: The third Number has Integers (whole Numbers), and belonging to them are two proper Fractions, which require to be reduced to a ſingle Fraction, by adding them together; and then it will be $\frac{71}{45}$ of $\frac{1}{2} = \frac{71}{90}$ which makes 24 *lb.* a mixed Number, that muſt be reduced to an improper Fraction: Then as 24, &c. will ſurely coſt more than 3 *lb.* &c. and ſhews the Proportion is Direct; therefore multiply the ſecond and third Fractions, and divide by the firſt for the Anſwer. *If* 3 oz. $\frac{2}{3} \times \frac{3}{5}$ *of* $\frac{4}{7}$ *coſt* $\frac{3}{8} - \frac{1}{18} \div \frac{3}{7}$: *What coſt* 20 lb? Here the third Number muſt be made an improper Fraction; in the ſecond Number you have one Fraction to take from another, and the Remainder to be divided by another Fraction; in the firſt Number you have a whole Number, and beloning to it two Fractions to be multiplied together, and then it is $\frac{6}{35}$ of $\frac{4}{7}$ to be reduced, and alſo the whole Number to be made an improper Fraction, and then find your fourth Number, or Anſwer to your Queſtion.

And now let me prevail on the worthy Youth to form many ſuch Queſtions from theſe four; if he will reſolve to do ten at leaſt of each Sort, making a Variation every Time in the Figures, but keeping to the Form of each Queſtion, he will be heartily pleaſed with himſelf at the Conqueſt, as he will moſt aſſuredly overcome all Difficulties.

Inverſe Proportion.] *If by Experience I know* $4\frac{7}{8}$ *Yards of Broad Cloth, which is* $1\frac{7}{8}$ *Yard wide, will make me a Suit of Clothes: What Quantity of fine Plane, which is* $1\frac{1}{8}$ *Yard wide, will do for my Purpoſe?* Surely I muſt have more of the latter; and therefore multiply the firſt and ſecond Numbers together, and divide by the third, that the leſſer Diviſor may give a greater Quotient; that is, $1\frac{7}{8}\times4\frac{7}{8}\div1\frac{1}{8}=$ the fourth Number x; then as $x : 4\frac{7}{8} :: 1\frac{7}{8} : 1\frac{1}{8}$; and $x\times1\frac{1}{8}=4\frac{7}{8}\times1\frac{7}{8}$. *What Quantity of Shalloon,* $\frac{3}{4}$ *Yard wide, will line* $4\frac{7}{8}$ *Yards of Cloth* $1\frac{7}{8}$ *wide?* Here again there muſt be more Yards of Shalloon than of Cloth; therefore multiply the firſt and ſecond together, and divide by the third as a leſſer Diviſor, in order to obtain a greater Quotient; that is, $1\frac{7}{8}\times4\frac{7}{8}\div\frac{3}{4}=$the fourth Number z; then as $z : 4\frac{7}{8} :: 1\frac{7}{8} : \frac{3}{4}$; and $z\times\frac{3}{4}=4\frac{7}{8}\times1\frac{7}{8}$. *An Upholſterer was employed to hang ſeveral Rooms, requiring* $576\frac{3}{4}$ *Yards of Paper* $\frac{1}{2}$ *Ell wide, but the Gentleman afterwards ordered the Hangings to be of Damaſk* $\frac{9}{16}$ *Yard wide: How many Yards of Damaſk muſt there be?* Here, before you can determine whether the third is greater or leſs than the firſt, you muſt conſider there are 36 Inches in a Yard, and 45 in an Ell; then $\frac{9}{16}$ of $\frac{36}{1}=\frac{324}{16}=20\frac{1}{4}$ Inches; and $\frac{1}{2}$ of $\frac{45}{1}=\frac{45}{2}=22\frac{1}{2}$ Inches; therefore the Damaſk is the leſſer Number, and requires more Yards than the Quantity of Paper; that is, $576\frac{3}{4}\times22\frac{1}{2}\div20\frac{1}{4}=$ the Yards of Damaſk y; then as $y : 576\frac{3}{4} :: 22\frac{1}{2} : 20\frac{1}{4}$.

Practice.] In ſome Circumſtances there is more Elegance and Expedition in working a Sum by Vulgar Fractions than by any other Method, wherein a Youth has any Light to direct him to the Theory, which affords ſome Degree of Pleaſure both to the Maſter and Scholar, when the latter has Abilities to be rouzed with the plain Reaſon of what he is about: This is the true Cauſe I have hitherto in this Treatiſe rejected, when poſſible, ſome ſhorter Rules of Operation, where the Theory does not lie within the Comprehenſion of young Heads. The Youth will be pleaſed to have his Eye on the Practice Table, or aliquot Parts, that he may go along with me, and underſtand what ſhall be ſaid to him.

1. *Bought* 1876 lb. *of Cochineal at* $\frac{7}{8}$ l. Here it is a compound Fraction $\frac{7}{8}$ of $\frac{1876}{1}=1641$ *l.* 10*s.* for $\frac{7\times1876}{8}$ is the Rule and Theory; or otherwiſe, for another Theory, as 8 *pts* : 1876 : : 7 *pts*. Here, as formerly, the Numerator of the Dividend is divided into 8 Parts, to find the Value of 7 of thoſe Parts; for 1876 is the known Denomination of the Numerator of the Fraction $\frac{7}{8}$.

2. *Sold* $\frac{5}{8}$ *of a Ship, the whole Value thereof being* 960 *l.* Here again, by the Theory of compound Fractions, we have $\frac{5}{8}$ of $\frac{960}{1}=600$ *l.* the Anſwer; that is, $\frac{5\times960}{8}$ or by the other Theory, as 8 : 960 : : 3 : 600.

3. *The Officers of a Ship, which took a Prize, valued at* 240,000 l. *are to have* $\frac{3}{8}$ *of the whole.* Then $\frac{3\times240000}{8}$ is both the Rule and Theory, as before.

4. *Bought* 1876 lb. *of Indigo at* $\frac{3}{8}$ l. Here, again, we have $\frac{3\times1876}{8}=$ the Value of the whole Parcel of Indigo.

5. *Sold* 1876 lb. *of any Thing, at* $\frac{3}{8}$ *s. or* $\frac{5}{8}$ *s.* Then $\frac{3\times1876}{8}=$ the Value in Shillings; and $\frac{5\times1876}{8}=$ the Value in Shillings alſo.

6. *Bought* 1876 lb. *of any Thing, at* $\frac{3}{4}$ *d.* Here, $\frac{3\times1876}{4}=$ the Value in Pence.

7. *A Partner's Capital in Compound Fellowſhip is* 5750 *multiplied into his Time, and his Gain is* $\frac{3}{16}$ (3s. 9d.) per *Pound: What is his Share of Gain upon his Stock?* Here, $\frac{3\times5750}{16}=$ to his Share.

8. *A Merchant failing can pay* 12s. 6d. *in the Pound, and A's Demand on the Effects is* 176l. 17s. 6d. *What is his full Dividend?* Here, 12*s.* 6*d.* is $\frac{5}{8}$ *l.* then we have $\frac{176:17:6\times5}{8}=$ his full Dividend. So if the Effects will pay 15*s.* in the Pound, or $\frac{3}{4}$ *l.* we have $\frac{176:17:6\times3}{4}$ for the Dividend; or 17*s.* 6*d.* we have $\frac{176:17:6\times7}{8}$ for the Dividend; or 7*s.* 6*d.* we have $\frac{176:17:6\times3}{8}$ for the Anſwer.

9. *Brokerage, or Commiſſion, on* 176l. 17s. 6*d.* *at* $\frac{3}{8}$*l.* per Cent. at 1 *per Cent.* it is $\frac{176:17:6}{10\times10}=1l.\ 15s.\ 4\frac{1}{2}d.$ for 100*l.* : 1*l.* : : 176*l.* 17*s.* 6*d.* that is, $\frac{100}{1}$ of $\frac{176:17:6.}{1}$ Then $\frac{1}{8}$ of $\frac{1l.\ 15s.\ 4\frac{1}{2}d.}{1}$ that is, $\frac{1l.\ 15s.\ 4\frac{1}{2}d.}{8}=4s.\ 5d.$ So at $\frac{1}{4}$ *l.* *per Cent.* we have $\frac{1l.\ 15s.\ 4\frac{1}{2}d.}{4}=8s.\ 10d.$ At $\frac{3}{8}$*l.* *per Cent.* it is $\frac{3}{8}$ of $\frac{1l.\ 15s.\ 4\frac{1}{2}d.}{1}$ that is, $\frac{1l.\ 15s.\ 4\frac{1}{2}d.\times3}{8}$ for the Anſwer. At $\frac{1}{2}$ *l.* *per Cent.* $\frac{1l.\ 15s.\ 4\frac{1}{2}d.}{2}$ At $\frac{5}{8}$*l.* *per Cent.* $\frac{1l.\ 15s.\ 4\frac{1}{2}d.\times5}{8}$ At $\frac{7}{8}$*l.* to be managed in the ſame Manner.

10. *Commiſſion on* 176l. 17s. 6d. *at* $2\frac{1}{2}$ *per Cent.* Here, as 100 : $2\frac{1}{2}$: : 176l. 17s. 6d. to the fourth Number. Bring the firſt and ſecond Numbers into Halves, and we have $\frac{200}{5}=40$ for a Diviſor to the third Number, as was ſhewn in the *Rule of Three*; and then it will be $\frac{176l.\ 17s.\ 6d.}{4\times10}=$ the Anſwer. So, at 4 *per Cent.* Intereſt, we have $\frac{100}{4}=25\ (5\times5)$ for a Diviſor to the Principal. And at 5 *per Cent.* Intereſt, it is $\frac{100}{5}=20\ (5\times4)$ for a Diviſor to the Principal.

DECIMAL FRACTIONS.

THE Learner may perhaps think there is, in general, ſome Trouble in managing Vulgar Fractions; but we ſhall now ſhew him the Manner of ſhortening his Operations, by finding another Fraction with a Denominator 10, equal to any he diſlikes; thus, to find one equal to $\frac{2}{5}$, we have, as 5 : 2 : : 10 : 4; then $\frac{4}{10}=\frac{2}{5}$, which you may prove to be true, by multiplying the Terms acroſs, as you were taught before; thus, $4\times5=10\times2$. But inſtead of expreſſing $\frac{4}{10}$, you need only write . 4, and this is called a Decimal Fraction, becauſe its Denominator, expreſſed by the Point, is 10. In like Manner, if we would find a Fraction, whoſe Denominator is 100, to be expreſſed by a Point, that ſhall be equal to, ſuppoſe, $\frac{1}{4}$; then, as 4 : 1 : : 100 : 25; Here, $\frac{1}{4}=\frac{25}{100}$, but we need only expreſs it . 25 alone. If you want a Fraction, whoſe Denominator is 1000, that ſhall be equal to $\frac{1}{8}$, we have, as 8 : 1 : : 1000 : 125, which is expreſſed . 125

as before. So, by the ſame Theory, we may find the equal Decimal to $\frac{1}{2}$, whoſe Denominator ſhall be 10; for, as 2 : 1 : : 10 : 5; that is, . 5 inſtead of the Denominator. In like Manner we may find the Decimal to $\frac{3}{4}$, whoſe Denominator ſhall be 100; for as 4 : 3 : : 100 : 75; that is, .75 without its Denominator. And hence you may turn any Vulgar Fraction into a Decimal, by adding Cyphers to the Numerator, and dividing by the Denominator: Thus, $\frac{1}{8}=\frac{1000}{8}=.125$, as before.

Addition.] Let us now add all theſe Decimal Fractions together, the *Vulgar* ſtanding by themſelves, and the *Decimals* by themſelves, as in the Margin, that the Learner may form a Judgment of Decimals. You muſt be here told that the Fraction of the greateſt Value here is $\frac{3}{4}=.75$, and that the leaſt in Value here is $\frac{1}{8}=.125$; that the Figure next to the Decimal Point is of more Value than the ſecond on the Right Hand, and the ſecond of more Value than the third, and ſo on: For ſuppoſe them all Fractions of a Pound, then $\frac{1}{2}=5\times 20s.=10s.$ that is, $\frac{5\times 20s.}{10}=10s.$ But if the Fraction had been .55; (that is, $\frac{55}{100}$) we have $\frac{55\times 20}{100}=11s.$ For as 100 *pts* : 20*s* : : 55 *pts* : 11*s.* ſo that you ſee the Value decreaſes, as . 5 was 10*s.* and now $\frac{5}{100}$ is only one Shilling: So had the Fraction been . 555; that is, $\frac{555}{1000}$ then we ſhould have $\frac{555\times 20\times 12\times 4}{1000}=11s.\ 1d.\ \frac{800}{1000}$ of a Farthing: So that the 5, in the third Place from the Point, is only 1 *d.* and $\frac{800}{1000}$ of a Farthing. Now add up the Decimals, beginning at the firſt Place on the Right Hand, ſaying 5 is 5; then go to the ſecond Place, ſaying 5, 10, 12; that is 2 and carry 1 to the Tens, 8, 13, 15, 16, 20; that is, 0 and carry 2 on the Left of the Point, and the Total is 2 . 025. And, by this Time, you ſee the Neceſſity of keeping the Tens under Tens, Hundreds under Hundreds, and Thouſands under Thouſands, &c. for next to the Point they are Tens of a Pound, (yds. oz. &c.) in the ſecond they are Hundreds of a Pound; in the third, Thouſands of a Pound, &c. Thus, in . 125 it is $\frac{1}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, &c. But you muſt, in reading the Decimals, as in the laſt Margin, call them *four Tenths, a hundred and twenty-five Thouſandths, twenty-five Hundredths, five Tenths, ſeventy-five Hundredths,* &c.

Value in Decimals.		
$\frac{2}{5}$	=	. 4
$\frac{1}{8}$	=	. 125
$\frac{1}{4}$	=	. 25
$\frac{1}{2}$	=	. 5
$\frac{3}{4}$	=	. 75
		2 . 025

Now you muſt be very ſenſible of the Convenience and Expedition of Decimals, for you know it will take you up ſome Time to add all the Vulgar Fractions together, as you muſt reduce them to a common Denominator firſt. But you will be pleaſed to remember, that 1, 2, 3, *&c.* Cyphers before a Decimal, advances it ſo many Places forward to the Right Hand, whereby it is made 10, 100, 1000, *&c.* Times leſs; thus, ſuppoſe .5, .05, .005, *&c.* read *five Tenths, five Hundredths, five Thouſandths, &c.* for you ſaw before that .5 is 10*s*. .05 is 1*s*. .005 is not quite 5 Farthings; and if you would have them ſtand as Vulgar Fractions, they are $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, *&c.* If you were required to go on a great Way farther in thus decreaſing 5 ten Times, and as in Vulgar Fractions to reduce them to a common Denominator, in order to add all ſuch Fractions together, how great would be your Toil! But how ſoon this is done by ſetting Points inſtead of the Denominators.

And let me now hope you will again read over and pauſe on what I have ſaid and done for you in Decimal Fractions, and then form ten, twenty, or thirty Sums in Addition, and ſo let each Sum conſiſt of ten Lines, with whole Numbers and Fractions, carrying ſome Figures out to nine Places of Decimals, ſome to eight, ſome to ſeven, ſix, five, four, three, two, and ſome to have only one Figure next to the Point, now and then placing 1, 2, 3, *&c.* Cyphers before a Decimal, and then you will be well grounded in *Notation* and *Addition of Decimal Fractions.*

Subtraction.] You may remember what Trouble you had in Subtraction of Vulgar Fractions; but now you need only obſerve how you place your Figures as in Addition of Decimals, and you may ſubtract one Number from another, with as much Eaſe as in whole Numbers. Thus, ſuppoſe, you want to take .025 from .5, you ſay, 5 from 10, 5 and carry 1, and 2 is 3 from 10, 7 and carry 1, and 0 is ſtill 1 from 5, 4; ſo that the Remainder is .475. Here you ſee it is only ſuppoſing Cyphers affixed in the Minuend, where there are no Figures to anſwer thoſe in the Subducend, and the Buſineſs is done. Now, ſet yourſelf ten, twenty, or thirty Sums from the following Example; from 75.8675, take 24.076895, but let both the Terms be altered in each Sum. You may perceive 75 and 24 are whole Numbers, and thoſe Figures on the Right Hand of the Point are Decimal Fractions; that is, Vulgar Fractions thrown into others of equal Value, as was proved in Addition of Decimals.

Multiplication.] You will mind what was ſaid of the Theory of multiplying Vulgar Fractions, and then proceed as in Multiplication of whole Numbers, only when you have done reckon how many Decimal Points you have in the Multiplicand and Multiplier, and let the Product conſiſt of as many, which if it cannot, prefix Cyphers to make up the Number. Thus, multiply .7863 by .00005, when you will have 39315 for a Product; but becauſe there are nine Decimal Places in the Multiplicand and Multiplier, you muſt have this Number in the Product, which will ſtand thus, .000039315. And now increaſe the above Example gradually, and work ſeveral Sums, and let there be at leaſt a whole Number in .7863 before you give over.

Diviſion.] What has been ſaid of the Theory of Diviſion of Vulgar Fractions, muſt be equally underſtood of Decimal, and then proceed as in whole Numbers, only Regard is to be had to the Points; but in this you have nothing more to do than to let the Quotient conſiſt of the Difference between the Diviſor and Dividend; for if the Dividend has, ſuppoſe, four Decimal Places, and the Diviſor one, then the Quotient will have three. You may affix what Number of Cyphers you pleaſe to the Dividend, in order to have a proper Number of Figures in the Quotient. Thus, divide 25 by .05; here you will have $\frac{25.00}{.05} = 500$; and becauſe the Number of Decimal Places (.00) in the Dividend (25.00) is the ſame as in the Diviſor (.05) the Quotient will be without a Decimal; that is, it will be a whole Number 500. But ſuppoſe you were to divide .05 by 25; here we have nothing to do but affix a Cypher to the Dividend .05, when we ſhall have .050, and this divided by 25, as if all were whole Numbers, we have 2 in the Quotient; but becauſe the Number of Decimal Places in the Dividend is three, (.050) and the Diviſor 0, as it is a whole Number, therefore we muſt prefix 00 to the Quotient 2, and we have the Difference .002 for the Anſwer, and $.002 \times 25 = .050$ the Dividend, which is a Proof that you are right. And now vary theſe two Examples in all the poſſible Ways, ſometimes as above, and ſometimes letting both Diviſor and Dividend conſiſt of Integers, (meaning whole Numbers) by which you will find nothing in Diviſion of Decimals too hard for you; but uſe yourſelf to prove the Operations, by multiplying the Quotient by the Diviſor, as this will be a great Means of fixing your Attention to the Nature of the Number of Points.

Reduction.] We have before ſhewn you how to *reduce a Vulgar to a Decimal Fraction,* and ſtill keeping its Value; but if you want to reduce Shillings, Pence, or Farthings to the Decimal of a Pound Sterling, for the Convenience of working Sums in the *Rule of Three,* you are to conſider there are 20 Shillings, 240 Pence, and 960 Farthings in a Pound, ($20 \times 12 = 240 \times 4 = 960$) which are your Denominators; and if the Example is, ſuppoſe, 17*s.* 6¾*d.* which make 843 Farthings, ($17 \times 12 + 6 \times 4 = 843$) we have this Vulgar Fraction $\frac{843}{960}$ to be reduced to a Decimal thus, $\frac{843.000000}{960} =$ 878125; that is, $\frac{878125}{1000000} = \frac{843}{960}$, but expreſſed with the Decimal Point for Eaſe and Expedition. If you had only 17*s.* to find its Decimal, it would be $\frac{17.00}{20} = .85$; if the Decimal of 6*d.* is required, you have $\frac{6.000}{240} = .025$; in like Manner had you only ¾ *qrs.* given to find its Decimal, it would be $\frac{3.000000}{960} = .003125$: And now if you add theſe together, they make juſt he ſame as when you brought the 17*s.* 6¾*d.* into Farthings; for $.85 + .025 + .003125 = .878125$ as before, and proves the Operations. And thus you may do to find the Decimals of *Yards, Cwts. qrs. lbs.* or any given Part of a Denomination, by conſidering how to find the Numerator and Denominator of the Vulgar Fraction.

2. *To find the Value of a Decimal Fraction.* This is nothing more than a Stating in the *Rule of Three,* thus: *What is the Value of* .85 *l?* Here we have, as 100 *pts* : 20*s* : : 85 *pts* : 17*s.* that is, $\frac{85 \times 20}{100} = 17$*s.* *What is the Value of* .025 *l?* Anſwer, as 100 *pts* : 20*s.* × 12*d* : : 25 *pts* : 6*d.* *What is the Value of* .003125 *l?* Anſwer, as 1000000 *pts* : 20*s.* × 12*d.* × 4*qrs* : : 3125 *pts* : 3 *qrs.* *What is the Value of* .878125 *l?* Anſwer, as 1000000 *pts* : 20*s.* × 12*d.* × 4 *qrs* : : 878125 *pts* : 17*s.* 6¾*d.* which proves the reſt.

3. *To find the Value of the Decimal of a Pound Sterling at Sight.* This will prove of great Eaſe to you, by committing what ſhall be ſaid to Memory. *What is the Value then of* .003125 *l?* Now, look attentively at the Figures. The Value of the third Place from the Point is always *Farthings*; therefore the Anſwer is ¾, for you cannot value at Sight the fourth Place of any Decimal. *What is the Value of* .025 *l?* The Value of the ſecond Place is *Tens of Farthings*; that is,

5 in the ſecond Place is 50 Farthings, but for each 25 Farthings abate one, ſo that we have 48 Farthings or one Shilling: And here obſerve, when the ſecond Place is exactly 5, call it one Shilling, if more it is one Shilling and ſo many Tens of Farthings over, but if under 5 it is ſo many *Tens of Farthings:* So that in .025 we have 25 Farthings, and, abating one, the Value is juſt 6 *d.* And you are to fix it in your Mind, that the Double of the firſt Place will always be Shillings. *What is the Value of* .85 *l?* Here, twice 8=16 *s.* and becauſe the Figure in the ſecond Place is exactly 5, it is one Shilling more; ſo that .85 *l.*=17 *s.* *What is the Value of* .878125 *l?* Now, look carefully on the Fraction; the firſt Place doubled is 16 *s.* and becauſe the ſecond Figure from the Point is above 5, it is 1 *s.* more, and 20 Farthings over, and 8 in the third Place is 28 Farthings, and, abating 1=27 or 6¾ *d.* as above.

4. To reduce the Decimal of a Shilling, a Penny, or a Farthing to that of a Pound Sterling: Thus, for Inſtance, ſuppoſe .5 *s.* then $\frac{.5}{20}$ = .025 *l.* and alſo .75 *d.* then $\frac{.75}{12}$ = .0625 *s.* and $\frac{.0625}{20}$=.003125 *l.* So if .5 *qrs.* it will be brought into the Decimal of a Pound, by dividing by 4; that is, $\frac{.5}{4}$=.125 *d.* and $\frac{125}{12}$ = .0104166 *s.* and $\frac{.0104166}{20}$= .0005208 *l.*=.5 of a Farthing. You ſee theſe Operations are only the Converſe of finding the Value of any Decimal Fraction of a Shilling, a Penny, or of a Farthing; for in one Caſe you multiply, but in the other you divide.

And here let me recommend it to you to ſet yourſelf many Sums from the Examples I have given you, that you may be perfect in this uſeful Part of Arithmetic; and when you have done this, find the Value of the Decimals of *Yards*, *Inches*, *Cwts. qrs. lbs. ozs. Miles, &c.* and then reduce the Decimals of *Inches*, *Feet*, *Yards*, *&c.* to that of a *Mile*; and alſo reduce the Decimals of *ozs. lbs. qrs. Cwts.* to that of a *Ton.*

The Rule of Three.] This is nothing more but multiplying the ſecond and third Numbers in Decimals together and dividing by the firſt; or firſt and ſecond, and dividing by the third, juſt as the Quotient requires from the *Direct* or *Inverſe* Proportion. But it uſually happens that the Learner is at a Loſs how to proceed, when the Queſtion conſiſts of both Vulgar and Decimal Fractions: Thus, *if* ⅞ *lb. coſt* .525 *l. What will* 8¾ *lbs. coſt?* A Perſon ſkilled in Fractions will ſtate it thus, $\frac{7}{8}$: 10*s.* 6*d.* : : 8$\frac{6}{8}$: 105 *s.* Here the $\frac{3}{4} = \frac{6}{8}$; then as the firſt and third are both in Eighths, we have 7 : 10*s.* 6*d.* : : 70 : 105 *s.* that is, $\frac{70 \times 10\frac{1}{2}}{7}$; for by refuſing the Denominator 8 in the firſt and third, we ſtill preſerve the Proportion, as we have multiplied them by 8. Or thus, $\frac{7}{8}$: $\frac{21}{40}$: : 8$\frac{3}{4}$ to the Number required. But, by Decimal Fractions, we have as ($\frac{7}{8}$=) .875 *lb.* : (10*s.* 6*d.*=) .525 *l.* : : 8.75 *lbs.* : 5.25 *l.* So that you ſee you muſt determine by your Eye, whether working the Queſtions by Decimals will be the ſhorter Way: Undoubtedly it often will, if the Numbers are given in Decimals. You may vary the above Queſtion to your own Fancy, and invent ſome others, to exerciſe your own Skill in this Part of Knowledge, ſometimes trying whether Vulgar or Decimal Fractions will ſooneſt help you to the Anſwers.

But when you meet with any of the Forms in the Margin, after a little Attention, you will be able to make ſo many Queſtions of them in the Rule of Three, by making ſome the ſecond, and others the third or firſt Numbers. Let us ſuppoſe *n* the firſt, *a* the third, and *e* the ſecond Number; you know the fourth Number will be like the ſecond, and that you muſt bring *a* into the ſame Denomination with *n*; that is, *lbs.* and Decimal Parts of a *lb.* and that when you have found your fourth Number, you are to divide that by 12 and 20, and to value the Decimal of the *l.* both by Sight and multiplying by 20, 12, and 4, and always dividing by the Denominator of the Decimal Fraction. Now, again, let us make *m* the firſt, *n* the ſecond, and *b* the third Number; then bring *m* and *b* into Pence, to make them both of a Name, and, by performing what the Rule of Three requires, the fourth will be *lbs.* and a Decimal of a *lb.* and ſo on for all the reſt in the Table, which you may eaſily do by yourſelf.

12 *C.* 3 *qrs.* 11 *lbs.*	13	—	*a*
7 *l.* 18 *s.* 11 *d.*	5	—	*b*
17 *s.* 6 *d.* $\frac{1}{4}$ *qr.*	13	—	*c*
5 *s.* 9 *d.*	8	—	*d*
11 *d.*	7	—	*e*
8 *Ells Eng.*	3	—	*f*
3 *l.* 12 *s.* 6 *d.*	9	—	*g*
5 *l.* 18 *s.*	4	—	*h*
54 *Tons*	75	—	*i*
5 *yds.*	6	—	*k*
3 *C.*	8	—	*l*
2 *s.*	7	—	*m*
3 *lbs.*	45	—	*n*
3 *qrs.* 25 *lbs.* 7 *oz.*	98	—	*o*

Practice.] You may remember when we taught you how to value a Decimal of a Pound Sterling at Sight, that you were to double the Figure next to the Point for Shillings; now you may eaſily perceive, if you halve any even Number of Shillings under 20, that you have the Decimal of thoſe Shillings. Thus, ſuppoſe 325 Yards at 2ſ. *per* Yard; here the Decimal of 2ſ. is .1, for $\frac{2.0}{20}=.1$, or as, 20 *d* : 2 *n* : : 10 *d* : 1 *n*; that is, $\frac{2}{20}=\frac{1}{10}$, or .1 *l.* without its Denominator. By the Queſtion then we have, as 1 *yd* : .1 *l.* : : 325 : 325 × .1 = 32.5; that is, 32 *l.* and the Double of the Decimal .5 = 10ſ. Hence then, Yards, Ells, Oz. &c. at 2ſ. is only pointing off the Units, and doubling the Figure for Shillings. And you may on many Occaſions take the Advantage of this *Principle*, (.1) whether the Price is below or above 2ſ. Thus, 325 Yds. at 8ſ. is 4 Times more than 32.5 *l.* the Price at 2ſ. And 4 Times, &c. leſs, is dividing 32.5 by 4, &c. The Table in the Margin may be of great Uſe to you, after pauſing a little on the Nature of it: Thus, 325 Yds. at 18ſ. Here we have 32.5 × 9 = 292.5 *l.* the Anſwer. So 325 Yds. at 6*d.* we have $\frac{32.5}{4}=8.125$ *l.* that is, 8 *l.* 2ſ. 6*d.* The 325 Yds. being at 2*d.* we have $\frac{32.5}{12}$; and ſo on for all the reſt in the Table. If it is 325 Yds. ſuppoſe at 18ſ. 9*d. per* Yd. we have 32.5 × 9, and then you take the Parts for 9*d.* accordingly. So if the Price is 19ſ. 6*d. per* Yd. you have 32.5 × 9; then 1ſ. is the ½ of 32.5; and 6*d.* is the ½ of what it comes to at 1ſ. and ſo on for any Price below 20ſ.

The Table of 2ſ.

18ſ.	*is*	9	*times*	2ſ.
16	—	8	—	2ſ.
14	—	7	—	2ſ.
12	—	6	—	2ſ.
10	—	5	—	2ſ.
8	—	4	—	2ſ.
6	—	3	—	2ſ.
4	—	2	—	2ſ.
2	—	1	—	2ſ.
1ſ.	*is*	½	*of*	2ſ.
6*d.*	-	¼	—	2ſ.
3*d.*	-	⅛	--	2ſ.
2*d.*	-	$\frac{1}{12}$	—	2ſ.

Intereſt.] We ſhall firſt give the general Rule for finding the Intereſt of any Principal, and then the Reaſon of the Operation. *Multiply the Rate, Time, and Principal together for the Anſwer.* What is the Intereſt of 875 *l.* for 4 Years at 5 *per Cent. per Annum?* Here .05 × 4 × 875 *l.* = 175.0 *l.* the Anſwer; but you are to remember .05 is the Intereſt of 1 *l.* for a Year; for as 100 : 5 (or any other Rate) : : 1 : $\frac{5}{100}$ or .05; then, as 1 Year : .05 *l.* : : 4 Years : .2 *l.* and as 1 *l.* (in 4 Years) gains .2 *l.* Intereſt, ſo 875 *l.* (in 4 Years) will gain 175 *l.* the Anſwer required. If the Time is $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$ Part of a Year, you muſt divide the Rate of 1 *l.* for a Year accordingly; But if the Time conſiſts of Days, they are to be turned into a Decimal; and if there are Shillings, Pence, and Farthings, you can now at Sight expreſs them by a Decimal of a Pound. Here be pleaſed, of the above Queſtion, to form ſeveral Sums, by varying it in all its Circumſtances.

Compound Intereſt.] This is ſooneſt performed by Decimals. The Rule is, *multiply the Amount of one Pound into itſelf for the Number of Years leſs one, and this Product by the Principal.* Thus, what is the Amount of 875 *l.* for 4 Years at 5 *per Cent.* Compound Intereſt? Here $1.05 \times 1.05 \times 1.05 \times 1.05 \times 875 =$ the Amount required. For as 1 *l.* : 1.05 : : 1.05 : 1.05 = at the End of the ſecond Year; and as 1 *l.* : 1.05 : : 1.05 × 1.05 : 1.05 × 1.05, : × 1.05, at the End of the 3d Year; and as 1 : 1.05 : : 1.05 × 1.05 × 1.05 : 1.05 × 1.05 × 1.05 × 1.05, at the End of the fourth Year, which call x. Then if 1 *l.* (in four Years) amounts to x, what will 875 *l.* amount to in the ſame Time? Call it z; then $z-875 =$ the Compound Intereſt. And here alſo, from this Queſtion, ſet yourſelf ſeveral more, altering every Figure in each new Queſtion.

Intereſt Tables.] Though I am no Friend to printed Tables of *Simple Intereſt*, as People ſhould know how to work the Queſtions with the Pen, and thereby avoid many Errors in printed Books; yet, to amuſe and ground a Youth, he may make a Table for Months, Quarters, and one Day. Thus, for Month, $\frac{.05}{12} = .004166$; $\frac{.04}{12} = 003333$; $\frac{.03}{12} = .0025$; and ſo on for all the intermediate $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ *per Cent. per Ann.* And for Quarters it will be $\frac{.05}{4}$, $\frac{.04}{4}$, $\frac{.03}{4}$, &c. But for one Day we have $\frac{.05\quad .04\quad .03}{365,\ 365,\ 365,}$ &c. Hence, by knowing the Intereſt of one Day, we may multiply it by the given odd Days. Now, multiplying the above Quotients, thrown into a Table, by any given Principal, we have the Intereſt required. In like Manner, if we would form a Table of the Amount of 1 *l.* at *Compound Intereſt*, for any Number of Years, we have a Method of doing it from the above Queſtion in Compound Intereſt. For the Amount at the End of the firſt Year is 1.05; the ſecond, 1.05 × 1.05; the third, 1.05 × 1.05 × 1.05; the fourth, 1.05 × 1.05 × 1.05 × 1.05; and ſo on as far as you pleaſe. You muſt alſo remember to calculate to the End of the ſame Period, at .04 and .03; by which your Tables will be at 3, 4, and 5 *per Cent. per Annum.* You know you are to multiply any Number in your Table by the given Principal for the Amount, and that $z-$the Principal, gives the Compound Intereſt of the Money.

EXTRACTION *of* ROOTS.

The SQUARE ROOT.

Roots.		*Squares.*
1	—	1
2	—	4
3	—	9
4	—	16
5	—	25
6	—	66
7	—	49
8	—	64
9	—	81
10	—	100
11	—	121
12	—	144
00	—	576

NOW any ſingle Figure, under Ten, being multiplied by itſelf, produces what is called a *Square*, and to find the Number producing it, is called *extracting the Square Root*; thus, ſuppoſe a Side or Root of a Die is 5 Parts, then 5×5 is the Square or Face of the Die; and if a Perſon ſays the Square is 25, you conclude, from the Table in the Margin, that the Root, or Side of the Square, is 5. So if a Perſon tells you he has been meaſuring a Piece of Ground which contains a Square of 144 Yards, and requires you to tell him how many Yards there are in the Side or Root; that is, deſires you to extract the Square Root of 144, you will, by the Table, tell him 12 Yards: But if he ſays the Square was 576 Yards, you muſt then go to work to find out the Root. In many of the Operations of *Menſuration*, the Rules talk to you about this Sort of Buſineſs, and therefore you muſt make yourſelf Maſter of the Manner of extracting the *Square Root*. Now caſt your Eye on the Table, and remember, that when the Square conſiſts of *two Figures only* the Root is a *ſingle Number*; but if the Square contains *three Figures*, the Root muſt contain *two Places*: You likewiſe ſee, in this laſt Caſe, there are two Points over the firſt and third Figures, to denote the Root has two Places, which you are to find thus, for Inſtance, in 576. But, to give you ſome Inſight into the Reaſon of what you are about, I muſt tell you the Root is 24; that is, 20+4×20+4=576. Now, as Multiplication has form'd the Square from the Root, ſo Diviſion muſt undo the Square to come at the Root, which you will eaſily ſee by being a little careful. You know, to multiply the Parts of a Root into ſelf, is the ſame as to multiply the whole Root into itſelf to produce the Square; thus, 24×24=576; and 20+4×20+4=576, as in the Margin. Let this Square be pointed as in the firſt Table, and then find by it the Root of 5, which you ſee is 2;

$$\begin{array}{r}
20+4 \\
20+4 \\
\hline
80+16 \\
400+80 \\
\hline
400+160+16
\end{array}$$

that is 20, for $20\times20=400$. We now conclude 176 is the next Dividend, and that there muſt be ſuch a Diviſor to it as will give a Quotient equal to 4; that is, $\frac{176}{40+4}=4$, (or the fours in 17) the ſecond Part of the Root: Now, conſider what this $40+4$ is. You know your firſt Root was 20, and the Double of this is 40; and then, to compleat the Diviſor, you muſt add 4 to $40=44$, which being multiplied by the Quotient 4, is equal to the Dividend 176. But to give you a nearer View of this, conſider attentively what you have in the Margin, which eſtabliſhes this Rule, *double* the firſt Figure in the Quotient *c*, and add ſuch a Figure (4) to it in the ſecond Diviſor *a*, as will go in the ſecond Dividend *b*, (or the fours in 17) and place the 4 alſo in the Quotient *c*; then multiply *a*, that is, 44 by 4 in *c*, and we have 176, which muſt be taken from *b*, and if nothing remains, 24 is the true Root or Side, whoſe Square is 576. In like Manner always keep doubling the Quotient *x* for a Part of the Diviſors *d*, *e*, *f*, when you are to proceed further with different Squares, or when there is a Remainder *g*, to which you muſt affix Cyphers in Pairs, to bring out a ſufficient Number of Decimal Places. Thus, let the Square be 595, and it is required to find the Root in whole Numbers, and three Decimal Places. You ſaw before that the Root was 24; but here there is a Remainder of 19 at *g*, to which you affix .00, and then doubling the Quotient 24 at *x*, we ſee by placing 3 (or the fours in 19) at *x* and *d*, the Diviſor will be 483, which multiplied by 3 in $x=1449$, which taken from *g* we have 451, to which affix .00. And now doubling 24.3 in *x*, we have 486, and putting 9 (or the fours in 45) to it, (and at *x*) as we find it requires it to make the Diviſor *e* go a proper Number of Times, we have $4869\times9=43821$ from *b*, and there remain 1279, to be made 1279.00 by affixing the two Cyphers to bring on the Decimal Places in *x*. Again doubling $x=24.39$ we have 4878 for Part of the new Diviſor *f*, and

```
           . .  c
           576 (24
           4
     a     —
     44)   176 b
           176
           ———
            0
```

```
             . .  x
             595 (24.392
             4
             ——
     44)     195
             176
     d       ————
     483)    19.00 g
               15 49
     e         ——————
     4869)     4 51.00 b
               4 38 21
     f         ———————
     48782)   1279.00 k
               975 64
              ————————
               303 36
```

affixing 2 (or the fours in 12) to it, (and alſo to *x*) as we find *k* requires ſuch a Diviſor, we have *f*=48782×2=97564 from *k*, the Remainder is 30336, and we have three Decimal Places in *x*, which =24.392 the Root or Side of a Square 595. You perceive you may go on as long as you pleaſe, affixing two Cyphers to any Remainder, and forming your new Diviſors. The Proof that your Work is right, will be *x*×*x*, and taking in the Remainder. But let us extract the Root of a higher whole Number, ſuppoſe of Eight Places, 87689542. Here point every other Figure, beginning at Units, as in the Margin, and for every Point you will have a Place in the Quotient, in all four. I need not trouble you with the Manner of doing this, as it is in all Reſpects performed like the laſt, which conſiſted of nine Places, by affixing three Couple of Cyphers.

. . . .
87689542(9364
183) 668
1866) 11995
18724) 79942

5046

For as Proof.

9364×9364
+5046=87659542

Fractions.] From the Manner of affixing the Cyphers by Couples to the Remainders, you may conceive how you muſt point off Decimal Fractions: Thus, ſuppoſe .81250000: Here you have four Points, which ſhew you will have four Places in the Decimal Root, and that you muſt begin to point off from the ſecond Figure next to the point to have Pairs, becauſe there muſt be four Places in the Root; nay indeed, if it was required, you may have any Number of Points, provided you continue the Cyphers in Pairs. When you have extracted the Square Root of this Fraction, in the ſame Manner as whole Numbers, you will have .9013×.9013+15831=.81250000. If there is a Decimal Fraction, for Inſtance, .8125 belonging to a whole Number, ſuppoſe 5765, they will both ſtand thus, 5765.8125, &c. Now view the Forms, and you need no further Inſtruction, but only to obſerve that Vulgar Fractions may be abbreviated and turned into Decimal Fractions, in order to have their Roots extracted for the more uſeful and common Purpoſes of Life. The Learner will find this Subject reſumed in the Algebraic Part, where we ſhall endeavour to let him ſee (if he travels ſo far) the Reaſon of what could not be here underſtood; but he will do well to form ſeveral Sums of what he has done, by varying the Figures, and carrying the Decimals out to a greater Number of Decimal Places; by this Means he will be ready at extracting the Square Root, when he comes to Menſuration, &c. without which it is ſcarcely worth while learning to extract Roots at all, though placed here, which is done purely for the Sake of the Algebraic and Geometrical Parts further on.

CUBE ROOT.

THIS is but troubleſome Buſineſs at beſt to Youth, but I ſhall endeavour to avoid being tedious, and yet give them ſufficient Inſtructions how to extract the *Cube Root.* What a Cube is, the Learner may conceive from a Die, one Side whereof being ſuppoſed 5 Parts, and if we multiply this Side into itſelf, we have the Square or Face 25, and by multiplying this Square by the Side 5, we have the Cube 125 Parts. Now, if you are told the Cube conſiſts of three Figures, ſuppoſe 125, and you are required to tell the Side or Root of the Cube, you may do it by the Table in the Margin; but if the Cube conſiſts of four or more Figures, you muſt reſolve within yourſelf to diſpute the Ground Inch by Inch to obtain the Conqueſt. Now, by the Table, what is the Cube Root of 216? Here you find the Anſwer is 6, for 6×6=36, and 36×6=216; and ſo on for any of the reſt of the Cubes form'd from a ſingle Figure; but you will do well to get the Table perfectly by Heart, by ſaying the Cube Root of 729 is 9, and in like Manner going upwards and downwards; and alſo form the Cubes from the Roots, by repeating thus, the Cube of 9 is 729, &c. Let us now proceed to the Conteſt, and ſuppoſe the Cube to be 614̇125̇, whoſe Root you are to find. The firſt Buſineſs is to point it into Periods of *three Figures,* which you perceive will require for every Point, by the Table, *one ſingle Figure,* and two for its Cube Root. Now, conſider a little; the higheſt Period 614, is by viewing the Table between 512 and 729, and

Roots.		*Cubes.*
1	—	1
2	—	8
3	—	27
4	—	64
5	—	125
6	—	216
7	—	343
8	—	512
9	—	729

$$
\begin{array}{rll}
 & 61\dot{4}12\dot{5} & (\overset{a}{80}+\overset{b}{5} \\
 & \underline{512000} & c \\
\overset{e}{19440)} & \underline{102125} & d \\
 & 96000 & \\
 & 6000 & \\
 & \underline{125} & \\
 & \underline{102125} & f \\
 & 0 & \\
\end{array}
$$

$$85 \times 85 \times 85 = 614125$$

therefore you pronounce the Cube Root to be between 8 and 9; therefore call it 8, that is, 80, becauſe $80 \times 80 \times 80 =$ 512000, which is the neareſt beneath 614000; had you made the Root 9, (90) you plainly perceive it would have been too much, as that would have been 729000; or had you made it 7, (70) this is too little, as it would be only 343000, which falls very ſhort of 614000. Now, view the Work in the Margin, and you will find *a* is cubed and makes *c*, and *c* taken from the given Cube makes *d*. Now pauſe a little; we muſt contrive a Diviſor for *d* before we can go further. The firſt Rule is, *triple the Square of* a + *triple* a *for your Diviſor*; that is, $80 \times 80 \times 3 + 80 \times 3 = 19440$, the Diviſor to *d*, whoſe neareſt Quotient (or Nineteens in 102) is 5, which place under *b*. Now, the Figure under *b* muſt always, upon Trial, be ſuch, as when properly prepared and combined with *a* will come neareſt to *d*, under this ſecond Rule, which is *the Triple of the Square of* a × *into* b + *the Triple of* a × *into the Square of* b + *the Cube of* b, as you ſee in the Margin of the foregoing Page; all which added together is juſt equal to *d*, and therefore nothing remains, and ſo you conclude 85 is the Cube Root. Thus you are ſufficiently enabled to manage this Part of Knowledge, when the Root conſiſts of two Places of Figures; but when there are more than two Places we muſt proceed as follows:

Let the Cube whoſe Root is required be 41421736: Here we muſt point off in Periods of three Figurs as before, and proceed as far as *f*; then the Remainder *g* will be 2117736; and now you are to look upon *a*+*b* as *a* only, and proceed as before, that is, firſt form the Diviſor *m* for the Dividend *g*, by the firſt Rule, which directs you to triple the Square of *a*, now *a*+*b* = 340 + triple *a*; that is, $340 \times 340 \times 3 + 340 \times 3 =$ *m*, which will go 6 Times in *g*. And now you are to perform the Conditions of the ſecond Rule, which di-

```
                .  .  . a    b    n
         41421736(300+40+6
         27000000 c
      e
  270900)14421736 d
         10800000
          1440000
            64000
         12304000 f
      m
  347820)2117736 g
         2080800 p
           36720 q
             216 r
         2117736 h
               0
```

rects you to triple the Square of a, now $a+b=340$, and multiply it by b, now called $n=6$ + triple a × into the Square of n; + the Cube of n; that is, $340\times340\times3\times6=p$; $340\times3\times36=q$; $6\times6\times6=r$; and $p+q+r=h$, which being taken from g, the Remainder is nought; ſo that the true Root is 346, which cubed is 41421736 the given Cube, whoſe Root or Side is 346. And were there more than three Places of Figures in a Cube, which will be known by the Points, the Manner of Operation muſt be continued from h, as it was before from g, by calling $a+b+n$ ſtill a, and obſerving the Direction of the two Rules, which, after varying theſe given Cubes, and forming ſeveral Sums of them, will be eaſily and effectually enough fixed in your Mind.

But as it frequently happens that a Decimal Fraction is annexed to a Cube, whoſe Root is required, we muſt next ſhew the Manner of Operation in ſuch Caſes: You know alſo if the Cube has a Vulgar Fraction annexed to it, you can eaſily reduce it into the Form of a Decimal Fraction of equal Value to the Vulgar Fraction. Now let the Cube be 25.625; here you may annex what Cyphers you pleaſe in Periods of Threes; the more you make Uſe of, nearer to the true Root you will approach in Decimals. Suppoſe the Cube ſtood thus, $2\dot{5}.62\dot{5}00\dot{0}00\dot{0}$; you are to proceed in all Reſpects as before directed, and you will find the Root to conſiſt of a whole Number and three Places of Decimals, which Fraction may be near enough for common Purpoſes. Do this by yourſelf, and then form other Sums from this Example.

The laſt Thing to be mentioned to you is; it frequently happens that when a Cube is given in whole Numbers, there will be a Remainder; when you cube the Root found, you muſt at laſt add the Remainder, otherwiſe the Work will not prove. But if you would, for more Accuracy, have the Root alſo conſiſt of Decimal Places, you are to annex three Cyphers to the Remainder for each Decimal Place, and proceed as before. Now vary the Cube 41421736 into ſeveral Sums, and you will have Remainders, to which annex the Cyphers. And if you ſhould meet with a Cube, conſiſting of Decimal Places only, ſuppoſe .75, (vary this alſo into ſeveral Sums); here you muſt annex four Cyphers, to make two Periods, as $.75\dot{0}00\dot{0}$, and proceeding as before, you will have .89 the Root, and a Remainder 45031, to which you may annex three more Cyphers, and go on as before.

Biquadrate Root.

THE Meaning is any Number involved *four* Times; thus, $9 \times 9 \times 9 \times 9 = 6561$ is a *Biquadrate*, and to extract the Root of this Form is firſt to find the Square Root; thus, let the *Biquadrate* be 6561, then the Square Root of this is 81, and the Square Root of 81 is 9, the Anſwer required. I ſhall not trouble you with any Thing farther on this Head, but muſt refer you to the Algebraic Part, if you would make yourſelf Maſter of the Reaſon of the Rules given you in the Extraction of the Cube Root; but if you ſhould not travel on with me, I here take my Leave; yet I muſt tell you, ſooner or later, you will repent your being left behind.

SYSTEM IV.

The ARITHMETIC *of* ALGEBRA.

THE very Name of *Algebra* is ſufficient to fill the Minds of many People with dreadful Apprehenſions of inſurmountable Difficulties; but I muſt tell the worthy Youth, that *Algebra*, to our preſent Purpoſe of ſhewing the Principles on which ſome valuable Parts of Arithmetic are founded, is not ſo terrible, and that what I have to ſay to him on this Subject, with its Application to Uſe, will be a pleaſing Introduction to a better Acquaintance with *Algebra*. Surely it is well worth while to be inſtructed in the Reaſons of many Rules in daily Uſe, and in the Manner of ſhortening the tedious Operations of others, even excluſive of the real Pleaſure of having Opportunities of frequently diſcovering new and better Methods of being uſeful: But when a Perſon only learns *Algebra* with a View of waſting his Spirits in the Solution of algebraic Queſtions, of no other Uſe than to gratify a Vanity of being thought extraordinary, this is no better than an Infatuation, or an ingenious Method of doing nothing. But the Study (if a Youth chuſes ſtill to purſue it) of *Algebra*, in order to underſtand the Theory of the wonderful Phenomena of God's Works in the material World, wherein we always find the Marks of his expreſs Will, is truly noble, and worthy of a rational Being, deſtined upon Terms to ſpend his Eternity with the eternal and glorious Firſt Cauſe.

Addition.] I mentioned before, that *Figures* may be repreſented by *Letters*: Thus, if *a* ſays he has 12*s.* in his Pocket, and c ſays he likewiſe has juſt as much; then ſurely $a=c$; that is, 12*s.*=12*s.* If I ſhould add a Number (ſuppoſe 6) repreſented by b to both Sides of the Equation, (which is this Mark =) the Equality is ſtill preſerved; that is, a will have as much as c, and c as much as a; and then we have $a+b=c+b$; that is, $12+6=12+6$. Now if b repreſents the Figure 6; then we have $12+6=12+6=18$; that is, a has 6*s.* more than he had before, and c has the ſame, and therefore, by adding 6 on each Side, the Equation is preſerved. You eaſily perceive that if you add on both Sides any other Letters, repreſenting Figures, you will preſerve the Equation.

Subtraction.] If $a+b=18$ as before, and we would ſubtract b from each Side of the Equation, this is ſoon done; for it is only tranſpoſing b with the Sign—; thus, $a=18-b$, which is $12=18-6$. Now let us put $a+b+d=20$, as a is 12, b is 6, and d is 2, and ſubtract b and d from each Side of the Equation; then by changing the Sign of b and d, we have $a=20-b-d$, which, in Figures, is $12=20-6-2$; that is, 6 from 20=14, and 2 from this=12; ſo that the Value of each Remainder, when the Subtraction is made, equals 12.

Multiplication.] Let the two Letters, as in Addition, be a, c, each=12, and we would multiply $a=c$ by b, which you know repreſents 6. Now, to multiply each Side of the Equation by b, or any other Letter, is only to join it to them thus, $ab=cb$; that is, $12\times6=12\times6=72$ on each Side. Suppoſe you would multiply $a=20-b-d$ by $d=2$, you muſt join d to every Member, to preſerve the Equation, thus, $ad=20-db-dd$; in Figures it is, $12\times2=20\times2-2\times6-2\times2=24$, and you know multiplying the whole or its Parts is the ſame Thing. So $a+b+d=20$, as above, being multiplied by d, will be in Letters, $ad+db+dd=20\,d$, and in Figures, $24+12+4=40$.

Diviſion.] You know that before it was $a=c=12$; but if we would divide each Side of the Equation by b, we have $\frac{a}{b}=\frac{c}{b}$; in Figures, $\frac{12}{6}=\frac{12}{6}$. Now, to divide $a+b+d=20$, by d, or any other Letter ſtanding for a Figure or Figures, we have $\frac{a+b+d}{d}=\frac{20}{d}$, or $\frac{12+6+2}{2}=\frac{20}{2}$, or, which is the ſame Thing, by dividing the Parts, $\frac{a}{d}+\frac{b}{d}+\frac{d}{d}=\frac{20}{d}$, or $\frac{12}{2}+\frac{6}{2}+\frac{2}{2}=\frac{20}{2}$. So $a=20-b-d$ divided by d, will be $\frac{a}{d}=\frac{20}{d}-\frac{b}{d}-\frac{d}{d}=6$, for $\frac{12}{2}=\frac{20}{2}-\frac{6}{2}-\frac{2}{2}=6$, or ſhorter $\frac{a}{d}=\frac{20-b-d}{d}=6$. However frightful the Appearance of theſe Diviſions may be, you ſee there is nothing eaſier to perform.

Involution.] Let $a=c=12$, as before, and it is required to ſquare each Side of the Equation, which is done thus, $aa=cc$, or $12\times12=12\times12$. So if their Cube is required, it will be $aaa=ccc$, or $12\times12\times12=12\times12\times12=1728$. Here you may obſerve that the Letters are only joined by *Twos* or *Threes*, ſignifying the Figures they repreſent are multiplied twice or three Times into themſelves, which you know is only ſquaring or cubing thoſe Numbers.

Evolution. This is only undoing what is done by Involution: Thus, if the Square Root of $aa=cc$ is required, we have $\sqrt{aa}=\sqrt{cc}$, or, in Figures, $\sqrt{144}=\sqrt{144}=12$, becauſe it is the Root of 144; or you pronounce at Sight that the Square Root of $aa=cc$ is $a=c$. So if the Cube Root of $aaa=ccc$ is required, you ſay, without Heſitation, it is $a=c$, becauſe the Cube was made by joining theſe Letters together by Threes. As to what relates to the Theory of extracting the Square and Cube Roots, and the practical Operations, you will meet with it at the End of Algebra.

The Single Rule of Three.] If the three Numbers are, ſuppoſe, 3 : 12 : : 4 in Letters, $x:z::r$; here multiply the ſecond and third together, and divide by the firſt; that is, $x:z::r:\frac{zr}{x}=16$, the Number required, or, in Figures, $3:12::4:\frac{12\times4}{3}=16$, as before. So if by the Senſe of the Queſtion the Proportion is Inverſe, we have $x:z::r:\frac{xz}{r}=9$. Suppoſe we would prove that $x:z::r:\frac{zr}{x}$ are Proportionals; you know the Way to do this, is to ſee if the Product of the two Means is equal to that of the two Extremes. In order to do this, by Letters, you are to multiply x the firſt Number into $\frac{zr}{x}$ the fourth; now remember that this 4th Number ſtands in the Form of a Fraction, and to multiply a Fraction is to multiply its Numerator; therefore $\frac{xzr}{x}=zr$; that is, $\frac{3\times12\times4}{3}=12\times4=48$, and conſequently the Numbers and their Letters are in Proportion. So in the Inverſe Proportion, $\frac{xz}{r}:z::x:r$; that is, $\frac{xzr}{r}=zx$, or $\frac{12\times3\times4}{4}=12\times3=36$; for you know in this Caſe, the fourth Number is to the ſecond, as the firſt is to the third; therefore it is we ſay, $\frac{xz}{r}:z::x:r$, or $\frac{36}{4}:12::3:4$.

Reduction.] Suppoſe I ſay $y+5-3=18$, and I do not know what y is, but want to find the Number this Letter ſtands for. Now if you, with a contrary Sign, tranſpoſe the Figures ſtanding on the ſame Side of the Equation with y, which is unknown, you will have this Expreſſion, $y=18-5+3$; Now you know the Equation is ſtill preſerved, and that y is equal to 16; for $18-5=13$, and 3 added to $13=16$. Pray, where is the Difficulty of finding out what Number y ſtood for? You ſee by tranſpoſing the Numbers you eaſily diſcover what you want to know. Let $\frac{2x}{3}=\frac{30}{5}$, required x. Here obſerve that, on both Sides of the Equation, you have the Form of two Fractions; in all ſuch Caſes, you remember that the Equality is preſerved, if you multiply the Numerator of one Fraction into the Denominator of the other; for $\frac{6}{3}=\frac{8}{4}$; here $6\times4=3\times8=24$: Thus, in like Manner, $10x=90$; and now you have nearly diſcovered what x is equal to. Look at it a little, and ſee if you do not pereceive a Method of letting x ſtand alone. You ſee x is multiplied into 10, and you know Diviſion undoes what Multiplication does; therefore, in all ſuch Caſes, divide both Sides of the Equation, and in this by 10, and you will have $x=9$; for the *Tens* in $10x$ will go x Times, becauſe $10\times x=10x$, and the Tens in 90 will go 9 Times. Is there any Myſtery or Difficulty in this? Suppoſe, $\frac{2x}{3}=30$, required x. Now conſider, you have the Form of a Fraction; that is, whoſe Numerator $2x$ is a *Dividend*, and whoſe Denominator 3 is a *Diviſor*, and whoſe *Quotient* is 30. You, ſurely, can come eaſily at the Value of x, as you know the Diviſor multiplied into the Quotient is equal to the Dividend; that is, $3\times30=2x$, or $2x=90$. Here you ſee x is multiplied into 2; therefore undo it; that is, divide both Sides by that Multiplier 2, and we have $x=45$. Suppoſe, $2x\frac{30}{5}=$; required x. Here, again you have a Quotient $\frac{30}{5}$(expreſſed like a Fraction); therefore multiply the Diviſor (or Denominator) 5 into $2x$, and you have $10x=30$; and now, as x is multiplied into 10, undo it by dividing by 10, and we have $x=\frac{30}{10}=3$. Suppoſe, $\frac{2x}{3}+4=\frac{7x}{12}+9$, required x. Now take Care; on each Side you have a mixed Fraction, therefore reduce each to an improper one, by multiplying the Integer by the Denominator of the Fraction, and taking in the Numerator, ſubſcribing the ſame Denominator, as you were taught in Vulgar Fractions; that is, $\frac{3\times4+2x}{3}=\frac{12\times9+7x}{12}$, or ſhortened, $\frac{12+2x}{3}=\frac{108+7x}{12}$; and here again you have two equal Fractions, and if you multiply the Denominator of one into the Numerator of the other, the Equality is ſtill preſerved: For ſuppoſe $\frac{4}{3}=\frac{8}{6}$; here, $8\times3=4\times6=24$; and therefore $144+24x=324+21x$. Now view the Equation a little, then tranſpoſe the Members with a contrary Sign, that you may have the Figures on one Side, and the Letter x on the other, tho' multiplied into ſome Number; that is, $24x-21x=324-144$. Shorten this Expreſſion, by ſaying $21x$ from 24, and there remains $3x$, and 144 from 324, and there remains 180, then it will be $3x=180$. Now you have fairly managed the Point, as you ſee 3 is multiplied into x; therefore undo it by Diviſion, in ſaying the *Threes* in $3x$ will go x Times, becauſe $3\times x=3x$, and the *Threes* in 180 will go 60 Times; conſequently $x=60$. Let me intreat the Learner to form ten, twenty, or thirty Examples of $\frac{2x}{3}+4=\frac{7x}{12}+9$, by altering the Figures, and he will be richly rewarded for all his Trouble. Among theſe Examples, let ſometimes the ſingle Fraction be on one Side; thus, $\frac{2x}{3}+4-9=\frac{7x}{12}$; then it will be $\frac{2x+12-27}{3}=\frac{7x}{12}$, and to clear each Side of the Fractions multiply each Numerator into the other Denominator, as before, and we have $24x+144-324=21x$; and then, with a contrary Sign, getting the Figures on one Side and the Letters on the other, you will have $24x-21x=324-144$, as before, to find x: Likewiſe let ſome Examples be

$2x=$

$\frac{2x}{3} = \frac{7x}{12} + 9 - 4$, and you will find they will, in all Forms, by taking Care of the contrary Signs, give you the ſame Value of x, and gratify you with a pleaſing Specimen of that Latitude of Expreſſion of which Algebra is capable. Laſtly, ſuppoſe, we have given $\frac{xx-12}{3} = \frac{xx-4}{4}$, to find x. Here, again, you have two equal ſingle Fractions, or Quotients, expreſſed by two Dividends, having their Diviſors ſet under them, in the Form of Fractions. Now clear the Equation of the Fractions, by multiplying croſs-ways, as before, and we have $4xx - 48 = 3xx - 12$; then it will be $4xx - 3xx = 48 - 12$; but ſhortened, we have this Square $xx = 36$. Now conſider x is multiplied into x, therefore undo it by Evolution; that is, extracting the Square Root of $36 = 6$, and then $x = 6$.

Simple Intereſt.] Here let $R =$ the Intereſt of one Pound for a Year, then we have $1\,y : R :: 2\,y : 2\,R$, and three Years : $3\,R$, &c. for any Number of Years $= t$ the Time; that is, : $1\,y : R :: t : t\,R$ the Time and Rate. Let the Principal, or Money lent out upon Intereſt, be $= P$, then we have $1\,l. : t\,R :: P : t\,RP$; that is, as one Pound is to its Intereſt $t\,R$ in a given Time, ſo is any Sum of Money P, to its Intereſt $t\,R\,P$ in that Time; the Principal and Intereſt added together is called the Amount $= A$; that is, $t\,R\,P + P = A$. Now tranſpoſe P, with a contrary Sign, and then we have $t\,R\,P = A - P$; that is, the Rule will be, *(t) the Time multiplied into* (R) *the Intereſt of one Pound, and this Product multiplied into* (P) *the Principal lent out is the Intereſt required.* And now with what Eaſe can you, as you may want it, find a Rule, whereby to anſwer any Queſtions concerned in Simple Intereſt: The Amount you have in $t\,R\,P + P$; the Intereſt you have in $t\,R\,P$; and if you want the Time, you have $t = \frac{A-P}{R\,P}$, obtained by dividing each Side of the Equation by $R\,P$; and when we would find R, we have $\frac{A-P}{t\,P} = R$; and if you want P, we have $\frac{A}{t\,R+1} = P$; for you know $t\,R + 1 \times P$ back again, is $P \times 1 = P$, and $t\,R \times P = t\,R\,P$; that is, $\frac{A}{tRP+P} = 1$, which is as much as to ſay, the Amount, divided by the Principal and Intereſt, is equal to one.

You will be pleaſed to remember, that t ſtands for not only a Year or more Years, but alſo for any odd Time, more or leſs than a Year; and therefore ſuch odd Time muſt be turned into a Decimal; thus, ſuppoſe there are 7 Years 8 Months and 13 Days; in all ſuch Caſes we are to find the Decimals of $\frac{8}{12}$, and $\frac{13}{365}$, and add them together, and call the Years and Decimal of a Year equal to t. And now let the Sum lent out for this laſt mentioned Time be 785*l.* 17*s.* 6*d.* at 4 *per Cent.* and let it be required to find not only the Intereſt tRP, but every Letter concerned in the Theorem $tRP+P=A$; and let there be ten, twenty, or thirty Sums formed, every Time varying the Circumſtances of the Figures. You cannot be at a Loſs to find the Intereſt of one Pound for a Year, repreſented by R; for, 100*l.* : 5*l.* : : 1*l.* : .05*l.* and 100*l.* : 4*l.* : : 1*l.* : .04*l.* &c. and if it is $\frac{1}{8}$, $\frac{1}{4}$, or $\frac{1}{2}$, &c. the ſecond Number will be 4.25, 4.5 4.125, &c. which will make the fourth Number R, .045, .0425, .04125, &c. and the ſame at 5 or 3 *per Cent. per Annum* with this odd Rate.

Compound Intereſt.] Here let R ſtand for one Pound Principal and its Intereſt for one Year; then, at 5 *per Cent. per Annum*, we have 1 : 1.05 : : 1.05 : 1.05×1.05; that is to the Square of R, to be conceived thus; as one Pound is to its Amount in one Year, ſo is that Amount put out to Intereſt to its Amount at the End of the ſecond Year; that is, 1*l.* : R : : R : RR, and 1*l.* : R : : RR : RRR, and 1*l.* : R : : RRR : $RRRR$, and ſo on, for any Number of Years; now let $t=$ the Number of Years; that is, how many Times R is multiplied into itſelf, ſquared, cubed, &c. that is, R^2 the Square, R^3 the Cube, &c. or R^t where t ſhews the Index or Power of the laſt Year of R. You cannot complain of any Difficulty ſo far in Compound Intereſt, as what has been ſaid is ſo plainly built on unerring Proportion, or *Rule of Three*, whoſe firſt Number is one; and therefore it was only $R\times R$ to the End of the ſecond Year, $RR\times R$ to the End of third Year, &c. Now 1*l.* : R^t : : P : PR^t; that is, as one Pound is to its Amount, (Principal and Compound Intereſt) in a given Number of Years, ſo is any Principal, or Sum of Money lent out, to its Amount (Principal and Compound Intereſt) in the ſame Time; that is, $PR^t=A$. From this Theorem you are prepared to anſwer all common Queſtions concerning Compound Intereſt; that is, to find the Intereſt, the Principal, the Amount, and the Time. For PR^t gives us the Amount, $\frac{A}{R^t}=P$, the Principal, $PR^t.-P$ gives us the Compound Intereſt, which the Principal gained in the Time, and $\frac{A}{P}=R^t$ which enables us to find the Time, which is only undoing what was before done; that is, as 1.05×1.05, *&c.* produced R^t ſo dividing continually back again, by 1.05, till nothing remain, we ſhall have t, by counting the Number of Times we uſed 1.05 for a Diviſor.

And now let 785 *l.* at 4 *per Cent. per Annum* be continued out at Compound Intereſt for the Term of 7 Years: And let me be indulged in the Favour of your finding all the above Particulars, of which the Theorem $PR^t = A$ is capable, as mentioned above; and let there be at leaſt ten of ſuch Sums formed of the Queſtion, every Time varying the Circumſtances of the Figures, in order to make you perfect in this Part of Knowledge, as you will have Occaſion for it when you come to Annuities.

The Double Rule of Three.] Let us take the 38th Queſtion, *Syſtem* 2, for our preſent Purpoſe. Let $P = 100$ *l.* $G = 4$ *l.* $T = 12$ Months; then to correſpond with theſe, let $p = 250$ *l.* $g = 2$ *l.* 10 *s.* and $t = 3$ Months, and we have $P : G :: p : \frac{Gp}{P}$, and $T : \frac{Gp}{P} :: t : g$, and, as theſe are in Proportion, we have $Tg = \frac{Gpt}{P}$, which will help you with great Eaſe to any Anſwer, which the *Double Rule of Three* can give you, that is in finding p, g, t. Now you ſee Gpt is a *Dividend*, P the *Diviſor*, and Tg the *Quotient*; therefore multiplying theſe two laſt together, we have $TgP = Gpt$ for our *general Theorem*. If we want g, we have, if we divide by TP, $g = \frac{Gpt}{TP}$, for TgP is conſidered as a *Dividend*, TP its *Diviſor*, and g the *Quotient*. If we want p, we have $\frac{TgP}{Gt} = p$, for the ſame Reaſon; and to find t, we have $\frac{TgP}{Gp} = t$, for the like plain Reaſon. And now you may be ſaid to be Maſter of this uſeful Affair, and which will ſave you much Time, and prevent a good deal of Vexation, which neceſſarily ariſes when we grope in the Dark. It only remains that I ſhould tell you p ſtands for a Principal, the Weight of Goods carried, Reapers cutting down the Products of the Earth, Workmen building, Horſes conſuming Corn, Perſons ſpending Money, or conſuming Proviſions, or any Thing elſe ſtanding for the Cauſe of what is done, or, in the Language of Grammar, a Nominative Caſe; that g ſtands for the Gain, the Money ſpent, or paid for Carriage, the Acres mowed, Work done, the Corn, *&c.*

[illegible]at, or whatever the Cauſe P produces; that t ſtands for the Time in which p performs, or the Diſtance the Goods are carried, or when the Cauſe produced g. Theſe Things being premiſed, the Learner will find much Satisfaction from his general Theorem $TgP = Gpt$; and he may now perceive the Reaſon of the Rules in working his Queſtion by the third Method in p. 85. If g is required, the Rule is, multiply the three laſt Terms for the Dividend, and the two firſt for the Diviſor; that is, $\frac{Gpt}{PT}$; but if p or t are required, the Rule is, multiply the third and fourth Terms for the *Diviſor*, and the other for the Dividend. According to theſe two Rules, the Statings will be $\begin{array}{l}P\text{——}T\text{—}G\\ p\text{——}t\text{—}\end{array}$ for the firſt, as the Blank falls under the third Place; and $\begin{array}{l}P\text{—}T\text{—}G\\ p\text{—}\quad\text{—}g\end{array}$ for the ſecond, as the Blank falls under the ſecond Place; and $\begin{array}{l}P\text{—}T\text{—}G\\ \text{—}t\text{—}g\end{array}$ for the ſecond Rule alſo, as the Blank falls under the firſt Place. Now you may perceive, by the ſecond Rule, that Gp and Gt are the third and fourth Terms; and you may obſerve that the Rules are contrived from the Theorem, $TgP = Gpt$, which delivers you from your Perplexity.

Diſcount and Equation of Payments.] In Simple Intereſt, *by Algebra*, we had $trp + p = a$, and $\frac{a}{tr+1} = p$, which call the Debt to be paid. Now, tho' the Goods were to be paid for at a particular Time; that is, p is the preſent Money of that Debt repreſented by a, to be diſcharged in t Time, and let the Diſcount be made at 5 *per Cent. per Annum*, or $r = .05$. Here then $\frac{a}{tr+1} = p$ is alſo a *diſcounting Theorem*; $tr + 1$ is the *Diviſor*; that is, (ſuppoſing $t = 12$ Months) $1 \times .05 + 1 = 1.05$; and let a, the *Dividend*, $= 200$ *l.* that is, if 1.05 one Year hence will be worth 1 *l.* preſent Money, 200 *l.* one Year hence, will be worth 190 *l.* .476, or 190 *l.* 9 *s.* $6\frac{1}{4}$ *d.* or cloſer, as 1.05 : 1 : : 200 : 190.476; therefore the Diſcounting Rule is, divide the Debt by the Amount of one Pound for the Time, and you have the preſent Money, and this deducted from the Debt gives the Diſcount. Now, let there be two other Debts to be paid in ſtill different Times, and find the preſent Money of each, and then add all the three preſent Sums together, and this Total, which call P, is what the Buyer muſt pay down if he would diſcount; that is, diſcharge theſe three Debts: But inſtead of diſcharging P, it muſt be continued, ſuppoſe, at 5 *per Cent. per Annum*, till P amounts in t Time to S, the Sum of the Debts. Theſe Things being premiſed, we have $trP+P=S$; that is, all the Rebates or Diſcounts, trP, added to the Total P of the preſent Sums, is equal to the Sum of all the Debts when they are reſpectively due. And now t the Time is eaſily diſcovered; for $trP=S-P$, and $t=\frac{S-P}{rP}$; that is, all the three Debts may be diſcharged in t Time, with Juſtice to both Parties. Hence then, we have, in *Equation of Payments*, theſe two Theorems concerned, $\frac{a}{tr+1}=p$, and $t=\frac{S-P}{rP}$. And now let me prevail upon the Learner to reſume the Queſtions he did in the Rules of Diſcount and Equation of Payments, by common Arithmetic, and he will be extremely pleaſed with himſelf, as he will make himſelf intirely Maſter of theſe uſeful Branches of Knowledge, and familiarize theſe algebraic Forms, which ſo happily eaſe the Memory.

Compound Fellowſhip.] I hope the Learner will take Pleaſure in the Arithmetic of Algebra, as he has hitherto ſeen what Sentiments he ought to form of its peculiar Advantages. Four Merchants, *A*, *B*, *C*, *D*, engage each 1000*l.* in Trade, for their mutual Profit, which is to be divided among them in Proportion to the Time each Capital was employed. *A*'s Time was 12 Months, *B*'s 9 Months, *C*'s 6 Months, and *D*'s 3 Months. They gained 500*l.* Required the Share of each. Let the Gain of $D=x$; then $C=2x$, $B=3x$, and $A=4x$, becauſe *A*'s Money was four Times longer in Trade. You know all their Gains muſt make 500*l.* that is, $4x+3x+2x+x=500l.$ therefore $10x=500$; and then $x=\frac{500}{10}=50l.$ *D*'s Gain; 100*l.* or $2x=C$'s Gain; 150*l.* or $3x=B$'s Gain; 200*l.* or $4x=A$'s Gain; then $200+150+100+50=500$ the whole Gain. But ſuppoſe one of the Partners, *A*, for Inſtance, put in 2000*l.* then he muſt have $8x$; then $8x+3x+2x+x=500$; and we have $14x=500$, and $x=\frac{500}{14}=35.714$; then *C*'s Gain is 71.428, and *B*'s Gain is 107.142, and *A*'s Gain is 285.712; and all theſe added together will make 499.996*l.* but if you make more Decimal Places than there are in *D*'s Share 35.714, you will ſtill come nearer the 500*l.* Gain: And thus, in general, you may conſider what Part one Partner's Share is of another, and increaſe or diminiſh x accordingly, though all their Capitals differ. And now give me

Leave to hope you will form many Sums, of more or leſs Merchants, thus uniting with very different Capitals and Gain, *&c.*

The Arbitration of Exchanges.] In the Arithmetical Part I have only taught you how to caſt up a Bill of Exchange; but now you muſt be acquainted with that ingenious Part of a Merchant's Buſineſs, called the *Arbitration of Exchanges.* Let there be three Merchants, *L* of *London*, *P* of *Paris*, and *A* of *Amſterdam*, united, for their mutual Advantage, to deal in Exchanges: For you muſt remember, that there is a continual Riſing and Falling of the Prices of Exchange, ſomething like what I told you of the Stocks or Public Funds. Suppoſe at one Time you find for 32 *d.* of our Money being paid in *London*, you could receive in *Paris* the Value of a *French* Crown; but at another Time you muſt give 33 or 31 *d.* for each Crown. You may look upon the chief Cauſe of theſe Alterations to be a great Plenty or Scarcity of Bills; for if many Merchants *(D)* apply to *L* to give them Money for the Bills on *Paris*, he takes the Advantage of the Demand, and for (ſuppoſe) 1000 *l.* will give them Bills on *P* at 33 *d. per* Crown, and then ſurely *D* cannot receive ſo much in *Paris* as when the Price is 31 *d.* For as 31 *d.* or 33 *d.* : 1 Crown : : the Pence in 1000 *l.* and therefore you receive more or leſs Crowns in *Paris*, in Proportion to the Price in *London*. But now let us conſider *L*, *P*, *A*, dealing in the Exchanges in Company.

The Exchange from *L* to *P* = 32 *d. per* Crown of 3 Livres, from *P* to *A* = 54 Groots *per* Crown; that is, 32 *d.* = 1 Crown, and 54 Groots = 1 Crown, then 32 *d.* or Crown is worth 54 Groots; What muſt the Exchange from *A* to *L* be to have neither Gain or Loſs? Then as 32 *d.* : 54 *g* : : 240 *d.* in a Pound : 405 *g.* the Courſe of Exchange from *L* to *A*. Now let the Sum negotiated be 1000 *l.* Then $\frac{1000\,l. \times 20 \times 12}{32} =$ 7500 Crowns; and 7500 × 54 *g.* = 405000 *g*; and $\frac{405000\,g.}{405\,g.}$ = 1000 *l.* So that if a Draught for 1000 *l.* were remitted to the Partner *P*, at 32 *d. per* Crown, it would be worth 7500 Crowns there; and if the 7500 Crowns were remitted to the Partner *A*, at 54 Groots or *Dutch* Pence *per* Crown, the Draught would have Credit there for $\frac{405000\,g.}{40} =$ 10125 Guilders; and if theſe Guilders were remitted to the Partner *L*, at 33 *s.* 9 *d.* *Flemiſh per* Pound *Sterling*, the Draught will be found to be worth juſt 1000*l.* for as 33*s.* 9*d.* *Flemiſh* × 12 Groots or *Dutch* Pence, is to 1*l.* *Sterl.* ſo is 10125 Guilders × 40 Groots or *Dutch* Pence, to 1000*l.* *Sterl.* Herein, throughout this Tranſaction, you perceive the Partnerſhip in Exchanges has gained nothing for their Trouble; but if the Exchange, from *A* to *L* had been under 33*s.* 9*d.* *per* Pound *Sterling*, there would have been a Gain, becauſe a leſs Diviſor than $\frac{405000}{405}$ would give a greater Quotient; that is, more than 1000*l.* Hence alſo you perceive, that if the Exchange from *A* to *L* is higher than the Equality; that is, when nothing is gained, there will be a Loſs, if Bills are negotiated upon ſuch Terms. I ſhall leave it to the Learner to ſuppoſe *P* to begin the Courſe, and to remit firſt to *A*, and *A* to remit to *L*, and *L* to remit to *P*, and to ſettle the Account, when it will be found nothing has been gained; and in like Manner let *A* begin the Courſe, and cloſe the Account, as you know there muſt be Charges in Poſtage, let who will have the Management of the Courſe. I muſt intreat the Youth to form many Sums in what has been ſaid, by varying the Circumſtances, that he may fully underſtand what follows.

Now let there be more Partners admitted, and let the Exchanges be as follow, to find the Equality between the *Flemiſh* and *Sterling* Pound, whereby there will be neither Loſs nor Gain to the Company of Merchants. Suppoſe by the preſent Courſe of Exchange,

64*d.* of *London* (*a*) = 1000 Rees of *Portugal* (*B*)
400 Rees (*b*) = 50 Groots of *Hamburgh* (*C*)
100 Groots (*c*) = 1 Ducat of *Venice* (*F*)
60 Ducats (*f*) = 100 Crowns of *France* (*G*)
1 Crown (*g*) = 56 Groots of *Holland* (*H*)
How many Groots of *Holland* (*h*) } = 240*d.* of *London* (*A*)?

Here you perceive, when the Money has been remitted from *London* through all theſe Places, and nothing gained or loſt, the Trial at laſt is, whether the preſent Courſe between *London* and *Holland* is above or below the Equality; if above, there will be a Gain; but if the Price is below, there will be a Loſs. And now you will never miſs your Way, if you take Care to let the firſt and third be both of one Name; and obſerve what the fourth, of the ſame Name with the ſecond, s in every Stating equal to. Now, then, as $B : a : : b : \frac{a\,b}{B} = C$; and as $C : \frac{a\,b}{B} : : c : \frac{a\,b\,c}{B\,C} = F$; and as $F : \frac{a\,b\,c}{B\,C} : : f :$

$\frac{abcf}{BCF} = G$; and as $G : \frac{abcf}{BCF} :: g : \frac{abcfg}{BCFG} = H$; and a $\frac{abcfg}{BCFG} : H :: A \frac{BCFGHA}{abcfg} = b = 437\frac{1}{2}$ Groots of *Holland* = 240 Pence of *London*. And here, if the preſent Price between *Holland* and *London* is above $437\frac{1}{2}$ Groots *per* £. *Sterling*, the Partner in *London* will direct the Remittance to *Holland*; but it means nothing to negotiate at $437\frac{1}{2}$ Groots, and worſe than nothing, if the Buſineſs is done under the Equality $437\frac{1}{2}$ Groots of *Holland*.

You plainly ſee that $BCFGHA = abcfgb$, and thereſore multiplying the Numbers a, b, c, f, g one into another, gives you a *Diviſor*, and multiplying B, C, F, G, H, A one into another, is the *Dividend*, and b is the *Quotient*, or Anſwer, $437\frac{1}{2}$ Groots of *Holland*, and is a much ſhorter Operation than forming the Proportions which give the Rule. As you are convinced $BCFGHA = abcfgb$, ſo you may ſtrike out c and G, each being 100, and the Equation, you know, is ſtill preſerved; and hereby the Labour, on many Occaſions, will be leſſened, as it may frequently be, by abbreviating the Parts of the whole on each Side: Thus $\frac{64}{8} = 8$; and $\frac{56}{8} = 7$; and $\frac{60}{60} = 1$; and $\frac{240}{60} = 4$. Here, then, we have $\frac{1000 \times 50 \times 7 \times 4}{8 \times 400} = 437\frac{1}{2}$, as before.

Again: Let us ſuppoſe the preſent Courſe of Exchange to be as follows:

56*d. Flemiſh* (a) = 1 Crown of *France* (B)
100 Crowns (b) = 60 Ducats of *Venice* (C)
1 Ducat (c) = 100 Pence of *Hamburgh* (F)
50*d.* of *Hamburgh* (f) = 400 Rees of *Portugal* (G)
1000 Rees (g) = 64*d. Engliſh* (H)
How many *d. Engliſh* (b) = 800£. *Flemiſh* (A)?

Here we have, as $B : a :: b : \frac{ab}{B} = C$; and as $C : \frac{ab}{B} :: c : \frac{abc}{BC} = F$; and as $F : \frac{abc}{BC} :: f : \frac{abcf}{BCF} = G$; and as $G : \frac{abcf}{BCF} :: g : \frac{abcfg}{BCFG} = H$; and as $\frac{abcfg}{BCFG}$, *Flemiſh* Pence, : H, *Engliſh* :: A, *Flemiſh* : $\frac{BCFGHA}{abcfg} = b$, *Engliſh* = $105325\frac{5}{7}$*d.* = 438*l.* 17*s.* 1*d.* $\frac{5}{7}$. Here you muſt remember, that A = 800*l. Flemiſh*, and therefore it will be $A = (800 \times 20 \times 12)$ 192000 *Flemiſh* Pence, becauſe the firſt Number is in the ſame Denomination. And now I ſhall leave you to abbreviate and ſtrike out, as before ſhewn, (to ſhorten the Work) and to form for yourſelf many Examples, both in Company and ſingle Remittances, *&c.* from thoſe already given, in order to be very perfect in this ingenious Branch of a Merchant's Buſineſs.

It may not be improper to mention ſomething relating to *Weights* and *Meaſures*, when their Proportions to one another are properly adjuſted: In the Courſe of Trade, if it were required to find the *Equality* between *London* and any given Place, the very ſame Theory and Practice holds good, as in Exchanges of Coins, Value for Value: Thus,

1. MEASURES.

35 *Ells of Vienna* (a) = 24 *of Lyons* (B)
3 *of Lyons* (b) = 5 *of Antwerp* (C)
100 *of Antwerp* (c) = 125 *of Frankfort* (F)
50 *of Frankfort* (f) = *How many of Vienna* (A) ?

2. WEIGHTS.

10 *lbs. Avoirdupois of London* (a) = 9 *of Amſterdam* (B)
45 *of Amſterdam* (b) = 49 *of Bruges* (C)
98 *of Bruges* (c) = 116 *of Dantzick* (F)
How many of Dantzick (f) = 112 *of London* (A) ?

1. *Meaſures.*] As $B : a :: b : \frac{a\,b}{B} = C$; and as $C : \frac{a\,b}{B} :: c : \frac{a\,b\,c}{B\,C} = F$; and as $F\ \frac{a\,b\,c}{B\,C} :: f : \frac{a\,b\,c\,f}{B\,C\,F} = A = 35$ Ells of *Vienna* equal to 50 of *Frankfort*, the Anſwer.

2. *Weights.*] As $B : a :: b : \frac{a\,b}{B} = C$; and as $C : \frac{a\,b}{B} :: c : \frac{a\,b\,c}{B\,C} = F$; and as $\frac{a\,b\,c}{B\,C} : F :: A : \frac{B\,C\,F\,A}{a\,b\,c} = f = 129,\ 92$ *lbs.* of *Dantzick* equal to 112 of *London*, the Anſwer.

Here you may perceive that, in the *Meaſures*, the Right Hand Column (when the Numbers are multiplied one into another) becomes the *Diviſor*, and the Left the *Dividend*; but in the *Weights*, from the Nature of the Queſtion, the Right Hand becomes the *Dividend*, and the Left the *Diviſor*; and that in both Caſes you are only to obſerve the Wants of the laſt Stating, whether to reverſe the firſt and ſecond Numbers.

Theory of Extracting the Square and Cube, &c. Roots.] It has been ſaid in the Arithmetical Part, that a *Square* is made of multiplying a Number into itſelf; but theſe Numbers may be repreſented by Letters, and involv'd (rais'd) to a Square, by what we have ſaid of Multiplication in Algebra. Thus, required the *Square Root* of $aa+2ab+bb$; this is involv'd from $a+b$. Here, as in the Margin, the Root of aa is a, which being ſquar'd and put under aa, and ſubtracted from it, the Remainder is 0; then bring down $2ab+bb$, the other Part of the whole given Square, whoſe Root conſiſts of two Places $a+b$. Now, the Diviſor $2a+b$ will give a Quotient b, which multiplied into the Diviſor $2a+b=2ab+bb$, and this taken from the Dividend $2ab+bb$, the Remainder is 0; ſo that $a+b$ is the true Root requir'd, as by the Proof in the Margin. And this Operation is nothing more than the Theory of the Rule, formerly given, to extract the Square Root in Numbers.

$$\begin{array}{lr}
 & aa+2ab+bb\,(a+b \\
 & aa \\
\hline
2a+b) & 2ab+bb \\
 & 2ab+bb \\
\hline
 & 0
\end{array}$$

Proof

$$\begin{array}{r}
a+b \\
a+b \\
\hline
ab+bb \\
aa+ab \\
\hline
aa+2ab+bb \\
\hline
\end{array}$$

Again: Required the *Square Root* of $aa+2ab+bb+2ac+2bc+cc$; this is involv'd from $a+b+c$. Here, as below, the Root of aa is a, which being ſquar'd and put under aa, and deducted therefrom, we have 0; then bring down the next Period, and the proper *Diviſor* you will find to be $2a+b$, and the *Quotient* b, becauſe theſe being multiplied together will produce the *Dividend* $2ab+bb$, and nothing will remain. Then bring down the next Period $2ac+2bc+cc$, and the proper *Diviſor* will be $2a+2b+c$, and the *Quotient* c, becauſe theſe, multiplied together, will produce the *Dividend*, and nothing will remain. And here again you ſee the Reaſon in *Arithmetic* of doubling the firſt and ſecond Figures in the Quotient for a new Diviſor, and of affixing the third Quotient Figure to it, and then multi-

$$\begin{array}{ll}
 & aa+2ab+bb+2ac+2bc+cc\,(a+b+c \\
 & aa \\
\hline
2a+b) & 2ab+bb \\
 & 2ab+bb \\
\hline
2a+2b+c) & 2ac+2bc+cc \\
 & 2ac+2bc+cc \\
\hline
 & \qquad 0
\end{array}$$

multiplying the Diviſor ſo form'd by that third Figure in the Quotient. And thus you may proceed, as long as you pleaſe, from any given Square, doubling the Members of the Quotient, for a new Diviſor, affixing ſuch a Letter as the new Dividend may require.

In like Manner let the *Cube Root* of $aaa+3aab+3abb+bbb$, be required; this is involv'd from $a+b$. Thus, the Root, of aaa is a, and this cub'd and taken from aaa, the Remainder is 0. Then bring down the other Members, $3aab+3abb+bbb$, and form the Diviſor thus; $3aa+3a$ will give b in the Quotient near enough to the Dividend; for when we ſay the Diviſor is $3aa+3a$, and the Quotient b, we are to ſuppoſe b, the Quotient, is likewiſe contained in ſuch a Manner in the Diviſor, as, when this is multiplied by it, the Product will be the neareſt to the Dividend; ſo that the true Diviſor ſhould be $3aa+3ab+bb$, and then the Work is almoſt ended: But, in order to diſcover b, we may ſay $3aa+3a$ will require b in the Quotient; and then ſupplying the Defect in the Diviſor by compleating it to $3aa+3ab+bb$, and multiplying it by the Quotient b, we have $3aab+3abb+bbb$; which being placed under the Dividend, and ſubtracted from it, there remains 0. And hence you ſee the Reaſon, in extracting the Cube Root in Figures, of firſt forming the Diviſor, then placing the ſecond Quotient Figure, and adding the Triple of $a\times$ into the Root $b+$ the Triple of $a\times$ into the Square of $b+$ the Cube of b, and then taking the Sum of all from the laſt Dividend. And thus we may proceed on to more Places from higher Cubes.

$$\begin{array}{r|l} & aaa+3aab+3abb+bbb\ (a+b \\ & aaa \\ \hline 3aa+3a) & 3aab+3abb+bbb \\ & \underline{3aab+3abb+bbb} \\ & \qquad 0 \end{array}$$

Poſition.] Poſition, or the *Rule of Falſe*, is more properly the Buſineſs of Algebra, though always inſerted in Books of Arithmetic, and never underſtood as it ought.

Queſtion 1. In a Bag of Money, the Half, Third, and Fourth all made 130*l*. How much was in the Bag? Here put a for what was in the Bag, then you are to add the Parts according to the Queſtion; but they muſt all be equal to 130, as in the Margin. You may perceive the

$$\frac{a}{2}+\frac{a}{3}+\frac{a}{4}=130$$

$$\frac{26a}{24}=130$$

$$26a=130\times24$$

$$a=\frac{130\times24}{26}=120$$

three Fractions muſt be added together, and they will make $\frac{26a}{24}$, which is ſtill equal to 130, as you have not alter'd the true Equation. You then, in order to come at a, multiply 24 into 130, as you have multiplied $26a$ by 24, by taking away the Diviſor 24, as $\frac{26a}{24}$ is a Quotient. And now you you have $26a=130\times24$; and dividing each Side of the Equation by 26, you have $a=\frac{130\times24}{26}=120$ *l.* in the Bag.

Queſtion 2. A Gentleman being aſk'd how old he was; why, ſays he, if you take 15 from my Age, and then add 21 to the Remainder, and then ſubtract 7 from that Sum, and add 19 to what is then left, the Sum will be 43. Requir'd the Age. According to the Queſtion, put a for the Gentleman's Age, then you have $a-15+21-7+19$, and the Reſult of all this muſt be equal to 43. You have nothing more to do in order to come at a, than to tranſpoſe $-15+21-7+19$ with a contrary Sign, and thereby you will find a, the Gentleman's Age, is equal to 25.

$$
\begin{aligned}
a-15+21-7+19&=43\\
a=43+15-21+7&-19\\
a&=25
\end{aligned}
$$

Queſtion 3. Three Perſons diſcourſed together concerning their Ages; ſays *A*, I am 20 Years of Age; ſays *B*, I am as old as *A* and Half *C*; and ſays *C*, I am as old as you both. Required the Age of each. Here put a for *C*'s Age; then, by the Conditions of the Queſtion, a muſt be equal to *A*'s and *B*'s Ages; that is, $a=40+\frac{a}{2}$; and now tranſpoſing $\frac{a}{2}$ with a contrary Sign we have $a-\frac{a}{2}=40$; and this being reduced to an improper Fraction, we have $\frac{2a-a}{2}=40$; and this multiplied by 2 we have $2a-a=80$: Here, taking once a from twice a, we have $a=80$ *C*'s Age, *B*'s Age $=60$, and *A*'s 20, as before.

$$
\begin{aligned}
a&=40+\frac{a}{2}\\
a-\frac{a}{2}&=40\\
\frac{2a-a}{2}&=40\\
2a-a&=80\\
a&=80
\end{aligned}
$$

Queſtion 4. Three Men, *A*, *B*, and *C*, would divide 100*l.* among them, as that *B* may have 3 *l.* more than *A*, and *C* have 4*l.* more than *B*. Required the Share of each. Put a

for what *A*'s Share comes to; then *B*'s Share is $a+3$; and *C*'s Share $a+7$: But all theſe muſt equal 100. So then, when the Expreſſion is contracted $3a+10=100$; and $3a=100-10$, by tranſpoſing 10; and to come at a we muſt divide both Sides by 3, and then we have $a=\frac{100-10}{3}=30$, and the other two will be 33 and 37, and all $=100l.$

$$a+a+3+a+7=100$$
$$3a+10=100$$
$$3a=100-10$$
$$a=\frac{100-10}{\cdot\cdot 3}$$

Queſtion 5. A Perſon left 100*l.* in Legacies; to *A* a certain Sum, to *B* twice as much, wanting 8*l.* and to *C* three Times as much as *A*, wanting 15*l.* Required the Share of each: For what the firſt was to have put a; then *B* will have $2a-8$; and *C* will have $3a-15$; and all theſe together muſt, from the Nature of the Queſtion, make 100*l.* Now tranſpoſe $-8-15$, and you will have $a+2a+3a=100+8+15$; then contracting this laſt Expreſſion, it will be $6a=123$; and dividing both Sides of the Equation by 6 to come at a, we have $a=\frac{123}{6}=20l.\ 10s.$ and *B*'s Share is $20l.\ 10s.\times 2-8=33l.$ and *C*'s Share is $20l.\ 10s.\times 3-15=46l.\ 10s.$

$$a+2a-8+3a-15=100l.$$
$$a+2a+3a=100+8+15$$
$$6a=123$$
$$a=\frac{123}{6}=20l.\ 10s.$$

Queſtion 6. Three Men, *A*, *B*, and *C*, built a Houſe which coſt 1000*l.* towards which *A* paid a certain Sum, *B* paid 10*l.* more than *A*, and *C* paid as much as *A* and *B*. Required what each Perſon contributed towards the whole Charge. Here, by the Conditions of the Queſtions, *A*'s Share is a, and *B*'s muſt be 10*l.* more; that is, $a+10$, and *C*'s Share is $2a+10$, as his muſt equal both theirs; then we have $a+a+10+2a+10=1000l.$ Here, by tranſpoſing $+10+10$, and contracting the Expreſſion, we have $4a=1000-20$, and dividing both Sides by 4, we have $a=\frac{1000-20}{4}=245$, *A*'s Share; and then *B*'s Share is 255*l.* and *C*'s is 500, being all equal to 1000*l.*

$$a+a+10+2a+10=1000l.$$
$$4a=1000-20$$
$$a=\frac{1000-20}{4}$$

Queſtion 7. I formerly delivered to a Banker a certain Sum of Money, to receive of him after the Rate of 5 *per Cent. per Annum*, and at the End of 12 Years he paid me 5000*l.* Principal and Intereſt together. Required the Sum firſt delivered to him. Here, put a for the Money depoſited; then $\frac{a\times 5}{100}+12+a=5000l.$ the Amount, but contracted, it will be $\frac{60a}{100}+a=5000$; then reducing the Expreſſion to an improper Fraction, we have this Form $\frac{60a+100a}{100}=5000$; and multiplying by 100 and contracting it, we have $160a=500000$; and dividing both Sides by 160, you then have $a=\frac{500000}{160}$ the Principal firſt left in the Hands of the Banker; that is, 3125 *l.* and the Intereſt is 1875 *l.*

$$\frac{5a}{100}+12+a=5000l.$$
$$\frac{60a}{100}+a=5000$$
$$\frac{60a+100a}{100}=5000$$
$$60a+100a=500000$$
$$160a=500000$$
$$a=\frac{500000}{160}$$

Queſtion 8. What Number is that, which, being multiplied by 10, and that Product divided by 2, alſo 19 added to this Quotient, and 99 ſubtracted from that Sum, the Reſult ſhall be equal to the Number ſought? Let $a=$ the Number ſought; then × by 10 we have $10a$, and divided by 2 we have $\frac{10a}{2}$, to which 19 being added, the Sum is $\frac{10a}{2}+19$, from which ſubtracting 99, the Remainder is $\frac{10a}{2}+19-99$; which being equal to a, the Number ſought by the Conditions of the Queſtion, we have this Expreſſion, $\frac{10a}{2}+19-99=a$; required a? Here tranſpoſe $+19-99$ and you have $\frac{10a}{2}=a-19+99$; and by multiplying each Side by 2, it will be $10a=2a-38+198$; and, to bring a on one Side, we have $10a-2a=198-38$; that is, $8a=160$, and dividing by 8, we have $a=\frac{160}{8}=20$, the Number ſought.

$$\frac{10a}{2}+19-99=a$$
$$\frac{10a}{2}=a-19+99$$
$$10a=2a-38+198$$
$$10a-2a=198-38$$
$$8a=160$$
$$a=\frac{160}{8}=20$$

It may be preſumed, that theſe few Queſtions will be ſufficient to give the Learner ſome Sort of Idea of what may be expected from Algebra; I have added the laſt, for him to ſee what he muſt expect, if he attempts to bring out the Anſwer by the common Rules in *Poſition,* as the Queſtion canot eaſily be reſolved without Algebra. If the Learner will vary each of the foregoing eight Queſtions, and will work them over by himſelf, he will be fairly prepared to anſwer any Queſtions, requiring only one unknown Letter; but when they require two, three, or more, as the Operations will frequently be very long in ſome of them, and require more Room than I can ſpare to explain them thoroughly, I am obliged to omit them; neither do they come within my Deſign, which is to introduce no more Algebra than is barely neceſſary for the Purpoſes of Arithmetic.

Annuities in Arrears.] Let it be requir'd to find the Amount of an Annuity of 50*l.* which was granted for 20 Years, but unpaid, for ſome Reaſon or other, 10 Years together, and let the Arrears be computed at 5 *per Cent. per Ann. Simple Intereſt.* The Youth muſt know that, to enjoy 50*l.* a Year for any fixed Time, there was a Sum of Money paid on ſuch a Bargain made; and therefore the Purchaſer, or his Family, has a Right to not only what is unpaid of the Bargain, but to the Intereſt as it becomes due.

Here then, by the Queſtion, there will be 50×10 Annuities due at the End of 10 Years: And for the Intereſt we have $r=2$*l.* 10*s.* at the End of the 2d Year, and $2r$ for the 2d and 3d Years, and $3r$ for the 2d, 3d, and 4th Years; and ſo on for 9 Years, as there is no Intereſt due the firſt Year: Hence we have a Series r, $2r$, $3r$, $4r$, $5r$, $6r$, $7r$, $8r$, $9r$. Now, if you add all theſe together, you will have all the Intereſt; that is, $r+2r+3r$, &c. as you know $r=2$*l.* 10*s.* But there is a noble Property in theſe Numbers, thus regularly increaſing by any Sort of *Addition*, as 2*l.* 10*s.* + 5*l.* + 7*l.* 10*s.* &c. which will ſave you that Trouble; that is, half the Sum of the two Extremes, 2*l.* 10*s.* + 22*l.* 10*s.* multiplied by the Series or Number of Years Intereſt, will give you the Sum of that Series; that is, $\frac{r+9r}{2}\times 9=$ all the Intereſt; to which add the Annuity 50*l.* × 10 Years, and we have $\frac{r+9r}{2}\times 9+50\times 10=612$*l.* 10*s.* the Amount of the Annuity, computed at 5 *per Cent. per Annum*, Simple Intereſt.

But in order to find the Time, Rate, or Annuity, we muſt have a more general Theorem, as the laſt is too confined for theſe Purpoſes. Before it was, the Half of the firſt and laſt Year's Intereſt, multiplied by the Number of Years, gives the Sum of all the accumulating Intereſt on each Annuity;

that is, $\frac{10r}{2}\times 9 = 112$ *l.* 10 *s.* For a general Theorem, $r =$ .05, the Intereſt of 1 *l.* and $r = 10$ Years the Annuity u was unpaid; then $\frac{10r}{9}\times 9 = \frac{10r-r}{2}\times 10$, ſuppoſing r on each Side of the Equation to be 2 *l.* 10 *s.* But becauſe $r = .05$, and $t = 10$ in the laſt Theorem, therefore we have $\frac{tr-r}{2}\times t$ = the Intereſt of 1 *l.* Annuity unpaid; that is, $\frac{ttr-tr}{2}$; then as 1 *l.* : $\frac{ttr-tr}{2}$: : u : $\frac{ttru-tru}{2}$, all the Intereſt that is due on an Annuity of 50 *l.* then *Theorem* 1, $\frac{ttru-tru}{2} + tu = A$ = 612 *l.* 10 *s.* the Amount requir'd, as before. And now, reducing the mix'd Fraction to an improper one, we have $\frac{ttru-tru+2tu}{2} = A$, and multiplying both Sides by 2, we have $ttru-tru+2tu = 2A$; and then, in order to find the Annuity, we muſt divide both Sides by $ttr-tr+2t$, and we have *Theorem* 2, $u = \frac{2A}{ttr-tr+2t}$, becauſe this, multiplied by the ſame Diviſor, will give the ſame Product on each Side. Again, from *Theorem* 1, we have $\frac{ttru-tru}{2} = A - tu$; that is, the Intereſt $= A -$ the Principal; and multiplying both Sides by 2, it will be $ttru-tru = 2A-2tu$; and now dividing by $ttu-tu$, we have *Theorem* 3, $r = \frac{2A-2tu}{ttu-tu}$, for the Reaſon given in finding u. And from *Theorem* 1, $\frac{ttru-tru+2tu}{2} = A$; whence $2A = ttru+2tu-tru$; that is, $2A = ttru + \overline{2u-ru}\times t$: Here the Expreſſion means that $2u-ru$ only is multiplied by t; divide by ru, and we have $\frac{2A}{ru} = tt + \frac{2u-ru}{ru}\times t$: Here as we are finding t, and as $\frac{2u-ru}{ru}$ is known, we may call this laſt x, and then we have $\frac{2A}{ru} = tt + xt$. Now, conſider; on one Side of this laſt Equation we have $tt+xt$; that is, a ſquare Number added to its Root, which is multiplied into the known Number x. Now, if we add $\frac{xx}{4}$ on each Side of the Equation, we have $\frac{2A}{ru} + \frac{xx}{4} = tt + xt + \frac{xx}{4}$, which is a compleat Square: You perceive you have the unknown Letter t on one Side of the Equation, and the other Side is known. The Way to come at t required is this; try what $t+\frac{x}{2}\times t+\frac{x}{2}$ will do; that is, whether the Root $t+\frac{x}{2}$ multiplied into itſelf will produce $tt+xt+\frac{xx}{4}$; if it does, you will conclude this laſt Expreſſion is a compleat Square. Now, $t+\frac{x}{2}$ is $\frac{2t+x}{2}$, an improper Fraction, which, multiplied into itſelf, is $\frac{4tt+4xt+xx}{4}$; and dividing each Member by 4 the Diviſor, (Denominator) we have $tt+xt+\frac{xx}{4}$, which is the Square compleated as above; becauſe when we have a Square Number and its Root multiplied into a known Number, we know the fourth Part of the Square of that known Number added will compleat the Square, by which the Root of the unknown Letter t is eaſily diſcover'd. Hence then, as above, $tt+xt+\frac{xx}{4}=\frac{2A}{ru}+\frac{xx}{4}$; then by extracting the Square Root out of each Side, we have this Expreſſion, $t+\frac{x}{2}=\sqrt{\frac{2A}{ru}+\frac{xx}{4}}$; now tranſpoſe $\frac{x}{2}$, and then we have *Theorem* 4, $t=\sqrt{\frac{2A}{ru}+\frac{xx}{4}}:-\frac{x}{2}$. Now, let us try in Figures the Iſſue of this troubleſome Affair to you in finding t the Time. Here $2A=1225$; and $ru=2.5$; then $\frac{1225}{2.5}=490$; and $x=\frac{2u-ru}{ru}$, as before; that is, $x=39$, and $xx=1521$; and this divided by $4=380.25$. Then $490+380.25=871.25$; and the Square Root of this is 29.5, from which take $\frac{x}{2}$, and we have $t=10$.

Now vary the Queſtion; that is, let u, r, t, be all different in ten ſeveral Sums fitted to each of the four Theorems, and you will never be at a Loſs to anſwer ſuch uſeful Queſtions, wherein Annuities in Arrears are concern'd, when *Simple Intereſt* only is allowed.

The preſent Worth of Annuities for Time.] By *Theorem* 1, we are enabled to find the Amount for any Number of Years *certain*; then to find the preſent Worth, at 5 *per Cent. Simple Intereſt*, we have the *Theorem in Diſcount* $\frac{A}{tr+1}=P$, the Anſwer. In the laſt Queſtion, the Amount in 10 Years was

612*l.* 10*s.* then, $\frac{612.5}{1.5}=408.333$*l.* the preſent Worth of an Annuity of 50*l.* for 10 Years certain: So that whoever pays down 408.333*l.* will find that he could make up his 612.5*l.* in ten Years at 5 *per Cent.* and the Annuity of 50*l.* being applied to Intereſt, during ten Years, will do the ſame. But to have a more *general Theorem*, we may join together $\frac{ttru-tru+2tu}{2}=A$, and $trP+P=A$, becauſe the Amount, A, in both is 612*l.* 10*s.* that is, it was found to be the Amount of the Annuity of 50*l.* and by dividing $\frac{A}{tr+1}=P$, by $tr+1$, we have $trP+P=A$. Hence then, $trP+P=\frac{ttru-tru+2tu}{2}$, which is no more than ſaying 612.5*l.*=612.5*l.* And now, multiplying each Side of the Equation by 2, we have $2trP+2P=ttru-tru+2tu$; and dividing this laſt Equation by $2tr+2$, we have *Theorem* 1, $P=\frac{ttru-tru+2tu}{2tr+2}$, becauſe $P\times\overline{2tr+2}=2trP+2P$. And now trying this Theorem, you will find $P=408.333$ *l.* the preſent Worth of the Annuity of 50*l.* as above. You are from hence enabled to find u, r, t, when the Queſtion requires you ſo to do.

For, as before, $2trP+2P=ttru-tru+2tu$, and dividing both Sides by $ttr-tr+2t$, we have *Theorem* 2. $U=\frac{2trP+2P;}{ttr-tr+2t}$ now try this, .and ſee whether u is not 50*l.* as we ſuppoſe the ſame Queſtion ſtill propoſed, and then $P=408.333$*l.* Again, to find r, we had before $2trP+2P=ttru-tru+2tu$; Now tranſpoſe $2P$; then $ttru-tru$, with their contrary Signs, and we ſhall have $2trP-ttru+tru=2tu-2P$; and dividing each Side by $2tP-ttu+tu$, we then have *Theorem* 3, $r=\frac{2tu-2P}{2tP-ttu+tu}$. Before you go any further, try this Equation alſo, whether r is not .05*l.*

The next and laſt Buſineſs is to find the Time t. In order to this, we had before $2trP+2P=ttru-tru+2tu$: Tranſpoſe $2trP$, and we have $2P=ttru-tru+2tu-2trP$; and now divide each Member by ru, and then the Equation will be $\frac{2P}{ru}=tt+\overline{\frac{2u}{ru}-\frac{2rP}{ru}-\frac{ru}{ru}}\times t$. Here, by Abbreviation, we have $\frac{2P}{ru} = tt + \overline{\frac{2}{r} - \frac{2P}{u} - \frac{1}{1}} \times t$. Now let $\frac{2}{r} - \frac{2P}{u} - \frac{1}{1}$ be called $= x$; and then it will be $\frac{2P}{ru} = tt + xt$. And adding $\frac{xx}{4}$ on both Sides, to come at the Value of t, the Expreſſion will be $tt + xt + \frac{xx}{4} = \frac{2P}{ru} + \frac{xx}{4}$; and then the Root of tt is t, and the Root of $\frac{xx}{4}$ is $\frac{x}{2}$; ſo that we have $t + \frac{x}{2} = \sqrt{\frac{2P}{ru} + \frac{xx}{4}}$, and by Tranſpoſition, the Reſult is the fourth *Theorem*, $t = \sqrt{\frac{2P}{ru} + \frac{xx}{4}} : - \frac{x}{2}$. Here alſo try whether t does not equal 10, as it ought by the Queſtion before propoſed.

And here again the Learner will do well to vary the laſt Queſtion, and adapt the ſeveral Variations to the four Theorems, concerned in the preſent Worth of Annuities, granted for a limited Time: As it is no mean Qualification to be Maſter of theſe uſeful Theorems, I hope the Youth will endeavour to underſtand the Reaſon of what he is about, as I have been very particular for that Purpoſe. He ſhould not let the Appearance of Difficulty ſtartle him, nor leſſen his Reſolution to acquire what may be juſtly called uſeful, nay genteel Knowledge.

Annuities in Reverſion.] The Meaning of this is, ſuppoſe the former Annuity of 50*l.* was granted for 10 Years *certain*, but is not to commence till after the Expiration of 10 Years, the Queſtion is, What muſt the Purchaſer pay down now, that either he or his Executors may enjoy the Annuity for 10 Years *certain?* You know the preſent Worth of the Annuity muſt be found, by what was taught you before, for the Term of 10 Years, at any Rate of Intereſt; and when you have found the preſent Worth, were the Annuity to be enjoyed now, you muſt at preſent only pay a Sum which in 10 Years will amount to the preſent Worth of the Annuity. Now vary the Queſtion, and of it form ten Sums, in order to make yourſelf very perfect in this uſeful Part of Knowledge.

Quarterly and Half-yearly Payments.] Hitherto we have ſuppoſed the Annuities to be paid Yearly, which is ſometimes the Caſe: But if, by Agreement, they are to be paid Half-yearly or Quarterly, the Learner muſt be told how this is to be done. Suppoſe *Half-yearly:* Let the ſame Queſtion be here again propoſed; then $t = 20$ Half Years; $u = 25l.$ $r = \frac{.05}{2}$. But if *Quarterly*, let $t = 40$ Quarters; $u = 12.5\,l.$ $r = \frac{.05}{4}$.

In either of theſe Caſes proceed as before directed, according to the Nature of the Queſtion, whether in *Annuities in Arrears, the preſent Worth of Annuities*, or *Annuities in Reverſion*, and you will find Half-yearly Payments are more advantageous than Yearly, and Quarterly more ſo than Half-yearly Payments. Now form many Sums of the ſame Queſtion, varying all its Circumſtances, and you will never hereafter be at a Loſs in anſwering what is required of you.

Obſervations.] The Law expreſly forbids taking Compound Intereſt on paying Annuities in Arrears, and therefore, in finding the Amount, we muſt uſe *Theorem* 1. for Annuities in Arrears, and compute at Simple Intereſt. But if the Purchaſer of the *Annuity certain* inſiſts upon being allow'd Compound Intereſt for laying out his Money, the Law has nothing to do with this, as the Parties are at Liberty to do what they pleaſe where the Law is ſilent. Undoubtedly, when the preſent Worth is conſiderable, the Purchaſer has a Right to have the Money he has to pay down ſuch as, when continued out at Intereſt upon Intereſt, will equal the Amount of the Annuity, and that this annual Income ſhould be conſidered as annually put out at Simple Intereſt, unleſs the Annuity is alſo conſiderable. And now ſatisfy yourſelf of the Reaſonableneſs of what is here ſaid, by ſuppoſing an Annuity pretty low, as 10 or 20*l.* and finding the preſent Worth; and an Annuity of 1000 or 2000*l.* and ſeeing what muſt be paid down in the Purchaſe of ſuch high Annuities for a *certain* Time.

Preſent Worth at Compound Intereſt.] There is a noble Property in Numbers, running on in Geometrical Progreſſion: Thus, ſuppoſe 3, 9, 27, 81, 243; here you ſee 3 is the Ratio, or common Multiplier, and the firſt and laſt Terms are 3, 243. The excellent Diſcovery is this; if you multiply the Ratio (3) into the laſt Term, (243) and from this Product take the firſt Term, (3) and divide the Remainder by the Ratio leſs one, you have the Sum of the Series; that is, $\frac{243 \times 3 - 3}{3 - 1} = A$, the Amount 363 of all the Series. Now let there be this Series, u, uR, uRR, $uRRR$, $uRRRR$. Here the laſt Term is uR^4, and the Ratio, or common Multiplier, is R, and the firſt Term is u; then, by the foregoing noble Property, we have $\frac{uR^4 \times R - u}{R - 1} = A$, the Amount of an Annuity forborn, or in Arrears 5 Years: Call the Number of Years t, and ſhorten the Expreſſion, and it will be *Theorem* 1,

$\frac{uR^t-u}{R-1}=A$, as before. And you ſaw formerly that $PR^t=A$; and therefore, without waſting your Time in applying the firſt of theſe Equations to find the Amount of what the Laws forbid to be inſiſted on, we ſhall find *P*, the preſent Worth, ſuppoſing the former Queſtion here reſumed; that is, $u=50l.$ but $R=1.05l.$ the Amount of 1*l.* inſtead of .05*l.* and $t=10$ Years.

In order to this, you know that if $\frac{uR^t-u}{R-1}=A$, and $PR^t=A$, then of Neceſſity $PR^t=\frac{uR^t-u}{R-1}$. Now, take Care; you ſee the Equation expreſſes a *Quotient* PR^t, a *Dividend* uR^t-u, and a *Diviſor* $R-1$; therefore multiply the Quotient into the Members of the Diviſor, and you will have what is equal to the Dividend; that is, $RPR^t-PR^t=uR^t-u$; now divide all by R^t and we have $RP-P=\frac{uR^t-u}{R^t}$. You perceive the Right Side of the Equation is an improper Fraction, and therefore reduce it to a mixt Fraction, which will then ſtand thus, $u-\frac{u}{R^t}$; but, for a ſmall Convenience, call $\frac{u}{R^t}=x$; then we have $RP-P=u-x$. And, in order to find *P*, divide all by $R-1$, and the Expreſſion will be *Theorem* 2, $P=\frac{u-x}{R-1}$, becauſe the Quotient $P\times R-1=PR-P$. Now, try the Event of this ſo far; that is, find *A* by *Theorem* 1, and *P* by *Theorem* 2; not that it is of any further Uſe here in finding *A*, than proving your Care in the Operations. Now, if you are attentive, you will find $P=386l.$ and a Decimal, and that $A=628.8946l.$ Here, if you try what *P* will amount to in the given Time, you will then be ſatisfied that the Sum 386*l.* and a Decimal is the preſent Worth required. That is, by having *P*, you can find *A* by the *Theorem* $PR^t=A$; and then it is a Thing indifferent whether a Perſon is in Poſſeſſion of the Money, *P*, or the Annuity, *u*, provided Compound Intereſt can be had in both Caſes. As thus, 386*l.* at 5 *per Cent.* will bring in the firſt Year 19*l.* 6*s.* and this Intereſt, put out into another Hand, will alſo bring in 19*s.* 3*d.* ½, and ſo on from Year to Year to the End of the Time. In like Manner the Annuity, 50*l.* will, when put out to Intereſt at the ſame Rate, bring in 2*l.* 10*s.* the firſt Year any is due; and this Intereſt will, if it was practicable to place it out in ſuch ſmall Sums every Year in different Hands, bring in 2*s.* 6*d.* Intereſt, and ſo on during the whole Period.

You ſaw before that the preſent Worth of the ſame Annuity, for the like Term of Years, when computed at Simple Intereſt, was 408*l.* odd, and that the Amount was 612*l.* odd; and you will conclude that there will be a Neceſſity of the Purchaſer's inſiſting on being allowed Compound Intereſt in the Computation, as there will be no Chance of his making Intereſt upon the Intereſt of the Annuity, unleſs it is a conſiderable Income; and that, without his making Compound Intereſt of the Annuity, the Seller will have the Advantage of him; that is, *P* will make more Money in the Time than *u*, which ought not to be done.

Annuities in Reverſion by Compound Intereſt.] What has been ſaid of Compound Intereſt applied to Annuities; that is, finding the preſent Worth, is the moſt uſeful; what remains, being ſeldom required, we ſhall not trouble the Youth with, leſt it tire his Patience, and diſcourage his Deſire to travel on; we ſhall therefore proceed to ſay ſomething of purchaſing Annuities in Reverſion by Compound Intereſt. Now then let the former Queſtion be here propoſed; that is, an Annuity of 50*l.* granted for 20 Years *certain*, but not to commence till after the Expiration of 10 Years. Here, as before, you are to find the preſent Worth for 10 Years, by the Theorem $P = \frac{u - x}{R - r}$, (you may remember that $x = \frac{u}{R^t}$); this being the Value of *P*, you are then to find, by the Theorem $P = \frac{A}{R^t}$, what Sum being now paid down, will, at Compound Intereſt, amount to *P*, the preſent Worth, in 10 Years Time, when the Annuity is to commence. You will do well to form ſeveral Sums of this, by varying the Annuity, Time, and Rate, in order to be extremely ready at ſuch uſeful Queſtions.

The Purchaſe of Real Eſtates.] You know the Annuities were for a *certain* Number of Years; but by the Rent, or annual Income, of a Real Eſtate, is meant, that it is to continue in a Family, from Father to Son, to the lateſt Poſterity, unleſs ſold to ſome other Family. Let us ſuppoſe the Eſtate to be 500*l.* a Year, clear of all Deductions, lett upon long Leaſes, and the Tenants, or Heirs, able to pay their Rents punctually. Now the Queſtion is, What Money muſt be paid down for ſuch an Eſtaſte? If you have 12500*l.* which brings you in 500*l.* a-year upon the beſt Land Securities, at 4 *per Cent.* Intereſt, and you give this Sum of 12500*l.* for the Eſtate of 500*l.* a-year, you perceive you have not mended yourſelf in this Purchaſe, even if you ſuppoſe the Rents will be punctually paid equally with the Intereſt of your Money. This being the beſt Caſe on both Sides of the Suppoſition, let $u = 500l.$ Rent, and $r = .04$, the Rent or Intereſt of one Pound; then as $.04 : 1 :: 500l. : 12500l.$ the Purchaſe-Money, or Value of the Eſtate; to be read thus, as the Rent of one Pound is to one Pound, ſo is the Rent of 12500*l.* to this Purchaſe; or, in general, *Theorem* 1, $\frac{U}{r} = P$, the Purchaſe. You know r may ſtand for any Rate, and that the greater the Diviſor r, the leſs will be P, and the leſs r is, the greater will the Purchaſe be. Indeed there is no fixing on the true Value of r, which ſhould determine P, the Purchaſe-Money; for the Owner of an Eſtate has often various Reaſons to keep r very low, as thereby he will have more Purchaſe-Money. But it is grown very common to value an Eſtate by the Number of Years Purchaſe; that is, ſo many Years Rent; and becauſe you ſhould not be ignorant what Intereſt is made of the Purchaſe-Money, you may caſt your Eye on the Margin. Now if the Owner inſiſts on $33\frac{1}{3}$ Years Purchaſe the Value will be $\frac{500}{.03} = 16666.666l.$ or $500l. \times 33\frac{1}{3} = 16666.666l.$ ſo if he inſiſts on 40 Years Purchaſe, the Value will be $500 \times 40 = 20000l.$ The Way of diſcovering the Rate, by the Years Purchaſe, is this; $\frac{100}{20} = 5$, $\frac{100}{25} = 4$, $\frac{100}{33\frac{1}{3}} = 3$, $\frac{100}{40} = 2\frac{1}{2}$, &c. that is, divide 100 by the Number of Years Purchaſe, the Owner aſks for the Eſtate, and you will ſee what Intereſt you will make of your Money, ſuppoſing there is no Diſappointment from the Tenure.

20	*Years*	*Purchaſe*	5	*per Cent.*
25	——	——	4	
$33\frac{1}{3}$	——	——	3	
40	——	——	$2\frac{1}{2}$	
50	——	——	2	

Now from *Theorem* 1, $\frac{u}{r} = P$, we have an uſeful Deduction; that is, if you would know what Eſtate any Sum of Money will purchaſe at any Rate of Intereſt, we have *Theorem* 2, $u = Pr$, or $500 = 12500 \times .04$; and if you would know the Rate, we have *Theorem* 3, $r = \frac{u}{P}$, or $.04 = \frac{500}{12500}$. And by theſe Means you are enabled to underſtand what is meant by purchaſing a *Real* or *Freehold Eſtate*, which no Gentleman can be ſuppoſed to be unacquainted with.

The Purchaſe of Houſes.] This Kind of Eſtate differs from Freehold Property, in that there muſt come a Time when the beſt Houſes will fall to Decay, notwithſtanding the Care taken in the Repairs to ſupport them. Houſes, like Land, are to be conſidered as an annual Rent, by which to find the Purchaſe; that is, $\frac{u}{r} = P$; but ſurely the Rate here ſhould be higher than in buying Land: Unleſs Fancy or Convenience is concerned, the Purchaſer ought to inſiſt upon r being pretty high, as a Houſe accidentally ſtanding empty proportionally affects the Intereſt. Let us ſuppoſe the Rent of good Houſes, in a fine Place of Trade, to be 500*l.* a-year, and you would know the Value, allowing the Purchaſer 5 *per Cent.* for his Money, we have $\frac{u}{r} = P$, or $\frac{500}{.05} = 10000$*l.* the Purchaſe-Money.

The Value of an Annuity for Life.] Before you were taught how to eſtimate an Annuity for *a certain* Term of Years, as thus; Suppoſe *A* buys of *B* an Annuity to continue with *A* and his Executors for the *certain* Term of 21 Years, at the End of which Period *B* ceaſes to pay the Annuity, as the Bargain between them is fulfilled; there is nothing but Equity and *Certainty* in theſe Sorts of Agreement, and there may be Convenience too; the Equity conſiſts in that the Money, ſo laid out in the Purchaſe, is truly equated with the Annuity, by allowing the ſame Rate of Intereſt to both Sides, whether computed at Simple or Compound Intereſt. But in purchaſing an Annuity for the Life of *A*, though the Equity is never ſo exactly aſcertained, the Certainty is abſolutely deſtroyed, and, except in one Caſe, either *A*'s Family or *B*'s muſt be Loſers. Let x be the Time, when, if *A* arrives at it, the Annuity he has received, and its accumulating Intereſt, will be equal to the Purchaſe-Money and its Intereſt, in the Hands of *B*; here is neither Loſs nor Gain to either Party: But in Proportion to the Deficiency of *A*'s not reaching to x, by ſo much will the Granter of the Annuity be the Gainer; and if *A*, in the ſame Proportion, ſurvives the Period x, *B* will as equally loſe by the Bargain. Hence then you ſee the great Uncertainty of Gain or Loſs in theſe Bargains; or, to expreſs it in other Words, there is Half a Certainty of Loſs, and Half a Certainty of Gain, both which, in equal Periods of *A*'s Life, act in Oppoſition to each other on the different Sides of x. If you do not readily underſtand what is meant by Half a Certainty, I ſhall expreſs it thus: When you toſs up a Halfpenny, and another Youth cries *Head*, before the Halfpenny comes to the Ground he has one Chance of winning and one of loſing, the Probabilities are equal on your Side and his, you may as equally win as loſe; that is, an equal Probability is Half a Certainty. So *A* has an equal Probability to live as many Years beyound x as he has to fall ſhort of that Period; and then, from the Notion of toſſing up, his Family has Half a Certainty of being Loſers, and Half a Certainty of being Gainers.

So then you ſee the Buſineſs is to find that Period in *A*'s Life, expreſſed by x, when the Purchaſe-Money paid down, and continued out at Intereſt, to the Time x, ſhall be equal to the Amount of the Annuity, running alſo at the ſame Rate of Intereſt, during the ſame Term of Years x. Not that x is the Years Purchaſe of *A*'s Life; for if he is, for Inſtance, about 56 Years of Age, his Years Purchaſe will, by *De Moivre*'s Tables, at 4 *per Cent.* be near 10, and then x will be ſomewhere about 13 Years, which ſhould be near the Half of the Years *A* may live; that is, $56+26=82$ Years of Age, which ſurely is a pretty advanced Time of Life, on which to build general Rules. Now an Annuity of 50*l.* in 13 Years, at 4 *per Cent.* will amount to ſomewhere about 800*l.* and the preſent Worth of this is about 525*l.* and this divided by the Annuity will give *A*'s Years Purchaſe, which, in this Caſe, will be about 10½ Years, which is Half the Annuity more for *A* to pay than by Mr. *De Moivre*. Now the Learner may eaſily perceive, that putting x higher than 13 Years will make the Purchaſe-Money higher; as thus, ſuppoſe $x=15$ Years, and then $56+30=86$, which we may call *A*'s enlarged Limit of Vitality out of the general Rule. The Annuity of 50*l.* in 15 Years, at 4 *per Cent.* will amount to 960*l.* and the preſent Worth of this Sum for *A* to pay down is 600*l.* that is, dividing the Purchaſe-Money 600*l.* by 50*l.* the Annuity, we have 12 Years Purchaſe for *A*'s Life of 56 Years of Age: But this is making *A* pay 100*l.* more than by *De Moivre*'s Table; yet let us conſider at what Rate *A* ſinks his Money. This paying 600*l.* for the Annuity of 50*l.* is ſinking the Principal for *A*'s Life at about 8½ *per Cent.* which, conſidering Money at 4 *per Cent.* is pretty well for both Parties.

But tho' this Method is chiefly uſeful for Lives from 30 Years of Age, and upwards, we ſhall now venture to ſhew how the following Table is conſtructed, and its Uſe on many Occaſions. The Youth muſt be firſt told, that this Manner of calculating Annuities for Lives is very different from the tedious and intricate Inveſtigation of Authors of great Characters; yet when he comes to be a Man, he may compare both Methods, and judge for himſelf; only, if I muſt ſay the Truth, I would never grant an Annuity upon a ſingle Life, by the Tables, from the Calculations of thoſe celebrated Authors, for this Reaſon, becauſe I am ſure the *Granter*, by theſe Tables, has a greater Chance of loſing by the Bargain than he ought to have. The Youth is to remember, that all depends upon the equal Chances mentioned above, and that theſe equal Periods were publiſhed by *Weyman Lee*, Eſq; of the *Inner-Temple*, in the Year 1751, on whoſe Authority I ſhall reſt the Point: He knowing the Government, in King *William*'s Reign, had a great Number of Nominees in their Books, was excited to ſee how, in the Courſe of Providence, theſe Nominees were called out of this World. The Reſult was, as you ſee in the Table; which, tho' it differs ſomewhat in particular Ages from thoſe who have formed their Tables on the Bills of Mortality, kept by the Pariſh Clerks in *London*, I ſhall ſhew my private Opinion of it, by inſerting it here, and then make my own Uſe of it. I ſhall alſo beg Leave to make a ſmall Alteration now and then, in order to prevent *Granters* being at a Loſs how to do themſelves a little more Juſtice, and to put them more upon the Footing with thoſe in 'Change-*Alley*, who ſell Annuities, but never by the beſt Tables now in Print, becauſe the Value is too low: But as I deſign this Table as a Specimen, and to exerciſe the Youth in ſome of the Theorems already eſtabliſhed in this ſmall Treatiſe, I ſhall begin with the Age of 20, and continue it to the Period of 70 Years of Age, beyond which Annuities are ſeldom *granted*, if at all. You are to underſtand the Table thus: Of all Perſons of 20 Years of Age, Half of them will be in their Graves in 31 Years, and the other Half will ſurvive that Period; of all of 40 Years of Age, Half will be in their Graves, and Half alive in 21 Years Time; of all

Ages.	*Equal Chances.*
20	31
25	28.5
30	26
35	23.5
40	21
45	18.5
50	16
55	13.5
60	11
65	8.5
70	6

of 50 Years of Age, Half will be in their Graves, and the other Half will remain alive; of all of 60 Years of Age, one Half will ſurvive 11 Years, and the other will not; and of all thoſe of 70 Years of Age now, Half will be in their Graves, and the other Half will remain alive; and ſo for the intermediate Ages between the above.

Now you are to remember, that the equal Chances, in the above Table, are what I call x, when the Purchaſe-Money in x Time will equal the Amount of the Annuity in the ſame Period. Now let the Annuity be 50*l.* the Purchaſer's Age 40; and then x, (which now call t) $=21$ Years; to find the Amount of the Annuity, at the Rate of 4 *per Cent.* Here the former Theorem for this Purpoſe is, you well remember, $\frac{ttu-tu}{2}\times r:+tu=A$, the Amount of the Annuity at the End of this Period. Then to find the Purchaſe-Money of ſuch an Annuity for Life, you have $\frac{A}{tr+1}=P$; that is, the Sum P in 21 Years will amount to A, and conſequently the Bargain is properly equated between the Parties, and $\frac{P}{u}=$ the Years Purchaſe for the Life of the Buyer. Now, when you have work'd theſe two Theorems, you will find $A=1470$*l.* and $P=798.9$*l.* and that this laſt Sum will amount to A in 21 Years at 4 *per Cent.* Here then the Purchaſer paying down 798.9*l.* ſinks his Money at the Rate of 6½ *per Cent.* nearly, which is as much as moſt People, who underſtand theſe Kinds of Bargains will give for a good Life of 40 Years of Age, when Money is conſidered only worth 4 *per Cent.* Now $\frac{P}{u}=15.0$ Years Purchaſe; that is, $15.9\times 50=795$, to which add a Remainder 3.9, and we have 798.9, as before. Inſtead of 15.9 Years Purchaſe, Mr. *De Moivre* makes it 13.2; then $13.2\times 50=660$*l.* the Purchaſe-Money, which means that the Buyer of the Annuity has ſunk the Principal at the Rate of more than 7½ *per Cent.* which ſurely is too much: And herein you evidently ſee the Difference between the foregoing Tables and that of Mr. *De Moivre*'s: But by Mr. *Simpſon*'s Tables, the Years Purchaſe is 11.5; and then the Purchaſe-Money is 575*l.* that is, the Buyer has for his Life 8¾ *per Cent.* for ſinking the Principal, which is a wretched Bargain for the *Granter*, when Money is conſidered as worth no more than 4 *per Cent.* upon Land Security.

Now try 60 Years of Age for the ſame Annuity. Here $t=11$ Years equal Chance, $u=50$, and $r=.04$, to find A, by the Theorem, as before. When you have alſo worked the two Theorems, you will find $A=660$ *l.* and $P=458$; that is, the *Purchaſer* has 10¾ *per Cent* for ſinking his Money. Then $\frac{P}{u}=9$ Years Purchaſe $\times 50=450+$ a Remainder of $8=458$ *l.* Mr. *De Moivre* makes it alſo 9.16 Years Purchaſe, which is the ſame as mine, when turned into a Decimal. Mr. *Simpſon* makes it $8.4\times 50=420$ *l.* which allows the *Purchaſer* 12 *per Cent.* but ſurely no *Granter* can agree to it, for a good Life of 60 Years of Age, unleſs he chuſes to let the *Purchaſer* have more than equal Chances in the Bargain.

Laſtly, try the Age of 20 for the ſame Annuity, at the ſame Rate, making $t=31$ Years. When you have uſed the two Theorems, as before directed, you will find that $A=2480$ *l.* and $P=1107$ *l.* which gives full 4½ *per Cent.* in ſinking the Money for this young Life of 20 Years of Age. But $\frac{P}{u}=22.14$ Years Purchaſe, and Mr. *De Moivre* makes it $15.89\times 50=794.5$ *l.* which is near 6½ *per Cent.* and Mr. *Simpſon*'s Value is $14.8\times 50=740$ *l.* which is ſinking the Money at the Rate of near 7 *per Cent.* for ſo young a Life as 20 Years of Age. You are to conſider that the *Purchaſer* has an equal Chance of enjoying the above 4½ *per Cent.* during the equal Period of Vitality for the Term of 31 Years, and when he ſurvives this Stage of his Life, the *Granter* runs the Hazard of paying the *Purchaſer* the dead Loſs of 50 *l.* a-year during another Period of 31 Years, as they both were neither Gainers nor Loſers at the Expiration of the firſt 31 Years.

I muſt now requeſt you will view the Table again, and reſolve to apply the two Theorems to every Age therein contained, and proceed in all Things in the Manner ſhewn you; that is, to find the Amount of the Annuity at the End of each equal Chance of Vitality, then to find what Principal, being now paid down, will be equal in that Time to the Amount of the Annuity; that is, you muſt put the Principal out at Intereſt for that Time, and if your Work is right, that Sum will be increaſed to the Amount of the Annuity. Then you are to ſee how much *per Cent.* the *Purchaſer* has for ſinking his Money, and how many Years Purchaſe it is, and how this laſt agrees with the two Authors Tables, mentioned before, which I ſhall inſert for that Purpoſe; and, laſtly, you are to find at what Rate the *Purchaſer* ſinks his Money, by the Calculations of thoſe two celebrated Writers, and carefully compare together the three different Rates of ſinking the Money. This Acquiſition of Knowledge in *Annuities for Lives* will be of great Service to you in your future Life, as it will enable you to judge what both the *Granter* and *Purchaſer* ought, in Point of Reaſon, to expect from each other.

Age.		*Value.*
20	—	15.89
25	—	15.31
30	—	14.68
35	—	13.98
40	—	13.20
45	—	12.32
50	—	11.34
55	—	10.24
60	—	9.01
65	—	7.63
70	—	6.06

Mr. *De Moivre* has grounded his Calculations on a Series of Numbers in arithmetical Progreſſion, by ſuppoſing 36 Perſons all of the Age of 50, and that in 36 Years they will all be in their Graves, by dying off at the Rate of one Perſon every Year; ſo that he makes the lateſt Limit of Life to be 86 Years of Age; whereas, you remember, we only put the Boundary of Vitality at 82 Years, as a general Period, without taking any Notice of now and then a Straggler, who may exceed ſuch an advanced Age. This Suppoſition of one in 36 dying every Year, is very near in Proportion to the Obſervations made at *Breſlau*, by Dr. *Halley*, from the Bills of Mortality there. This Syſtem of Experiments, on Vitality at that City, ſeems to ſhew that People there do not die off ſo ſoon as at *London*; but the chief Reaſon of the Difference appears to be this; there is a greater Reſort of People of all Nations at *London*, and conſequently more will ſeem to have died than the Proportion of Births requires. Mr. *De Moivre* then proceeds to eſtabliſh his Rule for finding the Value of a ſingle Life of any Age; that is, the Years Purchaſe multiplied into the Annuity; or, in other Words, what Sum of Money muſt be paid down for an annual Sum to be paid the Purchaſer for his Life: But were I to inſert the Demonſtration of his Rule, the Youth would be frighted at the Uncouthneſs of the Sight; beſides, it would require more Skill in Algebra than the Learner is at preſent Maſter of; and therefore we ſhall only give the Rule, which is as follows: $R=$ the Amount of 1 *l.* for one Year; $U=$ the Annuity of 1 *l.* $N=$ the Complement of Life, or what the Life wants of 86 Years; $P=$ the Value of an Annuity to continue during the Time N; then will $Y=$ the Years Purchaſe required; but firſt put $\frac{RP}{N}=x$, and the

Rule is, $\frac{U-x}{R-1}=Y$, which, in Words, runs thus; find *P* and multiply it into *R*, and divide this Product by *N*, and take this Quotient from *U*, and dividing the Remainder by the Intereſt of 1 *l.* gives the Anſwer. And thus you are enabled to calculate Annuities for any Age, from 1 to the lateſt Period of Life, by Mr. *De Moivre*'s Rule, whoſe Tables are more followed than any others. Now calculate your own, and your Father and Mother's, or any other Relation's Life.

We come now to Mr. *Simpſon*' Rule, which is alſo to find the Years Purchaſe for any Age, as before. This Author ſuppoſes, or rather found by Obſervation, that of 462 Perſons at the Age of 20, one Year after there would remain alive 455; ſo that the Chance of each of them to live one Year is, as 462 to 455, and then the Purchaſe, or preſent Worth of 100*l.* to be received at the End of the Year is, *Rule* 1. $\frac{A\times 455}{tr+1:\times 462}=P$; the 100*l.* to be received at the End of the ſecond Year is worth in preſent Money $\frac{A\times 448}{tr+1:\times 462}=P$; the 100*l.* to be received on this Chance, at the End of the third Year, is worth, in preſent Money, the Sum of $\frac{A\times 441}{tr+1:\times 462}=P$. And ſo on, by a proper leſſening the Numerator of the laſt Fraction $\frac{441}{462}$, from the Age of 20 to the 75th Year of Mr. *Simpſon*'s Boundary of his Calculations, we have $P+P+P$, &c. the Sum of all which will be 1480*l.* But if the 100*l.* (Annuity) is called 1 *l.* Annuity, the preſent Value of a Life of 20 Years of Age is $\frac{1480}{100}=14.8$ Years Purchaſe. When the Value of any one Life is thus known, the Value of the next younger Life, down to the Age of one Year old, is found thus: Add one Year's Purchaſe (the Annuity being 100*l.*) to 1480*l.* (the Age of 20) $=1580l.=A$; then *Rule* 2. is, $\frac{A\times 462}{tr+1:\times 468}=P=1500l.$ the Purchaſe-Money for a Life of 19 Years of Age, and the Years Purchaſe $=$ 15. And thus you muſt proceed to form *P*, down to a Life of one Year old, and to conſtruct Mr. *Simpſon*'s Tables, which you ſee are, upon his Principles, to be effected with ſome Labour: However, as I ſhall be very glad to have a worthy diligent Youth acquainted with Mr. *Simpſon*'s Rules, I ſhall enable him to calculate one whole advanced Life, and then, by the laſt *Rule* 2. deduce that of the next younger Life, and then the next to that, down to the Age of 20, which will reward him for his Labour.

Ages.		*Multipliers*		*Diviſors.*
75	——	41	——	45
76	——	38	——	45
77	——	35	——	45
78	——	32	——	45
79	——	29	——	45
80	——	26	——	45
81	——	23	——	45
82	——	20	——	45
83	——	17	——	45
84	——	14	——	45
85	——	12	——	45
86	——	10	——	45
87	——	8	——	45
88	——	6	——	45
89	——	5	——	45
90	——	4	——	35
91	——	3	——	45
92	——	2	——	45
93	——	1	——	45

Here then, we ſhall begin at the Age of 75, and having ſhewn the Method of calculating the Years Purchaſe for this Life, all the reſt may be found by *Rule* 2. down to the Age of 20. Now, by *Rule* 1. we have A the Annuity, t the Time A is to be diſcounted, r the Rate, ſuppoſe .04 for one Pound.——Then $\frac{A\times 41}{tr+1:\times 45}=P$, the Value of the Annuity, at one Year's Purchaſe; and $\frac{A\times 38}{tr+1:\times 45}=P$, at 2 Years Purchaſe; and $\frac{A\times 35}{tr+1:\times 45}=P$, at 3 Years Purchaſe. And in this Manner you are to proceed from Year to Year, always diſcounting A, and uſing the ſecond Column in the Margin as Multipliers to A, the laſt diſcounted Annuity, and always uſing the Diviſor 45, as you ſee in the Rule. When you have arrived in your Calculation at the Age of 93, add up all the preſent Worths every Year expreſſed by P, and dividing the Total by the Annuity 100*l.* you will have the Years Purchaſe the Age of 75 is worth. When you have found this, you may find the Age of 74 by *Rule* 2. and in like Manner every Year's Purchaſe down to the Age of 20; and, as was ſaid before, from thence to a Life of one Year of Age, if it was worth your Labour to go lower than 8 Years of Age. You may have obſerved that there is no Difficulty in underſtanding the Reaſon of thus calculating a particular Life, as P is only what muſt be paid now for a Sum due in t Time, and that the *Multipliers* and *Diviſor* only leſſen P each Time, on Account of the *Chances* the *Purchaſer* has of arriving at that Period. Thus, the Rule $\frac{A\times 35}{tr+1:\times 45}=P$, implies that the preſent Worth of the Money, expreſſed that Year by P, leſſened by $\frac{7}{9}$ ($=\frac{35}{45}$), becauſe the Event wants $\frac{7}{9}$ of being certain; that is, the Purchaſer muſt have an Abatement of $\frac{7}{9}$ of $\frac{P}{1}$, or $\frac{P\times 7}{9}$, when you only uſe $\frac{A}{tr+1}=P$.

I ſhall now inſert ſo much of Mr. *Simpſon*'s Table, as will be proper to compare with Mr. *De Moivre*'s, and the firſt Method of computing the preſent Value of Annuities at 4 *per Cent.* It was mentioned before, that the Reaſon why Mr. *Simpſon*'s Value, in the Margin, is lower than Mr. *De Moivre*'s, is chiefly owing to the greater Number of Foreigners at *London*, which has prevented Mr. *Simpſon* from raiſing the Value. I make not the leaſt Doubt of his Accuracy, had there not been ſuch an uncertain Reſort of People from all Parts. As was obſerved before, the moſt unanſwerable Reaſon that they are too low, is, becauſe by that Means he makes the *Purchaſer* ſink his Money at too high a Rate, which conſequently injures the *Granter*, if any can be found that will treat upon ſuch diſadvantgeous Terms. Moſt certain it is, People in *'Change-Alley* know better than to ſell Annuities at ſo low a Purchaſe, as they will frequently inſiſt upon at leaſt two Years more Purchaſe than Mr. *De Moivre*'s Tables allow.

Ages.		*Values.*
20	—	14.8
25	—	14.0
30	—	13.1
35	—	12.3
40	—	11.5
45	—	10.8
50	—	10.1
55	—	9.3
60	—	8.4
65	—	7.5
70	—	6.5

We ſhall now inſert Dr. *Halley*'s Multipliers and Diviſor, as we did Mr. *Simpſon*'s, that the Learner may compare them together, and find the Value of the Age of 75, and as many Lives lower down as he pleaſes. Mr. *Simpſon*'s Chances were $\frac{41}{45}$, and by this celebrated Gentleman they are $\frac{78}{88}$ for the Age of 75. If you try theſe Fractions you will find the Doctor's is $\frac{98}{3960}$ leſs than Mr. *Simpſon*'s; that is, the leſs the Fraction, the leſs is the Year's Purchaſe; and conſequently this, if we work the Queſtion by the Doctor's Rule, is jumping out of the Frying-Pan into the Fire, in ſome Caſes. As a Proof of this Aſſertion, I ſhall here give you ſome of the Doctor's

Ages.		*Multipliers.*		*Diviſor.*
75	—	78	—	88
76	—	68	—	88
77	—	58	—	88
78	—	49	—	88
79	—	41	—	88
80	—	34	—	88
81	—	28	—	88
82	—	23	—	88
83	—	20	—	88
84	—	15	—	88
85	—	11	—	88
86	—	8	—	88
87	—	5	—	88
88	—	3	—	88
89	—	1	—	88

Doctor's Valuation of Lives, that you may judge whether the Rates of ſinking the Money is what a *Granter* of Annuities ought to allow. But the Learner is to obſerve, that as the following Table is computed by the Doctor at 5 *per Cent.* which was the current Intereſt of Money, on good Security, at that Time; ſo Mr. *Simpſon*'s Table, calculated by his Rules, and hereto annexed, is computed at 5 *per Cent.* alſo, in order to compare it with Dr. *Halley*'s. The Youth will remember, that the higher the Intereſt is, the leſs is the Purchaſe-Money; for, at 5 *per Cent.* the Years Purchaſe, for the Age of 10, by Mr. *Simpſon*, is 14.3; at 4 *per Cent.* it is 16.4; and at 3 *per Cent.* it is 19.0; that is, the preſent Worth of 100*l.* Annuity is 1430*l.* 1640*l.* 1900*l.* The Youth perhaps may wonder to ſee ſuch Difference in the Value; but whatever is the current Intereſt of Money, on Land-Security, ought to regulate the Rate at which Annuities are computed: So that at one Time the Annuity may be worth, at 3 *per Cent.* 1900*l.* at another 1640*l*, being at the Rate of 4 *per Cent.* and at another 1430*l.* which is 5 *per Cent.* Now let us ſee at what Rate the Money is ſunk for an Age of 20 by this Table. The Purchaſe-Money, according to Dr. *Halley*, is 1270*l.* for 100*l.* Annuity; this is allowing the Purchaſer 7⅞ *per Cent.* The Value of the ſame Annuity, according to Mr. *Simpſon*, is 1300*l.* and this is ſinking the Money at the Rate of 7¾ *per Cent.* You may obſerve where the Aſteriſm is, there the Doctor is under the Value of Mr. *Simpſon*; and this ariſes from their different Methods of eſtabliſhing their own Rules. Thus much may be ſufficient to give the Learner a pretty good Idea of the Nature of Annuities for all Ages, by the various Rates of Intereſt. We come now to ſay ſomething of Annuities on two or three Lives, as they ſtand diſtinguiſhed from thoſe granted on a ſingle Life only.

Ages.	*Halley*'s.	*Simpſon*'s.
10	*13.4	14.3
15	*13.3	13.9
20	*12.7	13.0
25	*12.2	12.3
30	11.7	11.6
35	11.1	10.9
40	10.5	10.3
45	9.9	9.8
50	9.2	9.2
55	8.5	8.5
60	*7.6	7.9
65	*6.5	7.1
70	*5.3	6.2

Annuities on Two Joint Lives.] Thus, ſuppoſe *A* and *B* purchaſe an Annuity of 100*l.* to be enjoyed ſo long as they both continue in Being; that is, when one dies, the *Granter* of ſuch annual Income ceaſes to pay the Annuity to the Survivor. Let us ſuppoſe the joint Annuitants to be each 40 Years of Age. By Mr. *Simpſon*, the Purchaſe-Money muſt be 810*l.* which is allowing the two joint Annuitants near $12\frac{1}{2}$ *per Cent.* for their Money ſo ſunk. But inſtead of Mr. *Simpſon*'s intricate and tedious Calculations, from whence his Table of two joint Lives is conſtructed, let us take a much plainer Road, that I may give the worthy Youth a Method of ſeeing the Nature of *joint Lives* with infinitely leſs Labour, and yet have ſome Light to conduct him. You may remember that the equal Chance of Vitality for the Age of 60 was put at 11 Years, and that the two Theorems for the firſt Method of finding the preſent Worth of an Annuity for Life were $\frac{ttu-tu}{2}\times r : + tu = A$, and $\frac{A}{tr+1} = P$. Now, let the two joint Lives be 60 and 20 Years of Age, $u = 100$*l.* and $r = .04$. In order to anſwer this Queſtion, and by it all others of this Nature, find P for 60 Years of Age, which you will bring out $= 916$*l.* the Money he muſt pay down for $u = 100$*l.* Conſider now; this would be the Caſe were he to be alone; but becauſe an Age of 20 is joined with him, and the Annuity muſt go to the *Granter* on the Death of either the younger or older Life; therefore as the Probability of the younger Annuitant's living over the Years Purchaſe of the older Annuitant, is expreſſed by $\frac{394}{462}$, according to Mr. *Simpſon*'s Obſervations on the Bills of Mortality, we muſt make this juſt Deduction from $P = 916$*l.* that is, $\frac{916\times394}{462} = 781$*l.* the Value of theſe two joint Lives. The Way you find $\frac{394}{462}$ is this; $\frac{P}{u} = 9$ Years Purchaſe for the Life of 60, and the Tabular Number of 20 Years of Age is 462, and 9 Years further it is 394, which Fraction acts in the ſame Manner on P, as if you were calculating the Life of 20, and were arrived at his 29th Year, which is then equal to the Years Purchaſe of the Age of 60.

Proceeding in this Manner, but taking $P = 840$*l.* the Purchaſe-Money, according to Mr. *Simpſon*'s Value of a Life of 60, the joint Lives will be 716*l.* that is, $\frac{840\times394}{462} = 716$*l.* which is 65 leſs than the above Value. But let us ſuppoſe the two joint Lives were 11 and 68 Years; then, by Mr. *Simpſon*'s Table upon two Lives, and by his general Rule of uſing it, we have $\frac{12.9-4.6 : \times 2.3}{12.9} + 4.6 = P = 6.1$ nearly, or 610*l.* the Value ſought, according to this Author. Now, let us ſee how Mr. *De Moivre*'s Rule, deduced from very intricate Calculations, will agree with this Solution. By this Gentleman's Table, let $a=16.79$, the Years Purchaſe of the Age of 11; $b=6.71$, the Value of the Age of 68; then $\frac{ab}{a+b-abr}=5.93$, or 593 *l.* the Value ſought, according to this Author. Laſtly, let us ſee the Event by the two Theorems, $\frac{ttu-tu}{2}\times r:+tu=A$, and $\frac{A}{tr+1}=P$, moderated by the Chances of Vitality. Here then the equal Period as to Life or Death, for the Age of $68=7$ Years$=t$. The Solutions will ſtand thus; $P=612$, the Money a Perſon of 68 Years of Age muſt pay down; then $\frac{P}{u}=6$ Years Purchaſe; and the Chances of the Age of 11 to live 6 Years$=\frac{480}{517}$; then $\frac{612\times480}{517}=568$ *l.* which is allowing the advanced Age of 68, with ſo young a Life jointly concern'd, at the Rate of nearly 17¾ *per. Cent.* for ſinking 568 *l.* upon the Contingency of one of the two Lives dropping, it would be 16½ by Mr. *Simpſon*'s Rule, and 17 by Mr. *De Moivre*'s Calculations.

I am obliged, for Want of Room, to omit ſhewing the Youth how to ſind the Value of *three joint Lives*, then *two* or *three ſucceſſive Lives*; I am under the ſame Neceſſity of omitting the Manner of computing the Value of *Copyhold Eſtates*, *renewing Leaſes*, and valuing *ſucceſſive Fines*, and many other Particulars of Uſe, belonging to theſe Heads.

If what I have done ſhould prove of Service to Schools, for which this Treatiſe is chiefly intended, I may ſome Time or other purſue the above Subjects.

And now, if the worthy Youth is quite tired down, and cannot travel on any further, I ſhall here take my Leave of him, and proceed to the *Geometrical* Part of my Deſign; only I muſt inſert here ſomething that was forgot when we were upon two joint Lives both of an Age (40). Mr. *Simpſon*, as was ſaid before in that Queſtion, makes the Value 810 *l.* But of two Perſons of this Age, one in the Courſe of Nature (Bills of Mortality) will die in nearly 21 Years; and then we have $\frac{0+21}{2}=10.5=t$, by which, and the other Data, as before, to find $P=$the Value of two joint Lives, which you will find to be 855 *l.* and by Mr. *De Moivre*'s Rule it is 896 *l.* And in like Manner for any two like Ages, the Half of the Equal Chance of Vitality being$=t$, to find P, by the other

Data. But here you may obſerve that, from my Rule, 855 *l.* the Value, is near upon a Medium, in this Queſtion, between theſe two Authors, and that the Annuitants have 11¾ *per Cent.* for ſinking 855 *l.* on their joint Lives.

PART II.

GEOMETRY.

WE are now come to another very uſeful Part of Knowledge, as it is more or leſs ſo interwoven in almoſt every Concern of Life; the Mechanic, Tradeſman, Merchant, or Gentleman, in the Courſe of their various Stations, ſtand in Need, in ſome Degree or other, of this noble Science. To be ignorant of the *lower Parts* of Geometry, is to deprive ourſelves of what may be of real Benefit to Individuals, beſides the Pleaſure ariſing from the perfect Forms and Regularity of Geometrical Figures. To ſtudy this Accompliſhment, for its real Uſe in Society, is extremely conſiſtent in a Courſe of School Education, or preparing Youth for the buſy Scenes. Theſe *lower Parts* of Geometry give young Gentlemen ſome Idea of the noble Delight which the *higher Parts* of Geometry will afford them, if they ſhould purſue their Studies till they arrive at Phyſical Aſtronomy, which alone rewards them for all their Toil, and ſtrikes the Mind with the greateſt Awe of the Author of ſo many Syſtems of Wonder. It is true the Journey to the Theory of Phyſical Aſtronomy is long; but, when accompliſhed, the Soul is gratified with ſuch noble Concluſions, that Infidelity will never more attack him, unleſs the ſpiritual Faculties are clouded by unregulated Paſſions.

The Properties of a Circle, Angles, and their Parts.

How a Circle is generated. IF you imagine two right Lines, *rx*, *xz*, inclining to a Point *x* (ſee Figure 1) and the Line *xz* faſtened by its End *x*, but yet ſo as to turn quite round, till the End *z* arrives at the Point *y*, where it is ſuppoſed to have ſet off, the Line *xz* will deſcribe a Figure called a *Circle*; that is, all that Space or *Area* contained within the Boundary *y r a n*; but this Boundary is called the *Periphery*, or Circumference, of this Circle, and the greateſt Breadth, *ay* or *rn*, is called the *Diameter* of this Circle, and *x* is its *Center* or middle Point; and *xy*, the *Radius* of this

Circle, which Radius you may eaſily conceive to be juſt Half of the Diameter *ay*, becauſe the Line *xz*, being equal in Length to *xy*, when it ſet off from *xy*, and going half Way round made one right Line with *xy*, which is now only doubled. And from hence you may eaſily conceive, that all Lines drawn from the Center *x* to the Periphery are of equal Length, *(Radii)* becauſe *xz*, in moving round, was of the ſame Length as when it ſet off. Now, imagine the Periphery, *y r a n*, divided into 360 equal Parts, called *Degrees*, and a fourth Part, or Arch of a *Quadrant*, *ry*, of the Circle, will then be 90 *Deg.* a *Semicircle*, or, more properly, Half Periphery, *y r a*, is 180 *Deg.* Hence alſo you may perceive, when the Line *xz* has moved ſo as to ſtand quite upright on the End *x* of the Line *xy*, the End *z* has moved over juſt 90 *Deg.* which is called a *right Angle*; becauſe the Inclination of the two right Lines *xz* and *xr*, meeting in a Point *x*, is called an *Angle*, and is *meaſured* by the Arch *yr*. But an Angle may be *greater* or *leſs* than 90 *Deg.* Thus, it may be eaſily conceived that the further the Lines *xz* and *xr* are opened, the greater will be the Angle; that is, it will contain more *Degrees* of the Arch, which muſt meaſure the Angle or Opening of *xz* and *xr*. If the Angle does not meaſure 90 *Deg.* that is, if *z* has not arrived at *r*, it is called an *acute Angle*; but if more, that is, if *z* has paſſed by *r*, it is called an *obtuſe Angle*. Fix it therefore in your Mind how an Angle is meaſured, as you ſee the Arch of the Circle muſt determine the Opening or Meaſure of any two right Lines touching, or meeting in the Center *x* of a Circle; that is, you may make the Point, where two Lines meet, the Center of your Circle, and if your Arch is divided into Degrees, you will ſee the Meaſure of your Angle. You may alſo conceive every *Degree* divided into 60 other Parts, call'd *Minutes*, for more Exactneſs.

The Angles deſcribed.] Now, after this particular Account of the Manner of generating a Circle, and of the Nature of meaſuring Angles, take your Compaſſes, and deſcribe or draw them. Thus, draw two Lines any how, and then ſet one Foot of your Compaſſes on the Point where they meet, and with any Opening *(Radius)* draw a Periphery with the other, which will make both your Lines of equal Length; and if your Periphery could be graduated, (divided into Degrees) you would ſee how many Degrees your Angle contains that is the Meaſure of your Angle.

FIG. 1. *Scale of Chords.*] But this great Conſumption of Time in graduating your Periphery on every Occaſion, in

order to find the Meaſure of your Angle, that is, Number of Degrees contained between the two Lines, is happily prevented by laying down a Scale, called *a Scale of Chords*, by which any Angle may ſoon be meaſured, provided you uſe the *Radius* of that *Circle* on the Scale. The Method of conſtructing this Scale is to divide the Arch *ry* of the Quadrant *r y x* of about 3 Inches Radius, into 90 *Deg.* which you may conceive to be done with your Compaſſes; then draw the Line *r s y*, which you may call the *Chord of* 90 *Deg* .and conceive one Foot of your Compaſſes to be ſet in the Point *y*, and transfer every Degree from the Arch *ry* on the Line *r s y*, and number them, juſt as if you were going to draw ſo many (90) Circles. By this Means you have a *Scale of Chords*, as you find them mark'd *C*, but of various Sizes, on common Scales uſed in drawing Geometrical Figures. Thus, for Inſtance, in the Uſe of this Scale in meaſuring an Angle, draw your two Lines *BC*, *BA*, *Fig.* 2, and take the Extent of 60 *Deg.* on your *Scale of Chords*, and with that Opening of your Compaſſes ſet one Foot in the very Point *B*, where your two Lines meet, and deſcribe the Arch *Ax*; then taking *Ax* between your Compaſſes, and applying one Foot thereof to the firſt *Deg.* of the *Scale of Chords*, the other Foot will ſhew you the Meaſure of the Angle in Degrees.

You muſt be content to be told at preſent, that the Reaſon why you take the Extent of 60 *Deg.* to deſcribe the Arch *Ax*, is becauſe the Chord of 60 *Deg.* is equal to the Radius of that Circle whereof it is a Chord; and hence it is, that very Scale will meaſure all the Angles greater or leſs than 60 *Deg.* But had you at Random deſcribed the Arch *Ax* from the Point *B*, and then applied to the Scale for the Meaſure of the Angle, you would have been diſappointed. As to the Chords themſelves, they bear a Proportion to the Arches they are drawn againſt; thus, a Line drawn from 1 to 60 *Deg.* *Fig.* 1, is its proportional Chord; ſo from 1 to 30 *Deg.* a Line drawn is its Chord; and in like Manner for all the Degrees of a Quadrant.

Scale of Equal Parts.] To the ſame *Scale of Chords* belongs alſo a *Scale of Equal Parts*, which is the Radius of that *Circle*; that is, the Chord of 60 Degrees divided into 100, &c. Equal Parts. The Manner of doing it is this: Let *AB* represent the Radius of that Circle belonging to ſome Scale of Chords; draw the Line *BC* at Pleaſure, and make the Angle *DAB*=to the Angle *CBA*. Here obſerve that

FIG. 2. *Dividing a Line into Equal Parts.*

Laying down Equal Angles.

that in ſpeaking of an Angle, when three Letters are mentioned, that in the Middle always ſtands for the Angular Point. In order to make the Angles *DAB* and *CBA* equal, with any Opening of the Compaſſes, deſcribe from the Point *B* an Arch *ef*, ſo as to croſs the Lines *AB*, *BC*; then with one Foot in *A* deſcribe *gh*; and, taking *ef* in your Compaſſes, ſet off that Extent from *g* on the Line *AB*, and mark where the other Foot falls, as at *h*; then draw *AD* through the Point *h*; and thus you have laid down the Angle *DAB*=*CBA*. With any Opening of the Compaſſes ſet off from *B* as many equal Parts upon the Line *BC*, as you intend to divide the Line *AB* into, ſuppoſe *five*; for the Method is the ſame, whether five or a hundred, *&c.* From the Point *A* ſet off the ſame Number upon the Line *AD*; and, joining the Points of Diviſion, the Line *AB* is divided into five equal Parts. You ſee you firſt mark 1, 2, 3, *&c.* from *B*; then you mark 1, 2, 3, *&c.* from *A*; then you lay your Ruler on 5, in *BC*, and on the Point *A*; then on 4, in *BC*, and on 1, in *AD*, *&c.* and mark 1, 2, 3, *&c.* on *AB*, from the Point *A*. In this Manner you have a *Scale of equal Parts*, mark'd *EP* on common Scales; the Uſe of which, you plainly perceive, is to ſave you the Trouble of actually dividing a Line, as above, into equal Parts.

Meaſuring a Circle.] As the *Chord* of an Arch, that is, a Line drawn from the firſt Degree of your Circle to any other Degree, bears a Proportion to that Arch it ſubtends or is drawn againſt, ſo the Diameter of that Circle bears alſo a Proportion to its Periphery. Geometricians have demonſtrated, that when the Diameter is 1, the Periphery is 3.1416, and that *the Peripheries of Circles are in Proportion one to another, as their Diameters are*; that is, by having the Diameter, ſuppoſe 20 Inches, to find the Periphery of that Circle, we have this Rule, as 1 : 3.1416 : : 20 to the Periphery of that Circle whoſe Diameter is 20; or by having a Periphery, ſuppoſe 62.8320 Inches, to find the Diameter of that Circle, we then have this Rule, as 3.1416 : 1 : : 62.8320 : 20 the Diameter required. Hence, if either the Diameter or Periphery of any Circle is given, the other may be found by one of the Proportions, by multiplying the ſecond and third Numbers together, and dividing by the firſt. It is likewiſe demonſtrated, that *the Areas of Circles are in Proportion one to another, as the Squares of their Diameters*. By *Area* you are to underſtand all that Space within the Boundary of the Periphery, and this Space is more or leſs in Proportion to the opening of your Compaſſes: So if the *Radius* is ½ an Inch, 20 Inches or 987 Millions of Miles, your *Area* will be leſs or more *in Meaſure*. We raiſe this Proportion for your Rule; but you muſt always conſider, that when the Diameter is 1, the *Area* of that Circle, which is a Part of the Proportion, is .7854; for $1 \times 1 : .7854 :: 20 \times 20$ to the *Area* or Meaſure of that Circle, whoſe Diameter is 20 *Inches*, *Yards*, *Miles*, &c. But to cut the Rule ſhort, ſay *the Square of any Diameter multiplied by* .7854 *gives the Area.* So likewiſe, if the Area a of any Circle is given to find the Diameter D, you have, as $.7854 : 1 \times 1 :: a : D \times D$; that is, your Anſwer will be 400; but you muſt remember this 400 is the Square of the Diameter; and therefore, in all ſuch Caſes, you muſt find the Square Root of the Anſwer for the Diameter required.

So far you are enabled to find different Parts from thoſe given; that is, the Periphery P, by having the Diameter, for $D \times 3.1416$ is the Rule; or the Diameter D, by having P, for $\frac{P}{3.1416}$ is the Rule, or the Area a, by having $D \times D$, for $.7854 \times D \times D$ is the Rule; or the Diameter, by having a, for $\frac{a}{.7854}$ is the Rule. And becauſe the Peripheries of Circles are in Proportion to one another, as their Diameters are, and as the Squares of their Diameters are in Proportion to their Areas; ſo likewiſe muſt *the Squares of their Peripheries be in Proportion to their Areas*; that is, as $p \times p : a :: P \times P : A$; here $p = 3.1416$; and $a = .7854$; and $P =$ (ſuppoſe) 62.832; then we have, as $3.1416 \times 3.1416 : .7854 :: 62.832 \times 62.832 :$ 314.16 the Area of that Circle, whoſe given Periphery is 62.832 Inches, Yards, &c.

Gauging a Circle.] Having found the *Area* of a Circle in Inches, we are prepared to gauge it; that is, to find how many Gallons are contained in its *Area*, which muſt be done before we can tell what any Veſſel, having a circular Bottom, holds. Now, it is known that the Standard for the *Beer or Ale Gallon* is 282 ſolid Inches, the *Wine Gallon* 231, and the *Corn Gallon* 268.8; and then we have, (ſuppoſe as in the laſt Example) as 282 *Inches* : 1 *Gal.* : : 314.16 *Inches* : the Gallons of Beer or Ale; and as 231 : 1 : : 314.16 : the Gallons of Wine; and as 268.8 : 1 : : 314.16 : the Gallons of Corn: So likewiſe, by knowing how many Gallons x, we may find the *Area* in Inches; for, as 1 *Gal.* : 282 *Inches* : : x *Gal.* : 314.16 *Inches*, &c.

Surveying a Circle.] Let us, for a little Amuſement, ſuppoſe a Cloſe to be a perfect Circle; that is, the Hedge to be the Periphery of a Circle, and let us ſuppoſe it meaſures 628.32 Yards$=P$: Here we have, as $p\ (=3.1416) \times p : .7854 :: P \times P$: the Area of the Cloſe in Yards; and dividing this by 4840 Yards, we have the Number of Acres, and multiplying the Remainder by 4, and ſtill dividing by 4840, we have the Roods, and multiplying this Remainder by 40, and ſtill dividing by 4840, we have the Perches. But here it may be obſerved, that finding the Diameter will ſhorten the Work, eſpecially by uſing the Proportion 7 and 22 in this and all other Caſes; that is, thoſe Numbers come near enough to the Proportion 1 and 3.1416; then we ſhall have, as $22 : 7 :: P : D$; then $D \times D \times .7854 =$ the Number of ſquare Yards in the Cloſe. But the Diameter may likewiſe be found thus: Set a Mark, and meaſure the fourth Part of the Periphery each Way, and again ſet Marks; the meaſuring a-croſs, from the ſecond and third Marks, will give the Diameter of the Cloſe.

FIG. 1. *The Senſible Horizon.*] Beſides the leſſer Circles concerned in the Affairs of Society, let us conſider the greater Circles of Nature, of Uſe in Geography and Aſtronomy; the leaſt of which is that called the *Senſible Horizon*, which is taken Notice of by a Spectator looking round him from an Eminence, ſuppoſe at the Center x, and obſerves the Sky and Earth ſeemingly touching each other. Let it be required to find how many Miles a Man muſt travel before he can go round the Periphery, $r\,y\,n\,a$, of this *ſenſible* or *viſible Horizon*. You muſt be told that, by Calculation, the Part where the Earth and Sky ſeem to meet in a clear Day, and when the Eye of the Spectator is properly ſituated, is ſomewhere about 90 Miles, for a round Number. Here then, we have the Radius xz, which is the ſame with xy, of this large Circle, and twice this is the Diameter ya, 180 Miles; then $180 \times 3.1416 =$ the Number of Miles the Man has to travel before he ends his Journey, or comes to the Place from whence he ſet off. And now to find the Area of this vaſt Circle, we have, as $1 \times 1 : .7854 :: 180 \times 180$: the ſquare Miles contained in this *viſible Horizon*, $r\,y\,n\,a$.

Points of the Compaſs.] Now let the Spectator at x, *Fig.* 1. face the Sun exactly at Noon-Day, or Twelve o'Clock; then that Part of the *Horizon* (n) he now faces is the *South*; that directly behind him (r) is the *North*; that on his Right Hand (y) the is *Weſt*, and that on the Left Hand (a) is the *Eaſt*; that

Part *(t)* exactly between the South and the Weſt is the *South-Weſt*; beween the North and the Weſt is the *North-Weſt (i)*; and that beween the South and Eaſt is the *South-Eaſt (u)*; and between the North and Eaſt *(w)* is called the *North-Eaſt*. Now you perceive theſe Points have divided your *viſible Horizon*, firſt into Quadrants, or four Quarters, and then into eight Parts. The Uſe is, when you ſee the Sun at any Time of the Day, you can tell whether he is in the South, South-Eaſt, South-Weſt, *&c.* So when the Wind blows in your Face, you can now pronounce whether it comes from the South, South-Eaſt, North, North-Weſt, *&c.* In like Manner, when you ſtand on the Coaſt and ſee a Ship ſailing directly from the South, you pronounce ſhe is ſteering to the North; or when you perceive a Ship coming directly from the Eaſt, and making towards you, you ſay ſhe ſteers directly Weſt.

Parts of the Horizon.] *Suppoſe I travel South* 10, 15, 20, 25, 30, &c. *Miles from the Center* (x, Fig. 1.) *of my viſible Horizon, What are the Areas of each of thoſe Circles? And if I travel Eaſt* 4, 8, 12, 16 *Miles, What are each of theſe Areas?* The Youth will be pleaſed to raiſe the Proportions in every one of theſe Queſtions, by having the Diameters given, and then he will be in no Danger of forgetting his Rules to work this intereſting Part of Geometry, as a Circle and its Parts are ſo univerſally uſed in all the Concerns of Life.

Fig. 1. *The Moon's Orbit.*] When the Youth, at *E*, ſees the Moon *M* directly in the South, Weſt, or Eaſt, or in the intermediate Points of the Compaſs, he is now enabled to eſtimate the Diameter of the Diſtance, the Periphery of her Journey round the Earth, and the Area of this great Circle. Thus it is computed, that the Moon is about 240,000 Miles from the Spectator's Eye: Here then the Youth is thoroughly prepared to find the Parts of the Circle required; for 240,000 is his Radius, and twice this is *D*, the Diameter of the Moon's Orbit near enough for our preſent Purpoſe, and $D \times 3.1416$ is *P*, the Miles the Moon travels round the Earth in a Lunar Month, and $D \times D \times .7854$ the Area, or Space of the Heavens contained within the Moon's Orbit or Periphery.

Fig. 1. *The Earth's Orbit.*] You muſt know that whether we call *r y n a* the Earth's or Sun's Orbit, the Concluſion is the ſame; but it is found by undeniable Demonſtration, that the Sun, in the Center of the Heavens, as at *x*, ſtands ſtill, and that the Earth *E*, and all the moveable Globes in the Sun's Neighbourhood, perform their Journies round him, to bring

about the Viciſſitudes of Summer and Winter, particularly the Earth *E* we live upon. Now, it is reckoned we at *E* are at leaſt eighty Millions of Miles from the Sun at *x*: Here then is *the Radius* xy *to find the Diameter* ya, *Periphery of the Earth's Journey, and the Area* r y n a, or Space of the Heavens contained within this vaſt Circle *r y n a*: The Rules for finding theſe Things are the ſame as in the Moon's Orbit. But as vaſt a Circle as you may think the Earth's Orbit is, it is but ſmall, when compared with thoſe whoſe Center is your Eye, and whoſe Peripheries are thoſe drawn through the amazingly remote Fix'd Stars.

The Properties of a Square, Triangle, Parallelogram, and their Parts.

FIG. 3. *To draw a Square.*

BUT firſt *to raiſe a Perpendicular at the End of a Line given.* Upon the given Line *AB* raiſe a Perpendicular *BC* of any Length; thus, with any Radius *Br*, (opening of the Compaſſes at Pleaſure) deſcribe a Circle, *hBw*, whoſe Periphery ſhall paſs thro' *B*; then draw a Line, *wrh*, that ſhall paſs through the Point of Interſection *w*, and the Center *r*, and then the Point *h* is what you wanted to find; now lay your Ruler over the Points *h* and *B*, and draw a Line *BC* at Pleaſure, and taking the Line *AB* between your Compaſſes, ſet off that Extent from *B*, and you will have *BC*=*AB*; with the ſame Opening deſcribe the ſmall Arches at *D* from the Points *A* and *C*; then laying your Ruler on the Points *D*, *C*, draw *DC*, and doing the ſame on *D*, *A*, draw *DA*, and you will have your Square *ABCD*, whoſe Sides are all equal, according to the Nature of a Square. Now, when you had raiſed a Perpendicular at the End of the Line, you muſt needs make the Figure *ABCD* of equal Sides, becauſe you had the ſame opening of your Compaſſes, which reached from *C* to *D*, and from *A* to *D*.

Reaſon of raiſing a Perpendicular.

But perhaps you do not ſee the Reaſon of raiſing the Perpendicular *BC*; you muſt know that the Figure *w h B* is a *Triangle*, and you ſee it contained in the Semi-Circle *w h B*, whoſe Diameter is *w r h*, and Geometricians demonſtrate that ſuch a Triangle has one Angle 90 *Deg.* and you ſaw formerly,

Right-angle Triangle.

that to have an Angle of 90 *Deg.* one of the Legs or Lines, forming the Angle, muſt be quite upright; that is, perpendicular; and this was the Reaſon you made *Br* Radius, in order to ſet *BC* upright. Here then you have alſo the Method and the Reaſon of laying down a *Right-angle Triangle*, when three Sides, *wB*, *Bh*, and *wh*, are given: For you need only make *wB* your *Baſe*, and raiſing your Perpendicular *BC*, ſet off *Bh*, and laying your Ruler over the Points *w* and *h*, draw *wh*, and your Triangle is formed: But you muſt remember to let the Periphery of your Radius (after two or three Trials) touch the two extreme Points *w* and *B* of your *Baſe wB*. Or you may draw your Baſe at Pleaſure, and only raiſe the *Perpendicular* as you did in the Square, then take the ſhorteſt Leg between your Compaſſes, and ſet it off from *B* towards *C*, and then take the Baſe between your Compaſſes, and ſet it off from *B* towards *A*, and joining thoſe Points with a Line, this will be the third or longeſt given Line, called the *Hypothenuſe*.

To lay down a Right-angle Triangle.

FIG. 4. *To meaſure a Square.*] Suppoſe one Side *BC* is 9 Feet, and you divide all the Sides by a Foot-Rule into 9 equal Parts, and then draw Lines croſs and croſs, from a Foot in Length to a Foot in Breadth, you will thereby divide your given Square *ABCD* into many leſſer Squares. Now add all theſe together, and as you will find 9 from *B* to *C*, and 9 from *B* to *A*, then $9+9+9$, &c. $=81$ Squares of a Foot each; that is, the given Square meaſures 81 ſquare Feet, the Anſwer required. But Multiplication is a ſhorter Method of performing this Addition; therefore $9\times 9=81$ ſquare Feet: And hence this Rule, *multiply one Side* BC *by the other* BA *for the Content* 81 *of the Square.* If the Side of the Square had been called 9 Inches, there would have been 81 ſquare Inches; or in Yards, Miles, Leagues, &c. there would have been ſquare Yards, Miles, Leagues, &c. Now conſider; as you know the Method of meaſuring a Square, you will ſoon find out a Rule to meaſure a Right-angle Triangle *ACB*, for you ſee the *Hypothenuſe AC* divides the Square into two equal Parts; and ſurely if the Square contains 81 Squares, its greateſt Right-angle Triangle muſt needs contain $40\frac{1}{2}$ Squares; and to obtain this, is to multiply $\frac{1}{2}$ *BC* into *AB*; that is, $4\frac{1}{2}\times 9=40\frac{1}{2}$: And hence we have this Rule to find the Meaſure, *multiply* $\frac{1}{2}$ *the Perpendicular* BC *into the Side* AB *the Triangle ſtands upon.* And now treaſure up in your Mind a noble Property in a Right-angle Triangle, which is, that the Square made upon *AC* is equal to the Sum of the Squares made upon *BC* and *AB*; and hence, by having any two Sides given, we can find the

To meaſure a Right-angle Triangle.

Noble Theorem.]

Square of the third, whoſe ſquare Root is the Anſwer. Thus, ſuppoſe $BC=9$, and $AB=9$, as the circumſcribing Figure is a Square; then $9\times 9=81$ and $9\times 9=81$; and $81+81=162$, and the Root of this is 12.7, *&c.*$=AC$: So if the Hypothenuſe *AC* of ſuch a Triangle and the Perpendicular were given, by taking the Square of the Perpendicular from the Square of the Hypothenuſe, we have the Square of the Baſe; and ſo for the Square of the Perpendicular when it is required.

Fig. 4. *Scale and Compaſſes.*] But if *BC* and *AB* are given, and *BC* is raiſed perpendicularly, and *AC* is drawn, and the Meaſure is required, open your Compaſſes from *A* to *C*, and ſee by the ſame Scale of equal Parts from which you took the given Meaſure of $AB=BC$, how many of thoſe Parts on the Scale *AC* meaſures, and you have the Length of the Hypothenuſe of the Triangle *ACB*. So if the Angle *BAC* is given, and the Legs *AC* and *BA*, you are to lay down the given Angle as before taught, and joining the Points *B*, *C*, the Angle *ACB* is known, becauſe the Right Angle, always 90 *Deg.* is known; and when you know two Angles, you muſt remember if you take them from 180 *Deg.* you have the third Angle, as Geometricians demonſtrate, that the three Angles of every Triangle are equal to 180 *Deg.* But when you have joined all the three Legs, and would meaſure the unknown Angle with your Compaſſes, take the Radius, or 60 *Deg.* and from the unknown angular Point *C* ſweep an Arch a-croſs the two Legs, and taking the Arch between thoſe two Legs in your Compaſſes, ſee on your Scale of Chords the Degrees that Angle contains. In like Manner, if *ACB* is given, and you would find the Angle *BAC*, ſweep an Arch from the angular Point *A* as before, and find the Meaſure on the Scale of Chords.

Fig. 5. *To draw a Parallelogram.*] A Figure called a Parallelogram *ABCD*, is formed from any two unequal Lines, *AB*, *BC*, given; thus, upon *B* raiſe a Perpendicular equal to *BC*, with the Radius *AB*, on *C* deſcribe *gh*; and with the Radius *BC*, on the Point *A*, deſcribe *xy*; then joining the Points *D*, *A*, and *D*, *C*, the Rectangle or Parallelogram *ABCD* will be formed.

Fig. 5. *To meaſure a Parallelogram.*] Suppoſe *BC* is 6 Feet; therefore by your Scale, by which you laid down the given Lines, divide *BC* into 6 equal Parts, and let *AB* be 9 Feet; then divide this alſo into 9 equal Parts; and do the ſame by the Lines *AD* and *DC*, and draw Lines croſs and

croſs from each Diviſion, by which you will have many ſmall Squares of a Foot each, which you may add thus, $6+6+6$, *&c.* nine Times together $=54$ Squares; but ſhorter by Multiplication, $6\times9=54$ ſquare of a Foot each; and hence you have the Rule to find the Content of a Parallelogram, *multiply a Perpendicular by a longer Side.*

FIG. 5. *To meaſure a Parallelogram's Triangle.* Now draw the Diagonal *AC*, and you have the Parallelogram divided into two equal Triangles, one whereof *ACB* you are to meaſure; but undoubtedly you know how to do this, for the $\frac{1}{2}$ of $54=27$ Squares; that is, you have the Rule, *the Perpendicular multiplied into* $\frac{1}{2}$ *the Baſe gives the Content* $=27$; *or* $\frac{1}{2}$ *the Perpendicular multiplied into the whole Baſe gives the Content* $=27$ *ſquare Feet. Now let* AB *and* BC *be given to form a Right-angle Triangle:* As before, raiſe upon *B* a Perpendicular $=BC$, then join the Points *C* and *A*, by drawing the Line *AC*, and you have the Right-angle Triangle *ABC* required.

To raiſe a Right-augle Triangle.

FIG. 5. *Scale and Compaſſes.*] You cannot now be at a Loſs how to meaſure any Side you want to know; for ſuppoſe you are to find the Length of *AC*, you are from your Scale of equal Parts to lay down on your Paper the Baſe *AB*; then raiſing *BC* perpendicularly at Pleaſure, as before taught you, ſet off its given Length from *B* to *C*; then join the Points *A*, *C*, and you have the Hypothenuſe *AC*. Now take *AC* between your Compaſſes, and ſee on your Scale of equal Parts how much it meaſures, and this is the Anſwer. Now, ſuppoſe *BC* is required, and *AB*, *AC*, and the Angle *BAC* are given, you muſt lay down the Angle as before taught, and ſet off the given Sides, and then join the Points *B*, *C*, and meaſure *BC* as before: So if *AB* is required, you are to proceed in the ſame Manner, which you will find eaſy to be performed. Find their Contents in Inches, and divide by 282, 231, 268.8 for Ale and Beer, Wine and Corn Gallons, as was mentioned before. A Square is 25 Inches in the Side: Required the Gallons of the three Sorts? A Parallelogram is 28 Inches in the ſhorteſt, and 52 in the longeſt Side: Required the Gallons of the three Sorts? A Right-angle Triangle is 20 Inches in the Perpendicular, and 45 in the Baſe: Required the Gallons of the three Sorts? It may here, as in the Circle, *&c.* before, ſeem odd to talk of Gallons in a flat Superficies,

To gauge a Square Parallelogram, or Right-angle Triangle.

Superficies, as it cannot hold Liquor in a Heap, but when we come to Solids we are obliged to find the Content of theſe Superficies before we can determine the Solidity. Find their Contents in Yards, and divide by 4840 to find the Number of Acres. A Square is 95 Yards in the Side: Required the Acres? A Parallelogram is 98 Yards in the ſhorteſt, and 125 in the longeſt Side: Required the Acres? A Right-angle Triangle is 85 Yards in the Perpendicular, and 273 in the Baſe: Required the Acres?

To ſurvey a Square Parallelogram, or Right-angle Triangle.

FIG. 6. *The greater Squares of Nature.*] To all the great Circles of Nature there is a proportional Square; that is, when the Area, ſuppoſe of the Earth's or Sun's Orbit *ADCB* is given, the proportional Square, *HFEG*, which circumſcribes this Circle, *ADCB*, is the Diameter of the Orbit ſquared; for you ſaw before, on divers Occaſions, that the Proportion was, as $1 \times 1 : .7854$; that is, in a Circle whoſe Diameter is 1, the Square of the Diameter is in Proportion to its Area .7854; and this 1×1 is the circumſcribing Square, whoſe Side *GH* (FIG. 6.) is the Diameter of that Circle. Therefore find the circumſcribing Squares and the Sides to thoſe Circles of Nature, ſuch as the Horizon, Equator, Orbit of the Moon and the Earth, by this Analogy or Proportion, as $.7854 : 1 \times 1 :: A : DD$: Here $A =$ the Area of the Circle, and *DD* is the Square in Proportion to *A* the Area; then the Square Root of *DD* is the Side of that circumſcribing Square. It muſt needs be eaſy for the Learner to find the inſcribed Squares (and the Sides) to thoſe Circles alſo; for $IB = IC = \frac{1}{2} AC$ the Diameter of the Circle. Now then, as before, we have $IB \times IB + IC \times IC = BC \times BC$ the inſcribed Square, and the Square Root of this gives the Side *BC* of the Square *ABCD*. Hence you perceive that the Diameter $AC = GH$ of the Circle *ABCD* is the Side of the circumſcribed Square, and the Chord of 90 Deg. *BC* is the Side of the inſcribed Square *ADCB*. It is worth your Pains to find the inſcribed Squares (and their Sides) to the Horizon, Equator, &c. as above, and likewiſe to diſcover the Proportion between the two Squares, and then compare their Areas with that of their Circle. Thus, the Square *HGEF* contains more Space than the Square *ABCD*, by the Triangles $BEC + CFD + DHA +$

FIG. 6. *To find the Square of the Horizon,* &c.

To find the inſcribed Square of the Horizon, &c.

FIG. 6. *Difference of the two Squares.*

$DHA+AGB$; therefore the Difference is eaſily found by finding the Contents of thoſe Triangles, and taking their Sum from the circumſcribed Square, and the Remainder is the inſcribed Square, which will be equal to the Square of the Hypothenuſe *BC*, found as before directed. Then we have as the Square *HGEF* : the Area of the Circle *ABCD* : : the Square *ABCD* : its inſcribed Circle required; and therefore you may perceive a noble Proportion; for if any three of theſe four Terms are given, you may find the fourth. And now you may infer, that the two Squares are in the Proportion of 2 to 1; for the greater contains 8 equal Right-angle Triangles, and the leſſer only 4; that is, every inſcribed Square is equal to Half the circumſcribed Square; and that the Area of the greater Circle is twice the Area of the leſſer Circle, correſponding to the inſcribed Square; for as $2 : A :: 1 : \frac{A}{2}$ the Area of a Circle *ADCB* to which the inſcribed Square becomes a circumſcribed Square.

FIG. 7. *The greater Triangles of Nature.*] Let us take the Earth's Orbit and form ſome noble Proportions of Importance, in applying Right-angle Triangles to ſome of their greater Uſes in Nature, tho' we are confined at preſent to the Scale and Compaſſes. With the Radius of your Scale of Chords draw the Circle *ASB*; on *B* raiſe the Perpendicular *BO*; now we will give any Angle, ſuppoſe 50 *Deg.* therefore take 50 *Deg.* from your Chords, and ſet off the Arch *Bi*; then from *x* draw a Line through the Point *i*, till it meet with the Perpendicular *BO*. Now you have a Right-angle Triangle *BxO*, Angle $B=90$ *Deg.* becauſe *BO* is perpendicular, Angle $x=50$ *Deg.* and then ſurely Angle $O=40$ *Deg.* becauſe $90-50=40$, or $180-90+50=40$ *Deg.* You know $xB=80000000$ Miles, as we told you formerly: And here you perceive your Scale of equal Parts fails you, as you cannot well meaſure ſuch immenſe Diſtances as the Sides *xO* and *BO* are by it. But you are to remember, that *BO* is called the *Tangent* of 50 *Deg.* and *xO* the *Secant*; and that the Side, repreſenting the Secant, is in Proportion to the Angle of 90 *Deg.* as the Side, repreſenting the Tangent, is to 50 *Deg.* So that if you ſhould hereafter be taught to look into the Books that have theſe Sorts of Tables, you, by having three of ſix Parts of the Triangle, may ſoon learn to find the fourth; that is, by having the Radius, and all the Angles in the preſent Caſe, we have, 40 *Deg.* : *xB* : : 90 *Deg.* : *xO*; and as

Secant and Tangent.]

40 *Deg.* : *xB* : : 50 *Deg.* : *BO*. By theſe Calculations we have found the two unknown Sides, from having the Side *xB*, and the Angles given. The Learner will do well to give due Attention to what has been ſaid of the *Tangent* and *Secant*, and the Parts of the Triangle given, that he may the more radily retain the other Proportions now to be laid before him. But before we can proceed he muſt be ſhewn how *to let fall a Perpendicular upon a Right Line from any given Angle or Point*, ſuppoſe from the Point *i* in *Fig*. 7. With any convenient Radius on *i*, *Fig*. 8. deſcribe *d*, *f*; with the ſame Radius on the Points *d*, *f*, deſcribe the two ſmall Arches, interſecting in *R*, and join the Points *iR*, but let the Line fall ſhort at *e*. The Reader is deſired to remember, at preſent, this mechanical Method of dropping a Perpendicular, the Reaſon of which ſhall be ſhewn him in another Place. Now the Line *ie*, or Perpendicular let fall, is called the *Sine ie* of the Angle *x*, *Fig*. 7.=50 *Deg*. and this *Sine* is alſo the Half Chord of the Angle *x*, and bears, like the Tangent, a Proportion to the Angle it ſtands againſt or ſubtends. And now we have another Triangle *xie* formed, and though there are ſix Parts (three Sides and three Angles) in a Triangle, we have here only four given, the three Angles, and the Side *xi*, (Radius=*xB*) to find the two Sides *xe*, *ie*: Then we have, as 90 *Deg*. : *xi* : : 50 *Deg*. : *ie*; and as 90 *Deg*. : *xi* : : 40 *Deg*. : *xe*. But it muſt be obſerved that this *xe*=*ai* which is the *Sine* of the other Part of the Quadrant; that is, *ie* is the *Sine* of 50 *Deg*. and *ai* is the *Sine* of 40 *Deg*. called the *Sine Complement*, or what *ie* wants of 90 *Deg*. The worthy Youth, by ſeeing a ſmaller Triangle drawn with Part of the Baſe and Hypothenuſe of the larger, and having Part of the others perpendicular, but removed from it on the ſame Baſe, muſt needs ſee there is a Likeneſs or *Similarity*, and conſequently a Proportion preſerved between two ſuch like or *ſimilar Triangles*; that is, as *BO* : *xB* : : *ie* : *xe* : or as *xB* : *xo* : : *xe* : *xi*. In like Manner as the *Sine ie* has its *Secant* and *Tangent*, ſo has the *Sine Complement ai* its *Secant Complement xr*, and *Tangent Complement sr*, or ſecond Syſtem of correſponding Triangles; for *ai* is the *Sine*, *xr* the *Secant*, and *Sr* the *Tangent*, and *xi* the ſame as before, and theſe Triangles of the Complements are alſo in Proportion to the others, which you may perceive as the *Sine Complement* and the leſſer Hypothenuſe entered before into the Proportions.

Fig. 8. *To let fall a Perpendicular from a given Point.*

Fig. 7. *Sine and Sine Complement.*

Fig. 9. *The Triangles of the Horizon.* I obſerve a Village at a great Diſtance, directly South of me; I alſo diſcover the Steeple of a large Town which lies due Eaſt from the Village; we ſet off in an Angle of 50 *Deg.* of the Periphery of the Horizon, and find by our meaſuring Wheel, that when we come to the Town, we had travelled 20 Miles. The Queſtion is, *How far I was from the Village when I ſet off, and how far the Town is from the Village?* Here you have a Right-angle Triangle, *NSE*, which you muſt conſtruct thus; draw a ſtraight Line, *NS*, on your Book to repreſent the North and South Line; then at the North Point *N*, with your Compaſſes, ſweep an Arch of 50 *Deg.* taken from your Scale of Chords; you are now to draw the Hypothenuſe *NE* through the Eaſt Point *E* of the Arch, to form the Angle of 50 *Deg.* and to ſet off from your Scale of equal Parts, 20 Miles from *N* to *E*, at the End whereof, which is the Town, let fall your Perpendicular *ES* on the North and South Line *NS*. The Triangle being now conſtructed, take, with your Compaſſes, the Length of the Baſe (the North and South Line) *NS*, and applying that Extent on your Scale of equal Parts, you will find the Diſtance in Miles of the Village from your firſt Station; and taking the Length of the Perpendicular *SE* (the Diſtance of the Town from the Village) between your Compaſſes, and applying that alſo on your Scale of equal Parts, you have the Meaſure in Miles of the Diſtance between the Town and Village: But if you ſhould hereafter learn to uſe the *Tables of Sines and Tangents, &c.* you will have theſe Proportions, as 90 *Deg.* : *Hypothenuſe* : : 50 *Deg.* : *the Diſtance of the Town from the Village*; *and as* 90 *Deg.* : *Hypothenuſe* : : 40 *Deg.* : *the Diſtance of the Village from the firſt Station.* You know how I come by 40 *Deg.* becauſe in every Right-angle Triangle 90 *Deg.* made by the Perpendicular is always given; ſo when you have 50 *Deg.* given alſo, it is 90—50=40 *Deg.* for there muſt be 180 *Deg.* in every Triangle, and 180—90=90 for the other two Angles; and as 50 is given, it is 90—50= the third Angle. And thus you may form by yourſelf many uſeful and entertaining Queſtions, in ſurveying a Country, from the above Example.

Fig. 9. *Courſe and Diſtance of a Ship on her Voyage.* The worthy Youth, if he is deſigned for a Merchant, will be highly pleaſed with himſelf for having, with ſo little Trouble, acquired this truly uſeful and genteel Part of Knowledge, which will, excluſive of

all other Conſiderations, enable him to look into a Captain's Sea-Journal, of Conſequence to thoſe Merchants concerned in Ships. And ſurely if the Youth is to lead the Life of a Gentleman, this Accompliſhment cannot fail of rewarding him for a few Hours Application to the general Principles of conducting a Ship through the pathleſs Ocean. Now, draw your North and South Line *NS*, as before, and let the North Point *N* be the Place the Ship ſails from, in an Angle *SNE*, by the Mariner's Compaſs, of 50 *Deg.* and meaſuring her Way, *NE*, as ſhe goes to the Left or Eaſt *E*, by throwing the Log-Line, ſhe is found to have ſailed 20 Miles, or the Length *NE*. The Queſtion is, How far ſhe is now to the Eaſt *E* of the South Point *S* of the North and South Line, *NS*, and how far the South Point *S* of the North and South Line *NS* is from the North Point *N* of this Line? Here are given the ſame Parts you had before, in ſurveying the Country, to conſtruct, by Scale and Compaſſes, the ſame Right-angle Triangle *NSE*, and to find the ſame Parts. You now perceive the Perpendicular *ES*, when meaſured on your Scale, ſhews you how many Miles due Eaſt the Ship is from the South Point *S* of the North and South Line *NS*; and the Baſe *NS* ſhews you, when meaſured on your Scale, how far the South Point *S*, of the North and South Line *NS*, is from the North Point *N* of this Line *NS*, or Place from whence the Ship ſailed. This is the moſt general Problem in Navigation, wherein the *Courſe* (Angle *SNE*) and *Diſtance NE* ſailed, are always given, to find the other two Sides of the Triangle, which you perceive are eaſily found by your Scale and Compaſſes: and you ſee the whole is like your *Surveying Problem*, only you ſaw the Village before, but now you have only Sky and Water, which obliges you to draw a longer North and South Line than you want, as you cannot tell where your Perpendicular will fall on the Baſe, or North and South Line. And here again, when you come to uſe *Sines* and *Tangents*, &c. you will have the ſame Proportions, as in ſurveying the Country; that is, as 90 *Deg.* (or Radius) : *the Diſtance ſailed* : : 50 *Deg. the Sine of the Courſe* : *the Eaſtern Diſtance from the South Point of the North and South Line*; and as 90 Deg. : *the Diſtance ſailed* : : *the Sine Complement of the Courſe* 40 *Deg.* : *the Length of the North and South Line.*

Fig. 10. *Other Caſes of Sailing.*

But it ſometimes happens that the Mariner is obliged to find the Courſe *SNE*, and Diſtance *NE*, from having the Baſe *NS* and Perpendicular *ES* given. Now you know this is moſt readily performed by drawing your North and South Line *NS* at Pleaſure, and with your Compaſſes ſetting off the real Length from your Scale, and raiſing a Perpendicular *ES* at Pleaſure on the South Point of the North and South Line, and ſetting off its real Length alſo from that Point: Draw your Hypothenuſe *NE* from the Top of your Perpendicular to the North Point of the North and South Line, and you have formed the Right-angle Triangle *SNE*. Now take the Hypothenuſe between your Compaſſes, and you will find, on your Scale of equal Parts, the Diſtance the Ship has ſailed; and with the Radius of 60 *Deg.* on the Scale of Chords, deſcribe an Arch *Si* from the North Point *N* of the North and South Line; then take between your Compaſſes that Part of the Arch *Si*, interſected between the Hypothenuſe *NE*, and the Baſe *NS*, and applying this Opening to the Scale of Chords, you have the Angle *SNE*, or *Courſe* the Ship ſailed on. Here you will be puzzled to find the Proportions, in order to anſwer this Queſtion by *Sines* and *Tangents*, *&c.* for as you have but one Angle *NSE* (90 *Deg.*) given, that ſhould be your middle or ſecond Number in your Stating, or Queſtion in *the Rule of Three*; but as the Side *NE* (Hypothenuſe) oppoſite to it, is not known, we have not three Numbers in Proportion to find a fourth; and therefore we muſt take another Method. You know that the *Sine* of the Arch of the whole Quadrant muſt ſtand before 90 *Deg.* as it is the greateſt that can be drawn in a Quadrant; and that this 90 *Deg.* is neither more nor leſs than the Side *NS*, (Baſe) by which you draw a Circle; that is, it is both a Side of the Triangle *SNE*, and the *Sine* of 90 *Deg.* if you complete the Quadrant. Therefore, now you have ſufficient Numbers in *the Rule of Three*; for making the Baſe *NS* the Radius of the Circle, we have, as the Baſe : 90 *Deg.* : : the Perpendicular : the *Tangent* of the Courſe, becauſe the Perpendicular, by making the Baſe the Radius, becomes the *Tangent*, and the Hypothenuſe becomes the *Secant*. And now try your own Ingenuity, by having other Parts given, to conſtruct the Triangles, and to find the reſt by Scale and Compaſſes, and forming the Proportions to anſwer the Queſtions alſo by actual Calculations; that is, if you ſhould chuſe to uſe the Tables of Sines, Tangents, Secants, and Logarithms.

Surveying of Land.] From what has been ſaid of finding the Parts in *Right-angle Triangles*, the Learner cannot be at a Loſs to apply the Particulars to Uſe in Surveying of Land; for in the Variety of Triangles to be caſt up, when very large Diſtricts of Land are to be ſurveyed, the Contents are more accurately found by Calculation than by the Scale and Compaſſes: Thus, for Inſtance, if the *Perpendicular* and *Angles* are to be looked upon as only given, the Baſe on each Side may be found by Calculation; which, as the two Baſes in one are to be multiplied into ½ the Perpendicular, the Error may be leſs than by Scale and Compaſſes. Thus, as the Angle at the Baſe is to the Perpendicular, ſo is the Angle oppoſite the Baſe to the Baſe; and in like Manner for the Baſe on the other Side of the Perpendicular; then the Sum of theſe Baſes, ſo calculated, multiplied into ½ the Perpendicular, as it meaſures by the Scale, will turn out more exact than by the uſual Method of finding the Contents of the Triangles: For Inſtance, ſuppoſe the two Baſes ſhould, on the Scale, be taken for 1500½ Yards, and this ſhould be only ½ Yard too much; and let us imagine the ½ Perpendicular to be 500 Yards accurately taken by the Compaſſes, and meaſured by the Scale, here will be an Error of 250 Yards of Ground; and, in an irregular four-ſided Figure, the Error will be 500 Yards of Ground more than what will turn out by Calculation.

Fig. 11. *To deſcribe an Equilateral Triangle.*

Let the given Line be *xy*, with Diſtance *xy*, on the Point *x* deſcribe the Arch *ri*; and, with the ſame Opening of the Compaſſes, on the Point *y* deſcribe *nm*; then join the Points *xa*, *ya*, and the three Sides of the Triangle will be of equal Length; for you ſee when you have *xy* between your Compaſſes, that Opening is *ya*, *xa*, and then ſurely the Sides muſt all be *xy*.

Fig. 11. *To divide a Line into two equal Parts.*

Before we can meaſure the equilateral Triangle *xay*, we muſt change it into two Right-angle Triangles *xaſ* and *ſya*, by dividing the Line *xy* into two equal Parts. To do this is only to make an Addition to the laſt Performance. From the Ends of *xy*, with the Radius *xy*, as before, (or any Radius greater than Half *xy*) deſcribe the Arches *nm*, *ri*, (but this you did before) and *gh*, *kl*, below the Figure; then join the Interſections *a H*, but you need not draw this Line out beyond the Line *xy*, which is now divided into two equal Parts, becauſe the Points *a*, *H*, are ſituated exactly over that middle Point in *xy*.

Fig. 11. *To meaſure an Equilateral Triangle*

Deſcribe a Parallelogram *awqy* with dotted Lines about the Triangle *xay*, then *a ſ* is equal to the Side of the Parallelogram, ſo that $aq \times qy = 2\,aqy$; that is, two Triangles aqy; becauſe $a\text{ſ}yq$ contains two, and $aqy = ax\text{ſ}$; therefore $aq \times qy$ is the Meaſure of the given equilateral Triangle xay, and gives this Rule, multiply the Perpendicular into one of the Baſes of the two Right-angle Triangles. Let the Perpendicular, as by the Scale of equal Parts, be 20 Inches, and the given Side $xy = 16$ Inches; then 20×8 (or qy) $= 160$ Inches required.

FIG. 12. *Fruſtum of an Equilateral Triangle.* This is the Figure $xd\text{ſ}y$, which you muſt now meaſure. Let gq be 5 Inches, and xy call 9 Inches, and df 4 Inches; then form the Parallelogram $gqfy$, and you have the Half Fruſtum $gqfy$, and the Triangle fhy, and both together make up the Parallelogram $gqhy$. Now, $qf = 2$, and $gy = 4.5$; then $4.5 - 2 = 2.5$, which is the Perpendicular fh, of the Triangle fhy; and the Meaſure of this Triangle is the Half of $yh \times fh$; that is, $\frac{5 \times 2.5}{2} = 6.25$ Inches; and this taken from the Parallelogram $gqhy$ 22.5 (5×4.5) gives 16.25 the ſquare Inches in the Half Fruſtum $gqfy$. As you want to have a Figure $tqga$ that ſhall have as many ſquare Inches as the Half Fruſtum $qfyg$; therefore if you divide 16.25 by 5 ($= gq$) you have 3.25 the other Side tq of the Figure $tqga$, containing 16.25 ſquare Inches, becauſe $gq = 5 \times 3.25 =$ the Half Fruſtum; and then $16.25 \times 2 = 32.5$ ſquare Inches in the whole Fruſtum: From whence we have this Rule, *if the Difference of the two Ends* (fh) *multiplied by the Height* (gq) *is taken from the Product of the greater End by the Height, the Remainder gives the Content of the Fruſtum.* But let us do this by a ſhorter Rule, *multiply the Arithmetical Mean, between the Ends, by the Height, and it gives the Content of the Fruſtum:* Thus, $\frac{4+9}{2} = 6.5$ the Mean, and this $\times 5 = 32.5$ the Content as before. You ſaw before that 3.25 was ½ the Side of the new Figure, and twice $3.25 = 6.5$ the mean Side as by the ſhorter Rule, which you now prove to be true; that is, to find out a Parallelogram which ſhall be equal in ſquare Inches to your Fruſtum.

FIG. 13. *To deſcribe and meaſure an Iſoſceles Triangle.* This is a Figure that has only two Sides equal. Let us imagine that $df = 4$ Inches, and $da = fa$, and $ag = 5$ Inches. Draw a Line at Pleaſure, and ſet off $df = 4$ Inches. Then biſect or divide df into two equal Parts, and draw a Line through g at Pleaſure; and from g ſet off

$ag=5$, and draw *af*, *ad* meeting in *a*. Then to meaſure this Figure, you muſt conceive it circumſcribed with a Parallelogram, and then it will be $ſf \times df = 20$ Inches. And, as it was ſhewn before, that every Parallelogram *drſf* is equal to twice the Triangle *daf* it is circumſcribed about; therefore, $ag \times \frac{1}{2} df = 10$ Inches (the Perpendicular $ag = ſf$, the Content of the given Iſoſceles Triangles. Now, at Pleaſure, cut off the Top of this Triangle, and then you will have a *Fruſtum dxof*, and a *Triangle xao:* Meaſure the unknown Sides with your Scale and Compaſſes, and find, from what you have learned, the Contents of both theſe Figures, which you know muſt not vary conſiderably from 10 Inches.

Fig. 14. *To deſcribe and meaſure a Scalenous Triangle.* This is a Figure that has all the Sides unequal. Let the Sides be $AB = 5$ Inches, $CB = 4$, and $AC = 3$ Inches. Upon *AB*, with the Opening 3 deſcribe a ſmall Arch from the Point *B*; with the Opening 4 deſcribe another ſmall Arch from the Point *A*, which ſhall interſect the other Arch in *C*; join the Points *A*, *C*, with the Side *AC*, and the Points *C*, *B*, with the Side *CB*. Now let fall a Perpendicular *CD* from the Point *C* upon the given Side *AB*, then take *CD* between your Compaſſes, and find its Meaſure on the ſame Scale by which you laid down the Triangle. Then $\frac{1}{2}$ the Perpendicular × into *AB*, or $\frac{1}{2} AB \times CD$, is the ſquare Inches. Now again, at Pleaſure, draw a Line parallel to *AB*, by which you will divide *ACB* into a Scalenous Triangle, *xCr* and its irregular Fruſtum *AxrB*. Meaſure what Lines you have occaſion for, and find the Contents of the Triangle and its irregular Fruſtum, and give your Reaſons, which you are now enabled to do, as you ſee the Uſe of circumſcribing your Fruſtums and Triangles with Parallelograms, which you know will ever be double their inſcribed Triangles. But leſt you ſhou'd be at a Loſs to find the Content of this Fruſtum *AxrB* I ſhall point out the Method, which will alſo enable you to find the Superficies of the Fruſtum of the Iſoſceles Triangle in the laſt Fig. 13. Let the Parallelogram *DCzB*, Fig. 14. be compleated, and let $a =$ the Content thereof in Inches; then $\frac{a}{2} =$ the Meaſure of the Triangle *DCB*, and $\frac{a}{2} - oCr. = DorB$; and let $CD \times AD = e$ the other Parallelogram, ſuppoſed to be circumſcribed; then $\frac{e}{2} =$ the Triangle *ADC*, $\frac{e}{2} - xCo = Ax\ D$, the other Part of the Fruſtum. Hence, $DorB + AxoD = Axr$,.

$AxrB$, the Meaſure of the Scalenous Fruſtum required. So in the laſt Fig. 13, $\frac{sf\times gf}{2} - tao = gfot$; and $\frac{rd\times dg}{2} - xat = dxtg$: Then $gfot + dxtg = dxof$, the Content of the Iſoſceles Fruſtum required.

FIG. 15. *To ſurvey many ſided plain Figures.* There muſt needs be great Variety in this Problem; however, in all Caſes, they are ſubject to one Rule, which is, *to throw the whole Figure into Triangles, and then, letting fall Perpendiculars upon each Baſe, you proceed to find the Content of each Triangle, and then add them all up together, and you have the Meaſure required.* Thus, there is an irregular Figure which has ſix unequal Sides; then, to deſcribe this Figure upon Paper, there muſt be given *all the Lines CB, CD, CE, CF, CG, CH,* and the Angle of every two Lines drawn from ſome Point *C*, towards the Middle to *every Angle*; then draw *BD, DE, EF, FG, GH, HB,* by which Means you have incloſed all the Space the Figure contains, and have thrown the whole into *ſix Triangles*, which are to be meaſured by letting fall Perpendiculars from an Angle oppoſite the longeſt Line in each Triangle. Then with your Compaſſes take the Length of the Perpendiculars, and applying them to your Scale by which you laid down the Lines drawn from towards the Middle of the Figure, you are prepared to give the Contents of the whole Figure.

To gauge and meaſure theſe and all irregular Figures. Find the Area of Equilateral Iſoſceles, or Scalenous Triangles, and their Fruſtums, and alſo the laſt irregular Figures in Inches, and divide by 282, 231, 268.8 to find the Contents in Gallons of Beer, Wine, or Corn.

In meaſuring and gauging theſe irregular Figures, the general Method is to find the Diagonals with a Chalk Line. Thus, in Fig. 16. are drawn *BF, DF, BG*; then are let fall the Perpendiculars *Hi, Gb, Ef, Dc,* and taking theſe between your Compaſſes, and applying theſe Openings to your Scale, you are prepared to gauge or meaſure the Figures. You may obſerve that the Exciſeman and Carpenter uſe the ſame Method in finding the Diagonals, which indeed is the ſame when the Surveyor meaſures the Field with his Chain, according to Fig. 16; but Fig. 15. ſhews he makes Uſe of his Angular Inſtruments, by which he readily diſcovers the Angle between any two Lines, drawn from the Place from whence he chuſes to obſerve the Corners of an irregular Incloſure; and when he finds the Angle between every two Marks ſet up, and meaſures his Diſtance from him to each Corner, he is prepared with his Scale of Chords, and Line of equal Parts, to lay down theſe Dimenſions on Paper. And thus you have the Pleaſure of knowing how ſuch irregular Figures are *meaſured*, *gauged*, and *ſurveyed*, tho' your Station in Life may be ſuch as may not require an actual Performance of this Buſineſs.

The Properties of a Globe, Cylinder, Cone, and their Parts.

FIG. 17. *Globe, how generated.*] You may imagine the *Semi-Circle A B C* to be moved quite round on its Diameter *A B*, and it will deſcribe a Figure called a *Globe* or *Sphere ABCD*; then, as was ſhewn in the Circle, all Lines drawn from its *Center* to the Periphery are equal Radii; ſo all Lines, conceived to be drawn from the *Center* of a Globe to its *Surface*, ſo generated by the Motion of the *Semi-Circle ABC*, will be equal among themſelves, as the Surface is only an infinite Number of *Peripheries* formed in the Revolution of the *Semi-Circle ABC*. And hence alſo all Lines drawn from Surface to Surface of the Globe, and paſſing through its *Center*, will be ſo many *Diameters* of that *Semi-Circle* which formed the Globe; and hence it is, that the greateſt Circle of the Globe is that whoſe Diameter *AB* is the Diameter or *Axis* of the Globe *ABCD*, paſſing through its Center: Or conceive the Generation of a Globe thus; Spin a Halfpenny or Crown Piece, repreſented by the Circle *ABCD*, upon the Table, and you will ſee how the Circle generates the ſolid Globe, and not only one but many, as long as the Spinning continues; ſo that you ſee while the Motion laſts, the Diameter of your Crown Piece, which is a Circle, is the Axis *AB*, or Diameter of the ſolid Globe *ABCD*.

Cylinder, how generated.] Lay your Crown Piece *EHFG* (Circle) flat on the Table, and imagine it to be moved up to the exact Height of its Diameter *AB*, and you have generated a *Cylinder EILKMHFG*, whoſe *Axis*, or Height, *AB*, is the Axis of a Globe, the greateſt the Cylinder will contain; and hence you may conceive that your Crown Piece *EHFG* (Circle) is equal to a Circle, whoſe Diameter is the Axis of the Globe, and that your Cylinder is generated by the Motion upwards of one of the greateſt Circles of the Globe.

Cone, how generated.] Now lay your Crown Piece, *EHFG*, flat on the Table again, and imagine it to riſe up, as before, to the Height of its Diameter, or Axis *AB*, but that its Periphery *EHFG* gradually decreaſes, till it is ſhrunk up into its Center or Point *A*, being nothing but a Point, having in the Imagination neither Periphery nor Diameter; and you have a Cone *EHFGI*, whoſe *Axis* is the Diameter *EF* of its generating Circle, and its *Baſe EHFG* the Circle itſelf, which is alſo the *Baſe* of the Cylinder, and this *Baſe* of the Cylinder is alſo a great Circle of the *correſponding* Globe. And hence you ſee what a Connection there is between the Globe, Cylinder, and the Cone, when the Baſes of the two laſt are a great Circle of the firſt, and their Height equal to the Diameter of their Baſes; and you may even now conclude there muſt be a noble Proportion among theſe geometrical Figures, and that to know the Meaſure of one is giving Rules to find the Meaſures of the other two.

FIG. 18. *To deſcribe a Cylinder.*] Draw *cd*, ſuppoſe $1\frac{1}{2}$ Inch, and upon the End *d* raiſe a Perpendicular *df* of any convenient Length, ſuppoſe $2\frac{1}{4}$ Inches: With the Opening of your Compaſſes *cd* deſcribe from *f* an Arch at *e*; and with the Opening *df*, from *c* deſcribe another Arch croſſing that at *e*: Draw *ef*, *ec*, and then you have the Section *cefd* of the intended Cylinder *cegſhidk* cut directly through the Centers *t*, *b*, of the Circles of the Top and Bottom: Draw the Axis *tb* through the Centers *t*, *b*. Here again you have alſo an Opportunity of conceiving how the *Cylinder* may be generated; for if you imagine the Parallelogram *tfdb* to turn quite round on the Axis *tb*, you have the Cylinder *cegfhidk*. With any convenient Opening of your Compaſſes, as *ix*, deſcribe on *x* the Arch *cid*; then take *ix*, and ſet it off from *k* to *y*; and on *y* deſcribe the Arch *ckd*. With the ſame Extent *ix* of your Compaſſes, in the ſame Manner, proceed to deſcribe from the Points *z*, *r*, the Arches *ehf*, *egf*; that is, *zh*=*ix*, and *gr*=*ky*.

FIG. 19. *To deſcribe a Cone.*] Draw *ab*, ſuppoſe $1\frac{1}{2}$ Inch; biſect *ab* in *r*, and draw *rqx*, and with the Opening *rq*, ſuppoſe $2\frac{1}{4}$ Inches mark the Point *q*, and draw *aq*, *bq*. With the Diſtance *iz*, draw on *z* the Arch *aib*; then make *wo*=*iz*, and with the Diſtance *wo* draw on *o* the Arch *awb*. And here again you may conceive the *Cone* to be generated by the Motion round the Triangle *rqb* on the Axis or Side *rq*.

Meaſuring a Cylinder.] You know the Cylinder is a Solid, made up of an infinite Number of Circles, equal in their Dimenſions to, and parallel to the Circle, which is the Baſe; therefore, *find the Area of the Baſe, and multiply that by the Axis or Height of the Cylinder, and this will give the ſolid Content.*

Content. Thus, ſuppoſe the Diameter of the Baſe is $1\frac{1}{2}$ Inch, and the Axis of the Cylinder is $2\frac{1}{4}$ Inches, we have, as $1\times1 : .7854 :: 1\frac{1}{2}\times1\frac{1}{2}$: the Area a of the Baſe; then $a\times2\frac{1}{4}=$the ſolid Content of the Cylinder. Theſe ſhortened, and turned into Decimals, will ſtand thus, $1.5\times1.5\times.7854\times2.25=$the Content; but you may ſtill ſhorten the Work, by multiplying the Area of the Baſe by 2, and dividing the Area by 4, add the Quotient to the Product of 2, as was formerly taught you.

Meaſuring the Superficies of a Cylinder.} *The Rule is, multiply the Periphery of the Baſe by the Axis of the Cylinder, to which add twice the Area of the Baſe:* For if you conceive it to be a hollow Cylinder, made of thin Paſte-Board, and cut perpendicularly down and ſpread out, you have a Superficies, the Length whereof is the Periphery of the Baſe, and the Height the Axis of the Cylinder; therefore multiplying the two Dimenſions together, you have the Superficies of the Sides of the Cylinder, now in the Form of a Parallelogram: But you know the Top and Bottom ſtill remain Circles; therefore find the Area of one, and, doubling it, add it to the Meaſure of the Parallelogram. For an Example; ſuppoſe the Height $2\frac{1}{4}$ Inches, and the Periphery of the Baſe 4.7124 Inches; then $4.7124\times2\frac{1}{4}=$the Superficies (S) of the Sides, and the two Ends$=2a$; then $S+2a=$the Superficies of the Cylinder, or ſuch outward Part to be gilded or painted.

Meaſuring a Globe.] *The ſolid Content of a Globe is found by taking $\frac{2}{3}$ of the Content of its circumſcribing Cylinder.* Thus, let the Diameter of the Baſe of the Cylinder be 2 Inches; then, $2\times2\times.7854=a$ the Area of the Baſe, and $a\times2$ the Height or Axis of the Cylinder is 6.2832 Inches the ſolid Content of the Cylinder, and $\frac{2}{3}$ of $6.2832=4.1888$ Inches the ſolid Content of a Globe whoſe Diameter is 2; which you may prove thus, by this Rule, $.5236DDD=4.1888$, as before; that is, the Cube of the Diameter 2 (D) is 8, and $8\times.5236=4.1888$.

To meaſure the Superficies of a Globe.} *The ſolid Content of a Cylinder is equal to $\frac{1}{2}$ the Superficies of its inſcribed Globe.* Thus, as before, the ſolid Content of the Cylinder (whoſe Diameter at the Baſe and Axis is 2) will be 6.2832; then the Superficies of the Globe will be 12.5664. Again, by another Rule, *the Superficies of a Globe is equal to four Times the Area of its greateſt Circle.* The Diameter is, ſuppoſe, as before, 2;

then, $2\times 2\times .7854\times 4=12.5664$ as above: Or, by another Rule, *the Superficies of a Globe is equal to the Superficies of the Cylinder without its Ends.* Thus, ſuppoſe the Diamenter 2, then, by the former Rule, we have, $3.1416\times 2=P$; then, $P\times 2$ the Axis$=12.5664$, the Superficies of the Globe: Or again, ſince the ſolid Content of the Globe is equal to $\frac{2}{3}$ of its circumſcribing Cylinder, ſo *the Superficies of a Globe is $\frac{2}{3}$ of the Superficies of its Cylinder*; for the Diameter at the Baſe being 2, we have $3.1416\times 2=P$; and $P\times 2=12.5664$; and $2\times 2\times .7854=3.1416$, and twice this for both Ends is 6.2832; and $12.5664+6.2832=18.8496$ the Superficies of ſuch a Cylinder; and $\frac{2}{3}$ of this is 12.5664, agreeing with the above. And here the Youth is deſired to let the Diameter be 3, 4, 5, 6, and to work the Queſtion by all the Rules, that he may get by Heart that which he likes beſt.

The Superficies of a Cone.] *Half the Product of the Periphery of its Baſe into the Length of its Side, with the Area of the Baſe, gives the Superficies of the Cone.* Thus, ſuppoſe the Girt of the Baſe is 60 Inches, and the Length of the Side, from the Top of the Cone to the Periphery of the Baſe, is 40 Inches, then $\frac{40\times 60}{2}=1200$ Inches, the Superficies of the Sides of the Cone; to which muſt be added the Area of the Baſe to find the whole Superficies.

The ſolid Content of a Cone.] As a Globe is $\frac{2}{3}$ of its Cylinder, and as a Cone is $\frac{1}{3}$ of the ſame Cylinder, ſo a Cone of ſuch a Cylinder is $\frac{1}{2}$ the Solidity of that inſcribed Globe. Thus, as before, ſuppoſe the ſolid Content of the Globe is 4.1888; then the Content of its Cone will be 2.0944: Or, more generally, *every Cone is $\frac{1}{3}$ of its Cylinder.* Before the Cylinder was 6.2832; and $\frac{1}{3}$ of this is 2.0944, as in the other particular Rule: Or ſhorter thus, the Area of the Baſe $\times\frac{1}{3}$ of the Height of the Cylinder.

FIG. 19. *The ſolid Content of the Fruſtum of a Cone.*} Let the Fruſtum be the Figure *anomsibw*; $ab=D$; $nm=d$; $ru=h$; then we have this Rule, $DD+dd+Dd\times\frac{h}{3}:\times .7854=$the ſoild Content required in Inches, Feet, Yards, or Miles, *&c.* according as the Queſtion is propoſed. Thus, let $D=9$, $d=3$, $h=21$, then $9\times 9+3\times 3+9\times 3=\frac{21}{3}:\times .7854=$the Anſwer in Inches, Feet, Yards, Miles, *&c.* If you are to gauge ſuch a Veſſel, you know you are to find the Content in Inches,

and divide by 282, 231, 268.8 to find the Gallons of Ale, Wine, or Corn. A practical Meaſurer, by knowing how to find the Height of the whole Cone *aqbw*, when he has the Fruſtum before him, may, if he pleaſes, find its ſolid Content, and doing ſo by the leſſer Cone, *aqms*, he may take one from the other, and the Remainder will be the ſolid Content of the given Fruſtum.

FIG. 19. *The Superficies of the Fruſtum of a Cone.* } Suppoſe the Periphery of the greater End to be 22, and that of the leſſer End 7, and the Height $ru=21$, then Half the Difference between the two Peripheries, multiplied into the Height, and taken from the Superficies of the Cylinder is the Rule, to which add the Area of the Ends: Thus, $22\times21=462$ the Cylinder; and $462-\frac{15}{2}\times21=304.5$. By the common Rule we have $\frac{22+7}{2}\times21=304.5$ alſo. In both Caſes we are to add the Ends, which I ſhall leave you to compute, as alſo to find the Reaſon of the firſt Rule, which is only propoſed to try your Ingenuity. As to the Reaſon of the ſecond Rule, you ſee there is a mean Periphery found, which is multiplied by the Height; that is, by this you have reduced the Fruſtum to a Cylinder, the Periphery of whoſe Baſe is 14.5, and Axis or Height is 21, as before.

To gauge Globes.] Let the Diameters be 16, 17, 18, 19, 20.5, 24.6, 28.7 Inches, then finding the ſolid Inches by any of the Rules taught you for this Purpoſe, divide by the proper Diviſors 282, 231, 268.8 to find the Contents in Gallons of Ale, Wine, or Corn Meaſure, ſometimes of one Sort, and ſometimes uſing the different Rules before delivered.

To gauge Cylinders.] Let the Diameters of the Baſes be 12, 14, 16, 17, 18, 19, 20 Inches, and their Heights 24, 28, 32, 34, 36, 38, and 40 Inches. Here find the ſolid Content by the Rules given before, and then divide by 282, 231, 268.8, for the reſpective Sorts of either Wine, Ale, or Corn Meaſure, bringing all the Rules of meaſuring the Cylinder into Uſe.

To gauge Cones.] Let their Baſes be 13, 15, 17, 21, 23, 24.5 Inches, and their Heights 26, 30, 34, 42, 46, 49 Inches; then bring all the different Rules for meaſuring a Cone into Uſe, ſometimes finding the Contents in Ale, ſometimes in Wine, and now and then in Corn Meaſure. There is no

Doubt of your being extremely ready in applying the geometrical Rules concerned in gauging Globes, Cylinders, and Cones, if you can but prevail on yourſelf to do what is ſet you in theſe three laſt Articles.

The Meaſures of the Globe of the Earth.

We are now come to the greater Concerns of Nature, wherein the Application of Geometry is very ſtriking. If a Spectator was to take the Altitude of the Pole, near a fine Star, ſuppoſe 50 Degrees high, and then proceed directly North or South, meaſuring his Diſtance all the Way; and when he has travelled 69½ meaſured Miles nearly, and then again takes the Altitude of the Pole, it ought to be ſtill 50 Degrees high, if the Earth was only a flat, extended, vaſt Plane. But though, at his ſetting out, he finds the Altitude 50 *Deg.* yet if he travels North 69½ Miles, and finds the Altitude of the Pole to be 51 *Deg.* or if he goes South 49 *Deg.* he is convinced the Earth is not an extended Plane; but that he has travelled on the Periphery of a Circle, which has cauſed this ſtrange Alteration in the Height of the Pole. But if he ſhould keep travelling on to the North, till he ſees that very Pole directly over his Head, or in coming back again, and going ſtill more and more to the South, till he ſees the Pole in the North Point of his Horizon, what could he otherwiſe conclude, but that he has travelled a fourth Part of a Circle, or 90 Degrees at 69½ Miles to a Degree, making 6255 Miles? You may pretty well gueſs whereabouts the *North Pole* is, for within 2¼ Degrees of it there is a fine Star, which alters its Place leſs than any in that Neighbourhood of the Heavens. Now then you are prepared to find all the Meaſures of the Globe we live upon; but as vaſt a Globe as you will find it, there are others much ſuperior to it; neither need you wonder at it, as the glorious Maker can do more than we can conceive, nay every Thing, becauſe he is Almighty.

Here then we have 25020 Miles $= P$ to find the reſt. Therefore $\frac{25020}{3.1416} = D$ the Earth's Diameter; and $DD \times .7854 \times 4 =$ the Superficies or ſquare Miles upon the Face of the whole Earth; and $DD \times .7854 \times D =$ the ſolid Miles of the Earth's correſponding Cylinder, and ⅔ of this Cylinder is the Number of ſolid Miles the Earth contains; and the ½ of this vaſt Meaſure is the ſolid Content of the great Cone correſponding to this Globe and its Cylinder; or by the other

Rule, $\frac{DD\times .7854\times D}{3}$, is the Solidity alſo of this Cone; and the Superficies of this Cone you muſt find thus: The ſquare Root of $DD+\frac{D}{2}\times\frac{D}{2}=$the Side s, of the Cone; then $\frac{s\times P}{2}$ with the Area of the Baſe$=$the Superficies required.

The Meaſures of the Sun.] But how will you be ſurprized in eſtimating the Meaſures of the Sun, notwithſtanding his Appearance reſembles a Circle of ſo ſmall a Size; but you muſt remember he is at leaſt eighty Millions of Miles from us. You are to multiply the Diameter of the Earth by 83; that is, the Diameter of the Sun is 83 Times greater than the Diameter of the Earth; hence you have $D\times 83\times 3.1416$ to find P the Periphery of the Sun's Diameter, and all the other Meaſures mentioned above. When you conſider the Dignity of the Sun's Office, you will not be ſurprized that he ſo much ſurpaſſes all the Globes he has the Command of, in Size, as without this all would run into Confuſion and Deſtruction.

The Meaſures of the Planets.] Beſides the Earth and its Attendant the Moon, there are five other Globes, or *Planets*, *Saturn* at the Diſtance of $\frac{95}{10}$ the Diſtance of the Earth from the Sun, *Jupiter* of $\frac{52}{10}$, *Mars* of $\frac{15}{10}$, *Venus* of $\frac{7}{10}$, *Mercury* of $\frac{4}{10}$, all moving round that Globe the Sun, which, by Divine Appointment, keeps them all in their proper Orbits. Now, though I chiefly deſign you to meaſure the Planets themſelves, yet I could wiſh you would at leaſt find the Peripheries of their Orbits, as you did that of the Earth formerly. In order to this, I ſhall ſhew you how to find the Radii; thus, as 10 *pts.* : 80000000 the Earth's Diſtance : : 95 : the Radius of *Saturn*'s Orbit; and ſo on for all the reſt of the Planets Orbits. *Mercury*'s Diameter is $\frac{4}{1000}$ of the Sun's Diameter, *Venus* $\frac{12}{1000}$, *Mars* $\frac{6}{1000}$, *Jupiter* $\frac{181}{1000}$, *Saturn* $\frac{137}{1000}$; that is, the Sun's Diameter multiplied by 4, and divided by 1000, gives you *Mercury*'s Diameter; and ſo on for the other Planets. And I hope you will be rewarded for all your Pains in finding, as you have all their Diameters, the ſolid Contents, at leaſt, of all theſe Globes, and comparing their Sizes together, and thus viewing the aſtoniſhing Works of their great Creator. But beſides theſe few Globes moving in this Neighbourhood of the Heavens, there are other Globes called Comets, which can only be ſeen by us when they are at the latter End of their long Journey, and then return from whence they came about their deſtined Buſineſs: The reſt of the immenſe Regions of the Heavens contain thoſe amazingly remote

Globes, called Fixed Stars, becauſe they do not move, they only ſeem to riſe and ſet; but this is occaſioned by our Earth rolling quite round with you every 24 Hours. But I ſhall trouble you with no more Calculations concerning the Size of the Heavenly Globes.

The Properties of a Cube, Pyramid, and their Parts.

FIG. 20. *Cube, how generated.* } As before you imagined the Crown Piece to move up to the Height of its own *Diameter* to form or generate a *Cylinder*, ſo here conceive the *Square EGHF* to riſe up to the *Height* equal to one of its Sides *EA*, and while it is thus moving to that Height, you muſt conceive that it has formed or generated a ſold Body, called a *Cube*, (or *Die*) whoſe Height, Length, and Breadth are all equal; for this *Cube* is nothing more than an infinite Number of Squares, laid one upon another, and cemented together to the Height of a Side of the Square: In like Manner, by the Square riſing up to a given Height, beyond the Length of *EA*, *Fig.* 20. will be formed the ſolid *Parallelogram*, or *Parallelopipedon*, *Fig.* 21. whoſe Baſe may be that of a Square or *Parallelogram.*

FIG. 22. *Pyramid, how generated.*] So while the Square is thus riſing up to generate its *Cube*, if you conceive the Square gradually to decreaſe, till it is the Height of the Side of the Square, and ſhrunk up into a Point directly over the Center of the Square, you have a *Pyramid* whoſe Baſe is a Square, and Height equal to that of the *Cube*: For this Pyramid is alſo generated of an infinite Number of Squares decreaſing in a regular Progreſſion upwards to a Point, but increaſing in a regular Progreſſion downwards till it ends in a Square, whoſe Side is equal to the Height of this Pyramid. And here again, you may expect there is a Proportion between theſe geometrical Bodies, the *Cube* and its correſponding *Pyramid*, and to know the Property of one is to diſcover, from ſome Proportion, a Rule to meaſure the other alſo: So in like Manner, if you conceive the Parallelogram *IMLK*, *Fig.* 21. or *RTVS*, *Fig.* 22. to riſe up to a given Height *W*, you will have another *Pyramid RTVSW*.

FIG. 23. *Priſm, how generated.*] In like Manner, imagine the Triangle *YOZ* to riſe up to a given Height *x*, and there will be formed a Figure called a *Priſm*, compoſed of an infinite Number of ſimilar Triangles: So when the Triangle

YOZ begins to move, if you conceive it gradually to decreaſe, and at the laſt be ſhrunk up into the Point *x*, there will be generated the *Pyramid YxZO*.

Fig. 20. *To deſcribe a Cube.*] Draw *EF*, ſuppoſe 8 Inches, but contracted into one Inch; that is, take one Inch from your Scale, and ſet it off on the Line *EF* drawn at Pleaſure, and remember in what Proportion you have contracted your given Meaſure. Raiſe *FD*, at Pleaſure, as your Perpendicular on the Point *F*, and ſet off *EF* on it, then form the Square *EADF*. Now draw *EG* of a convenient Length to pleaſe your Eye, and the Angle *FEG* may be about Half a Right-angle to look the better on Paper; produce *EF* with a blank Line to *x*, and with the Diſtance *EG*, deſcribe the Arch *Hr*; then, where it interſects *Fx*, ſet off *Gi*=*Hr*; through *H*, draw *FH*=*EG*, and join *GH*, and you have formed *EGHF* *Rhombus.*] the Baſe, which, from its Method of Conſtruction, is called a *Rhombus*, as the Sides *EG*, *FH*, of the Square are not at Right-angles. Set off the Angles *DAB*, *zDC*, in the ſame Manner; that is, from *A* deſcribe the Arch *Gi*, and where it interſects the Line *AD*, from thence ſet off *Gi*, and through the Point, where the Arch ends, draw *AB*=*EG*; and doing the ſame on the Point *D*, of the Line *Dz*, draw *DC* (=*AB*), *BC*, *BG*, *CH*, and you will have compleated the Cube.

Fig. 21. *To deſcribe a Parallelopipedon.*} This is done in the ſame Manner as in deſcribing the Cube, only the Baſe *IMLK* is to be ſuppoſed either a Square or Parallelogram; if it is a Parallelogram, and as the Sides, *IM*, *KL*, do not ſtand at Right-angles, it *Rhomboides*] aſſumes the Name of a *Rhomboides*. Let the Parallelopipedon, to be deſcribed, be *IK*=nine-eights of an Inch, or 9 Inches contracted into $\frac{1}{8}$ of its Meaſure, and let *IN*=$1\frac{1}{2}$ Inch, or 12 Inches contracted into $\frac{1}{8}$ of the Height. You know the Dimenſions of a Parallelopipedon may be very various, as it may ever depart from the Cube in all its Meaſures.

Fig. 22. *To deſcribe the Pyramid of a Cube or Parallelopipedon.*} The Baſes are drawn in the ſame Way as before directed, only, to ſhew the Pyramid to more Advantage, you may enlarge *EG*, *Fig.* 20. or *IM*, *Fig.* 21. into *RT*, *Fig.* 22. You are then to draw *RV*, *TS*, and at the Point *X*, where the Lines croſs each other, raiſe the Perpendicular *WX*=*RS*, if it is to be the Pyramid of a Cube; but if it is to be the Pyramid from the

Parallelopipedon, *WX* may be of any given Height, ſuppoſe one and a Half of *RS*; draw *RW*, *TW*, *VW*, and *SW*, and you have the Pyramid *RTWVS*.

FIG. 23. *To deſcribe a Priſm and its Pyramid* } As the Baſe muſt be *triangular*, it may aſſume any Form, according to the Data, which may require either a Right-angle Triangle, Equilateral, Iſoſceles, or Scalenous; but we ſhall here ſuppoſe it Iſoſceles; that is, $YO=OZ$ $\frac{1}{2}$ an Inch and ſomething better. Let $YZ=1$ Inch, and then form *YZO* according to the Directions formerly given you in laying down Triangles. Then raiſe $ZF=1\frac{1}{2}$ Inch a Perpendicular on the Point *Z*, and complete the Parallelogram *YETZ*, and biſecting *YZ*, draw $OL=ZF$ thro' the Points *O*, *r*, and then from the Point *L*, draw the Sides *LE*, *LF*, which will complete the Priſm *YELFZO*. Draw *Yx*, *Zx*, and you have the *Pyramid* YxZOr *of the Priſm*.

FIG. 20. *The Menſuration of the Cube.* } You may remember the Way to meaſure a Square is to multiply one Side *FH* by the other *EF*; and you now perceive the Cube is nothing more than many Squares cemented together. Suppoſe *EA* is an Inch, divided into a Million of Parts; then as the Square *EG*, *HF*. contains a Million × by a Million $=a$, $a\times EA=$ the *ſoldid Content*, each infinitely ſmall Cube being the millionth Part of an Inch. In like Manner, if you conceive $EF=8$ Inches, then $EF\times EG\times EA=512$ ſolid Inches; that is, 512 little Cubes, each 1 Inch in its Dimenſions; ſo if $EF=8$ Yards, then there will be 512 Cubes, each one Yard in its Meaſures. Now view the Cube, and you will find there are ſix Squares on its Surface; and therefore to find the Meaſures of the Superficies, you muſt multiply one Square by 6; that is, ſuppoſe in Inches $8\times 8\times 6=$ the *Superficies of the Cube* in ſquare Inches, or ſquare Feet, Yards, Miles, &c. according to the Meaſure required.

FIG. 21. *The Menſuration of a Parallelopipedon.* } All that has been ſaid of the Cube, may be equally underſtood of the Meaſures of the Parallelopipedon; that is, to multiply the ſquare Inches, Feet, Yards, &c. of the Baſe by the Height for its *ſolid Content*: But to find its *ſuperficial Content*, we muſt conſider that the oppoſite Sides only are equal to one another; thus, $INQM=KOPL$; $NOKI=MQPL$; and $IMLK=NQPO$: And therefore finding the Meaſure of one Parallelogram gives you the oppoſite Side, and when all theſe Meaſures are

added together, you have the *Superficies of the Parallelopipedon*; or multiplying the Sum of the Sides of the Baſe into the Height, and adding the Area of each End, gives the ſame.

Fig. 23. *The Menſuration of a Priſm.* The Baſe of a Priſm is to be conſidered as Half a Square, and as ſuch the Content of the Baſe of the Priſm, multiplied into its Height, will give the *ſolid Meaſure* in Inches, Feet, Yards, *&c.* as the Queſtion requires. As to the Method of finding the *Superficies of the Priſm*, you cannot be at a Loſs, as you ſee the Back is a Parallelogram, and that the Sides muſt be meaſured as Parallelograms: The two oppoſite Sides will be always equal to one another, when the Baſe is either an Equilateral or Iſoſceles Triangle, and the Ends of the Top and Bottom will always be equal to one another; ſo that, in the preſent Caſe, the ſuperficial Meaſure will conſiſt of the Sum of the Back, two equal Sides, and two equal Ends, all added together.

Fig. 22. *The Menſuration of Pyramids.* Here we are to conſider two equal oppoſite Sides $WVS = WTR$; and $WSR = WTV$; all which added to the Baſe $RTVS =$ the *Superficies of the Pyramid*: Therefore, with a Chalk-Line, the Perpendiculars of theſe ſuperficial Triangles may be found, and the ſquare Inches may be added to the Content of the Baſe. But if we conceive the Sides of the Baſe to be continued out into one Right Line, and the Triangles of the Superficies placed on this Line, we ſhall find that they will make a Parallelogram whoſe Bottom is equal to Half the Sum of the Sides of the Baſe, and whoſe Top is the ſame, and whoſe Height is the ſlant Side *WV*; therefore the Rule is, *Half the Sum of the Sides multiplied into the Slant of the Pyramid will give the Content of the Superficies*, to which muſt be added that of the Baſe. But for the *ſolid Content* of the Pyramid, the Rule is, *multiply the Area of the Baſe by* $\frac{1}{3}$ *of the Height WX*, which is alſo the Height of its circumſcribing Cube or Parallelopipedon.

Fig. 23. *The Menſuration of the Pyramid of a Priſm.* We ſhall firſt ſhew how to eſtimate its *ſolid Content*. As a Priſm is to be conſidered as only Half a Parallelopipedon whoſe Ends are equal Squares, and as Pyramids are $\frac{1}{3}$ of their circumſcribing Cubes or Parallelopipedons, ſo the Pyramid of a Priſm is $\frac{1}{3}$ of its ſolid Content; that is, the Area of the Baſe *YOZ* muſt be multiplied into $\frac{1}{3}$ of its Height *ZF*. The Way of finding the *ſuperficial*

Content is this: You ſaw before that, in the Caſe of the Pyramid of a Parallelopipedon, the Superficies was found by multiplying Half the Sum of the Sides of the Baſe by the ſlant Side; ſo here, as theſe Triangles are Half their circumſcribing Squares, the Superficies of the *Pyramid of a Priſm* muſt needs be under the ſame Rule; that is, *to multiply Half the Sum of the Sides*, YO + OZ, *into the ſlant Side xZ*, for the ſuperficial Content, to which we muſt add the Area of the Baſe *YZO*.

FIG. 22. *The Fruſtums of Pyramids.* You ſaw before that the Rule for finding the ſolid Content of the Fruſtum of a Cone was $DD+dd+Dd\times\frac{h}{3}:\times .7854$; but this multiplying by .7854 was to reduce the Area of the Square to the Area of its inſcribed Circle; therefore in the Rule we muſt now omit .7854, and we have $DD+dd+Dd\times\frac{h}{3}=RTVS\ pqrs$, the ſolid Content of the *Fruſtum of a ſquare Pyramid*. And in order to find the *ſuperficial Content*, the Rule is the ſame as in the Fruſtum of a Cone; that is, Half the Sum of the Sides of the greater and leſſer End, multiplied into the Height, gives the *Superficies* of the Fruſtum of a ſquare Pyramid, to which add the Areas of the Ends.

FIG. 23. *Fruſtums of the Pyramids of Priſms.* And as the Fruſtum of a Priſm is Half the Fruſtum of a Parallelopipedon, therefore it muſt needs be ſubject to the ſame Rule; that is, *the Half of* $DD+dd+Dd\times\frac{h}{3}=$ the *ſolid Content* of the Fruſtum *YOZ lmn* of the Priſm: And for the *Superficies* of the Fruſtum of a Priſm, we have *Half* the *ſuperficial Content* of the Fruſtum of a ſquare Pyramid for a Rule, to which add the Back of the Fruſtum of the Priſm.

Fruſtums of Pyramids whoſe Ends are Parallelograms. Hitherto we have conſider'd theſe Figures as having the Ends either ſquare, or in the Caſe of the Fruſtum of the Priſm, as Half that Square; but when the Ends are *rectangled Parallelograms*, let the greater Length be *D*, and its Breadth *x*; the leſſer Length *d*, and its Breadth *z*; and the Height *h*: Then we have this Rule, $\frac{2Dx+dx}{2}+\frac{2dz+Dz}{2}:\times\frac{h}{3}=$ the ſolid Content, which in Words will run thus; the Half Sum of twice the greater

Length into the greater Breadth, added to the leſſer Length into the greater Breadth, muſt be added to Half the Sum of twice the leſſer Length into the leſſer Breadth, added to the greater Length into the leſſer Breadth; and this Total, being multiplied by a third Part of the Height, gives the ſolid Content of ſuch rectangular Fruſtums of Pyramids. And, from what has hitherto been ſaid, you may deduce Rules for finding the Meaſures of the Figures correſponding to the above Fruſtum, ſuch as the *Priſmoid*, or Parallelopipedon, whoſe Ends are equal Parallelograms, *Pyramid* of a *Priſmoid*, whoſe Baſe is a Parallelogram, the *Priſm* of a *Priſmoid*, whoſe Ends are equal ſcalenous Triangles, the Pyramid of this laſt Figure or its Fruſtum. You will do yourſelf no ſmall Service if you will conſtruct all theſe in your Book, and form your Rules from what has been ſaid in Geometry.

To gauge a Cube, &c.] Now as you are enabled to find the ſolid Content in Inches of the Cube, Parallelopipedon, Fruſtum of a Pyramid, and the Fruſtum of the Pyramid of a Priſm, *&c.* you have nothing more to do, when you find a Veſſel in any of theſe Forms, than to divide by either 282, 231, 268.8, to find the Number of Gallons of Ale, Wine, or Corn, ſuch Veſſels will hold.

I ſhall now go on to ſhew you how ſome of your Geometry may be applied to introduce you to ſome Acquaintance with that uſeful Part of Education, *Geography*, without which the Youth deſigned for a Merchant, or the Sphere of a Gentleman, will make but a poor Figure among Men of Knowledge.

The PRINCIPLES *of* GEOGRAPHY.

Of the Circles of the Earth, expreſſing Latitude, Longitude, &c.

Fig. 24.] On the Terreſtrial Globe the ſeveral Parts of the Earth are exhibited in their true Situations, Figures, and Proportions, which gives us an eaſy and ſpeedy Acquaintance with the Earth, becauſe hereby we have the true Repreſentation thereof ſet before our Eyes; ſo that this Globe may be ſtyled the Earth in Miniature, differing from it only in Size. That Circle on the Globe, which lies exactly between the North and South Poles *N* and *S*, is called the *Equator*, or Equinoctial Line; and if this Circle is divided both Ways, Eaſtward and Weſtward, from that Circle which is drawn thro' *London*, (ſtanding now under the Word *Meridian)* call'd the *Firſt Meridian*, into 180 Degrees or Parts, each 60 Miles, theſe are called the Degrees of *Longitude*; and if the firſt Meridian is divided both Ways, running to the Poles, into 90 Degrees, or Parts, each 60 Miles, theſe are called Degrees of *Latitude*.

If we reckon upon this firſt Meridian, we ſhall find *London* is about 51½ Degrees from the Equator, or 3090 geographical Miles, anſwering in general to computed, not meaſur'd Miles. If a Spectator faces the *Sun* at Noon-Day, the *South* is then before him, the *North* at his Back, the *Eaſt* on his Left, and the *Weſt* on his Right Hand. In the Morning the Sun will be in the eaſtern Parts of the Heavens, at Noon it will be in the South, exactly over the Meridian of *London*; and when it ſets it will appear in the weſtern Parts of the Heavens. All Places that are to the Eaſt of *London* will have Noon-Day before us, in Proportion as they are farther from, or nearer to, us; and thoſe lying due South, will have Noon-Day at the ſame Time; but all thoſe Places that are to the Weſt of *London*, will have their Noon later than us, juſt as they are nearer, or farther removed from us. Now, from hence, the young Reader will perceive, that before it is light with us in the Morning, ſome Places to the Eaſt will have the Sun above them, or riſen; ſome have got their Noon over, and to others he is ſetting, when riſing to us; and when it is Noon-Day with us the Sun is only riſing to ſome Places in the Weſt, and ſome have quite Night; and with ſome it is later, and with others it is earlier, in Proportion as they lie nearer or farther from us. And from this the young Reader will perceive, that when it is Twelve o'Clock at Noon with us, it is not ſo with any other Country, as is commonly imagin'd, unleſs it lies due South or North from us; for then the Sun ſtands over the Meridian of *London*, and over thoſe Places alſo at the ſame Time.

The Sky exactly over the Spectator's Head at *London*, is called the *Zenith*, and the oppoſite Place through the Globe, and extended to the Heavens in thoſe Regions, is called the *Nadir*; both which are expreſſed in the Plate, and appears extending to the Stars, or to what Height the Reader pleaſes. It may ſeem very odd to a Youth, unacquainted with Geography, that there ſhould be a Place *(the Nadir)* on the Earth where the Inhabitants appear with their Heads downwards; but he muſt remember the Earth is not endleſly broad, or deep without a Bottom, or reſts upon any Thing, and that Thing upon another, and ſo on; were it ſo the Earth could not be ſailed round, or a Ship return from the Left Hand, when ſhe had ſet off and gone towards the Right. Navigators know, by undoubted Experience, that if there was no Land to hinder the Ship's Track, and they were to ſail with a fair Wind continually to the Weſt, they would, without ever turning the Ship about, find themſelves at home again. It would be the ſame were they to ſail to the South, Eaſt, or North, or any of the intermediate Points between them, ſo there were no Land to hinder the Ship, and the Wind continued always fair, for they would in every Voyage be brought home again without turning the Ship about.

Theſe Navigators, while they were in the oppoſite Part of the Globe, called our *Antipodes*, or thoſe who ſeem to walk with their Heads downwards, beheld the Heavens over their Head, and ſaw the Sun and Stars perform their Courſes, and that Day and Night ſucceeded each other there, as well as here. The Sun then muſt move round the Earth every Day, and give Light to that Part of the World oppoſite to us; and thoſe Inhabitants have the very ſame Reaſon to ſay that our Heads are downwards, and to be afraid of us leſt we ſhould fall off from the Earth among the Clouds. Navigators there, as well as here, are in no Danger of falling upwards, as the Heavens were over their Heads; they knew the Weight of the Ship, when they were there, hindered it from tumbling upwards. If we ſuppoſe a Paſſage through the Earth from the oppoſite Part to us, and a Cannon-Ball ſuſpended at an equal Diſtance betwixt them and us, you may imagine it cannot come to us, that would be to fall upwards; and thoſe People, the Antipodes, will ſay it cannot come to them, as that would be alſo to fall upwards.

An Obſerver placed upon the Earth, (ſuppoſe at *London*) by extending his Sight every Way, thinks the World is bounded, and judges all remote Objects, (meaning the Stars) whoſe Diſtances are unknown, though really very unequally removed from one another, to be placed as it were in the ſame concave ſpherical Superficies: This Circle *vs*, which he views ſurrounding him upon the Earth, is called the *viſible ſenſible Horizon*, or Boundary of his Sight; and from this Situation he ſees in a clear Night all the Stars around him, ſome riſing in the Eaſt, others over his Head, and ſome ſetting in the Weſt; and that Circle which is even with it, but as low down as the Center of the Earth beneath his Feet, and extended out every Way to the Heavens about him, is called the *rational Horizon:* This Circle, like the Equator, or Equinoctial Line, divides the Earth and the Heavens into two equal Parts, one above, and the other below the Spectator's rational Horizon; the Stars above it at that Inſtant are ſeen, and thoſe below it do not appear to his Sight, till by their Motion round the Earth ſome diſappear, and others come into View

When the Spectator is extending his Sight every Way, from an Eminence near the Sea, and diſcovers Ships ſailing, theſe appear in his ſenſible Horizon; but theſe Objects cannot be ſeen when they are at a great Diſtance, becauſe of the globular Figure of the Earth: At the firſt Sight the Tops of the Maſts only appear, but as the Ships come nearer, then their Hulls are plainly diſcovered. So when a Ship is coming from the Sea, the People on board only ſee the Tops of the Mountains at a great Diſtance; then as they approach nearer to the Land, they plainly ſee the Shape and Form of nearly the whole; then the Tops of Steeples or Towers near the Coaſts, and afterwards the Churches appear in full View.

The young Reader may eaſily conceive that, whatever Way he travels to, he alters his Horizon, as there is, ſtrictly ſpeaking, a different ſenſible Horizon for every Yard of Sea or Ground he paſſes over; as for Inſtance, a Tower which he diſcovers from Sea will not be ſeen by him when he has ſailed many Miles from it; and the only Reaſon is, that he has had many new Horizons, and the Edge of the laſt is above the Tower; that is, he cannot ſee the Top of it, as it ſeems to have ſunk beneath that Part of his preſent Horizon; and new Objects before him, which were not to be ſeen by Reaſon of their Diſtance, and ſunk down as it were, are now on the Edge of this Horizon, becauſe he has approached within Sight of them. And in like Manner, when he is at the Antipodes, he will have an Horizon the fartherſt removed from the Place he ſailed from, and new Objects, the Sight of which he enjoys, after changing a vaſt Number of ſenſible Horizons.

So likewiſe he cannot travel any where, but he makes an Alteration in his Latitude, unleſs he goes directly Eaſt or Weſt, repreſented on the Globe by the Parallels or Circles, drawn even with the Equator both Ways, till he comes to the two Poles; becauſe one of theſe Eaſt and Weſt Circles, or Parallels of Latitude, is the ſame Diſtance from the Equator as the Place you travelled from: As for Inſtance; a thouſand

Miles Eaſt from *London* is the ſame Diſtance from the Equator as the Latitude is reckoned from thence to either Pole: But if we travel due North or South; for Inſtance, the Latitude is certainly altered, becauſe we approach nearer, or remove further from the Equator.

Neither can we travel any where without altering our Longitude, which is reckoned from that Place where the Meridian of *London* croſſes the Equator, unleſs we travel due North or South; becauſe all ſuch Places paſſed over have the ſame Meridian as the Place we left has, and therefore the ſame Longitude. As thus, if you bring *London* to the Braſs Circle, called the *Brazen Meridian*, all thoſe Places lying under it, or, which is the ſame Thing, all thoſe Places upon the Meridian of *London*, have the ſame Longitude, as we reckon Eaſt and Weſt on the Equator where theſe Circles meet: But if we travel North-Weſt, South-Weſt, South-Eaſt, or North-Eaſt, the Meridian of the Place arrived at, croſſing the Equator, as the Meridian of every Place does, the Longitude of that Place will be either to the Weſt or Eaſt of the Longitude of the Place departed from, and conſequently cannot be the ſame.

Geographers divide the Earth into Zones, two Frigid within 23½ Degrees of each Pole, bounded by a Circle at that Diſtance from each, called the Polar Circles; that in the North is called the *Arctic*, and that in the South the *Antarctic Circle*. It is commonly thought the Air under the North Pole is colder than at the Arctic Circle, as this is 23½ Degrees nearer the Equator, where we ſay the Air is hotteſt; but under the North Pole the Air is in all Probability warmer in Summer than about the Arctic Circle, becauſe the Sun continues longer above the Horizon at the Pole. For in our *March* the Sun juſt appears on the Edge of their Horizon, and moves all round them without ſetting, and continues to do ſo till *June*, when the Sun moves ſtill round them, and is as high as it is with us at Noon in *February*. It ſhould ſeem, therefore, that this continued Heat, as the Sun is as high at Midnight, as we ſhould call it, as it is at Noon-Day, muſt be very conſiderable. In our *September* the Sun begins to ſink down in the ſame Manner below the Horizon at the Pole, and they ſee no more of it till *March* returns; and this long Winter, or Abſence of the Sun for ſix Months, muſt occaſion there an intolerable cold Air, the Extreme to the Heat of their Summer. And in like Manner we may conceive the Heat and Cold, Winter and Summer, at the South Pole.

There are alſo two Zones called *Torrid*, on Account of the Heats: Theſe lie 23½ Degrees each Way from the Equator. It muſt be obſerved that, in the Middle of *March* and *September*, the Sun is at Noon-Day exactly over the Heads of the People living on the Equator; and the more upright the Rays of the Sun are, the more intenſe is the Heat, and therefore it ſhould be hotteſt there; and in Proportion as we travel Northward or Southward from the Equator, the Heats ſhould grow more moderate, unleſs ſandy Soils, or the like Cauſes, ſhould interfere. But though this is the Caſe in general, yet it muſt be hotteſt ſomewhere about 23½ Degrees on each Side of the Equator; becauſe as the Sun is twice, in his annual Courſe, over their Heads, and as he never is ſo to any Country but in theſe Torrid Zones, therefore the ſooner he returns to a Place where he was vertical, or over their Heads, there has been the leſs Time for the Heat to go off, and conſequently the Heat will be there the moſt intenſe. And yet Providence has ſo ordered Things, that, in theſe very Countries, the Weather in thoſe Seaſons is generally ſo cloudy as to ſcreen the Natives and Soils from the ſcorching Heats.

There are alſo two Temperate Zones, lying in a Space containing 43 Degrees, betwixt the Torrid Zones and the Polar Circles, on each Side of the Equator. Their Temperature ariſes from the Obliquity or Sloping of the Sun's Rays at Noon-Day; and, in general, the greater the Latitude, the longer is the Noon-Day Shadow caſt upon the Ground. Thus, under the Torrid Zones there is no Shadow when the Sun is over their Heads; but juſt beyond theſe Zones, in Latude 25 Degrees, there is a ſhort Shadow, and there the Heat is much greater than, for Inſtance, in Latitude 60 Degrees, where the Shadow is much longer. Under the Poles the Shadow is caſt every Way, becauſe the Sun moves quite round them; at the Equator their Shadow at Noon-Day is caſt to the South, from *March* to *September*, and from *September* to *March* the Shadow is caſt towards the North; beyond the Torrid Zone, on the South-Side of the Equator, the Shadow is always caſt towards the South; but on this Side of the Torrid Zone, on the North-Side of the Equator, our Shadow is always caſt Northward.

At the Equator the Days and Nights are always equal; under the Poles they have but one Day, as we may call it, of ſix Months long, and but one Night of the ſame Duration; that is, they have the Sun above their Horizon for ſix Months, and for ſix Months they are deprived of the Sight of him, as he is ſo long below their Horizon. In all other Places, in general, the Day in the Summer Time is longeſt the farther removed from the Equator, and the Nights ſhorteſt: But, at the Polar Circles, in their longeſt Day they have the Sun ſhining for 24 Hours above them, and at Midnight, inſtead of ſetting he only grazes on the North Point of their Horizon, but does not ſet; and in their ſhorteſt Night he only grazes on the South Point of their Horizon, but does not riſe; ſo that they have a Night of 24 Hours, though indeed it cannot be called dark, as they ſee the Rim of the Sun. And between the Polar Circles and the Poles they have continued Days, as the Sun does not leave them, and theſe Days continue in Proportion to the Diſtance from the Poles; and their Nights are, in the contrary Seaſon, in Proportion to their Days, always equal to one another, but in oppoſite Seaſons of the Year.

To draw the Circles of the Earth in their juſt Proportions to one another.

It will be a very uſeful and genteel Employment to apply the Geometry, you have ſo far acquired, to the Purpoſes of laying down the Circles by which Geographers divide the Surface of the Earth, in order to inſtruct their Pupils in this noble and entertaining Part of Knowledge, to be ignorant of which is to diſcover ſome Defect in Education.

FIG. 25. *To draw the proper Circles on a Meridian.*

On the Center *E*, with the Radius of your Line of Chords on the Scale, deſcribe the Circle *ABCD*, which is the Plane the reſt are to be drawn upon. Draw the Diameters *CA*, *BD*. The Line *CA* repreſents the *Equator*, and *BD* repreſents a *Meridian*; with the Chord of 23½ Degrees ſet off this Angle from *A* ♋, and from *C* to ♑, and draw the Diameter ♋ *E* ♑, and this Line repreſents the *Ecliptic*. But let us conſider the Uſe or Office of this Circle, called the *Ecliptic*, as no Mention has hitherto been made of it, only you ſee it is drawn upon the Globe in *Fig*. 24. You muſt know it is the Path or Periphery of the Sun's Orbit, (properly the Earth's) mentioned formerly, which you ſee is not level or parallel to the *Equator* produced out to the Sun, but croſſes it in an Angle 23½ Degrees. And in this Angle you ſee the gracious Appointment of your Heavenly Father; for were the *Path* to lie level with the *Earth's Equator* produced out, what would become of the Inhabitants in the more Northern Parts of the Earth for Want of Light and Heat; nay, what would become of us in *England*, and all Countries at the ſame Diſtance from the Equator? We ſhould want Heat to ripen our Corn and the Fruits of the Earth, as we ſhould then have no more Heat than you feel about the 20th of *March*, when the Day and Night would be perpetually equal.

To graduate the Meridian.] Take your Compaſſes, and with the Chord of 10 Degrees on your Scale of Chords, from the Point *A* ſet off the Arch *A* 10; then the Chord of 20 Degrees, and ſet off the Arch *A* 20; then take the Chord of 30 Degrees from your Scale, and ſet off *A* 30; and ſo on reſpectively till you come to *A* 80, where you muſt end, as *B* ſtands where the Chord *A* 90 ſhould ſtand, which is the whole Length of your Scale or Line of Chords. You cannot but remember how you formed your Line of Chords, and this Manner of dividing the Quadrant is only transferring the Numbers (Degrees) from the Chords back again to the Arch by which the Chords were formed. If you chuſe it, you may, when you have ſet off the Chord *A* 10, run your Compaſſes, with that Opening, on the Arch *AB*, and divide into the ſame Number of equal Parts as before; but I was willing to remind you of the Manner of ſetting off the reſpective Arches by their correſponding Chords. By the ſame Method you have divided the Arch *AB*, you may do the ſame by *CB*, *CD*, and *AD*, by which Means your two Semi-Circles *CBA*, and *CDA*, will be properly graduated.

Fig. 25. *Signs of the Ecliptic.*] If you ſuppoſe yourſelf placed in the Sun, which you muſt ſuppoſe fixed in *E*, you will take Notice that the Earth goes the ſame Track over again every Year, and you will particularly obſerve the Fixed Stars the Earth paſſes by in her Journey round her Orbit, which is the *Ecliptic* mentioned before, inclining to the Plane of the Equator in an Agle *AE* ♋ of 23½ Degrees, which you have juſt drawn. You will, whilſt you are viewing the Earth from the Sun *E*, ſuppoſed in the Center of the Orbit, imagine this Orbit (Ecliptic or Earth's Path) divided into 12 equal Parts, or Signs, each 30 Degrees, calling them by the Name of any neighbouring Conſtellation, or Figure thoſe particular Number of Fixed Stars ſeem to make. In this Caſe

FIG. 25. *To divide the Ecliptic.*

Caſe the Earth will move from ♈ (where *E* ſtands in the Plate) to ♉, and from thence to ♊, and ſo on by all the Signs, as they ſtand with their Names in the Margin. Now then divide the *Ecliptic* as follows: Take the Half Tangent of 30 *Deg.* or one Sign from you Scale, on which they are marked *ST, (Semi-Tangents)*, and ſet it both Ways from the Center *E*, and it will cut the Northern Signs, ♉ and ♍, and the Southern Signs ♓ and ♏, and the Half Tangent of 60 *Deg.* ſhews where to mark ♊ and ♌, ♒ and ♐; and thus may each Sign be divided into 30 *Deg.* if you had a large Scale for that Purpoſe. Now look on the Globe repreſented in *Fig.* 25. that I may be a little more intelligible to you concerning the Order of theſe Signs. You muſt remember, the Sun *ſeems* to move from *Aries* (the Point *E* in the Plate) about the 20th of *March*, to go to *Cancer*, in order to make our Summer; from thence he moves to *Libra*, (ſtill the Point *E*) becauſe he has done his Buſineſs in the Northern Hemiſphere, that is, on the North of the Equator, and is going to bleſs the other on the South of it with the Summer alſo; when he arrived at *Capricorn*, and we have had Winter, he returns to *Aries* again, makes the Day and Night equal, (as he did in *Libra*) goes to *Cancer* and gives us another Summer; and ſo on from Age to Age, till its glorious Author thinks proper to put a Stop to theſe wonderful Appointments. When you are told that the Sun, being in *Aries* and *Libra*, makes equal Days and Nights; the Reaſon is, he riſes exactly in the Eaſt, and ſets exactly in the Weſt, becauſe he is on thoſe Days over the Equator, and then his Rays juſt reach to both the Poles, which you know are exactly 90 *Deg.* from the Equator, and it is the Nature of a Globe that it can be only enlightened 90 *Deg.* on each Side of its Equator, or Circle that divides it into two equal Parts or Hemiſpheres.

Signs.	*Names.*
♈	*Aries.*
♉	*Taurus.*
♊	*Gemini.*
♋	*Cancer.*
♌	*Leo.*
♍	*Virgo.*
♎	*Libra.*
♏	*Scorpio.*
♐	*Sagitarius.*
♑	*Capricorn.*
♒	*Aquarius.*
♓	*Piſcis.*

FIG. 25. *To draw the Meridians.*] You are to be told, it is demonſtrated by thoſe who profeſſedly treat of Spherical Projections, that the Centers of theſe Meridians lie in the Plane of the Diameter *CA*, and are drawn by the Scale *(ST)* of Half Tangents. Take from your Scale the Half Tangent of 15 *Deg.* and ſet it off both Ways on the Equator *CA*, from the Center *E*, and draw *B* 75 *D*, and *B* 105 *D*; then take the Half Tangent of 30 *Deg.* and ſet it off both Ways from *E* on *CA*, and draw *B* 60 *D*, and *B* 120 *D*; then take the Half Tangent of 45 *Deg.* and proceed as before; and ſo on, till you have drawn *B* 15 *D*, and *B* 165 *D*, both which lie 15 *Deg.* from *C* or *A*; that is, *E* 15 is the Half Tangent of 75 *Deg.* and *E* 165 is alſo the Half Tangent of 75 *Deg.* but *EA*, or *EC* is the whole Scale; that is, the Half Tangent of 90 *Deg.* or Radius.

FIG. 25. *Hour Circles.*] Now you are to remember that theſe Spaces of 15 *Deg.* ſhew the Hours of the Day; for when the Sun, at an immenſe Diſtance, ſtands over one of theſe *Meridians*, or *Hour Circles*, it is then juſt Noon-Day with thoſe People, and when the Sun has moved 15 *Deg.* further, there is an Alteration of one Hour, and ſo on, making 24 Meridians, Hour or *Noon-Day* Circles, becauſe 15 × 24 = 360 *Deg.* quite round the Equator of the Earth, 15 *Deg.* making an Hour.

FIG. 25. *To divide the Equator.*} When you were drawing the Meridians, you were unknowingly dividing the Equator into its proper Degrees of *Longitude*; and you may now perceive that Longitude and Time are expreſſed by the Meridians; and when we ſay ſuch a City has its *Noon-Day* an Hour before *London*, we likewiſe underſtand that City lies in *Eaſt Longitude* 15 Degrees from *London*. So when we ſay *London* has its *Noon-Day* ſix Hours before ſome other Place, we likewiſe underſtand that Place lies in 90 *Deg.* *Weſt Longitude* from *London*, becauſe 6 Hours × 15 *Deg.* = 90 *Deg.* So likewiſe, if you ſuppoſe the Meridian *BCDA* to be drawn through *London* at *Noon-Day*; that is, (for Inſtance) the Sun then ſtanding on the 20th of *March* directly in a Right Line over *C*, its Rays will reach from Pole *(B)* to *(D)* making equal Day and Night: In this Situation it will be Midnight to all the Places on the Arch *BAD*, becauſe the Sun has been gone 12 Hours from their Meridian, and has paſſed over 12 Spaces, each containing 15 *Deg.* or one Hour; and all thoſe Places may be ſaid to have 180 *Deg.* Eaſt or Weſt *Longitude*, as they can have no greater either Way.

FIG. 25. *To draw the Parallels.* As every Place may be conceived to have a *Meridian* drawn through it, to determine its *Longitude*, or *Time*, when compared with the Meridian of *London*; ſo to determine how far from the Equator that Place lies, either North or South, called its Latitude, you muſt conceive a Circle, croſſing the Meridians, to be drawn through it: But actually to draw ſuch Circles, called *Parallels of Latitude*, you are to proceed thus: Take the *Half Tangent* of 10 *Deg.* and ſet it off on both Sides of the Equator from the Center *E*; then producing *BD*, as Occaſion may require, take the *Tangent Complement*; that is, what 10 *Deg.* want of 90, meaning 80 *Deg.* from the Scale of whole Tangents, and ſetting one Foot of the Compaſſes on each Point of the Half Tangents on *BD*, the other will give you, on the Line *BD* produced, the *Centers* whereby to draw the *Parallels* of 10 *Deg.* Latitude both Ways from the Equator. Then take the *Half Tangent* of 20 *Deg.* and ſet it off both Ways from *E* on *BD*, as before; and after this take the *Tangent Complement* (90—20) 70 *Deg.* and ſetting one Foot of the Compaſſes on each Point of the Half Tangents on *BD*, the other, as before, will give you both Ways on *BD* produced the Centers of thoſe two Parallels alſo. Then with the Half Tangent of 30 *Deg.* on *E*, ſet it off both Ways on *BD*, as before; and with the Tangent Complement 60 *Deg.* (90—30) on the Points of the Half Tangents on *BD*, find on *DB* the Centers, whereby to draw theſe two Parallels of Latitude: And thus proceeding on to ſet off the Half Tangents both Ways from *E* on *BD*, and finding their Centers by ſetting off on *BD*, the Tangent Complements both Ways from the Points of the Half Tangents, you will draw all the Parallels of Latitude at the Diſtance of 10 *Deg.* from each other.

FIG. 25. *To draw the Tropics and Polar Circles.* Now with the *Half Tangent* of 23½ *Deg.* and one Foot of your Compaſſes in *E*, ſet off on *BD* both Ways, the two Points, through which as above directed, by taking their *Tangent Complements* (90—23½) 66½, and ſetting off this Diſtance from each of theſe two Points of Half Tangents, draw the two dotted Parallels, repreſenting the *two Tropics*. Between theſe two Circles you may perceive the *Ecliptic* croſſing the Equator *CA*, in an Angle of 23½ *Deg.* is drawn; and therefore the Sun can never come more Northward nor go more Southward than 23½ *Deg.* on each Side of the *Equator CA*, becauſe the Sun (properly the Earth) always travels in the *Ecliptic* or Orbit. But as there muſt be Day and Night as well as Winter and Summer, either the Sun muſt mount up every 24 Hours, and travel alſo in the Periphery of another Orbit, equal to that of the *Ecliptic*; or elſe the Earth muſt have a Power to do this Buſineſs for herſelf, which ſhe can perform, by letting her go the annual Journey in the *Ecliptic*, and turn round 365 Times on her own *Axis BD* in the ſame Time, ſomething like a Bowl on a Bowling-Green, which takes a Circuit to the Jack, and turns round itſelf all the Way. With the *Half Tangent* of 66½ *Deg.* from *E*, on each Side, ſet off that Diſtance on *BD*; then with the *Tangent Complement* 23½ *Deg.* (90—66½) from theſe two Points of Half Tangents find the two *Centers*, whereby to draw the two dotted Parallels, called the *Polar Circles*, being within 23½ *Deg.* of each Pole *B, D*. You may remember when the Sun is in the Southern Tropic, his Rays fall 23½ *Deg.* ſhort of the *Pole B*, becauſe he is 23½ *Deg.* South of the Equator, and his Rays only reach 90 *Deg.* and then all that Region is in Darkneſs till the Sun moves again towards the Equator; and when he is in the Northern Tropic, then the Regions about the South *Pole D*, for the Space of 23½ *Deg.* are in Darkneſs, becauſe, as before, his Rays cannot reach farther than 90 *Deg.*

Thus much, I preſume, may be fully ſufficient to give the ingenious Youth a juſt Notion of the Projection of the Sphere on any Meridian. Places may be inſerted in ſuch a Map, by the Help of a Table of Latitudes and Longitudes; for by having the Latitude and Longitude of a City, &c. the Point of Interſection (in the Map) of the Meridian and Parallel of Latitude will be the true Situation of that City, &c. which may be inſerted accordingly. I hope the Learner will determine, by repeating the Example, to make himſelf Maſter of what has been ſaid of thus laying down a general Map, and then he cannot fail of being able to lay down a Map of a Part of the Earth.

The USE *of* MAPS.

A Map is the Reſemblance of the Earth on a Plane or Superficies, and does not fall ſhort of the Globe, becauſe it may, as we ſhall ſhew further on, exhibit the true Dimenſions of a ſpherical Body, the Figures, Diſtances, Poſitions, and Proportions of Places ſituated upon it, with the ſame Accuracy as a Globe; and therefore ſuch Maps have this

Advantage of a Globe, that we are not obliged to repreſent the whole Earth as we are on a Globe, but only as much as we think fit, from a Quarter of the World to a Kingdom, Province, or Diſtrict of the ſmalleſt Dimenſions: Then are they abundantly more particular, and give a more diſtinct View of Places; and indeed if too great a Part of the Earth be not exhibited, the Repreſentation even in common Maps will not fall very ſhort of the Globe for Exactneſs. Maps have a ſmall Compaſs uſually drawn upon them, by which the North at the Top is pointed out by the Needle; then the Eaſt Side of the Map is on the Right Hand, the Weſt on the Left, and the Bottom of the Map is the South. The Lines drawn from Eaſt to Weſt are the Parallels of Latitude, repreſenting thoſe on the Globe; the two extreme Parallels are divided into Degrees, and each Degree into Halves and Quarters, or into ſingle Minutes, according to the Largeneſs of the Map. The Lines drawn from North to South are the Meridians repreſenting thoſe on the Globe; the two extreme Meridians are divided into Degrees of Latitude, and each Degree ſubdivided as before.

PROB. 1. *To find the Latitude of a Place in a Map.*

Stretch a Thread over the Place, ſo that it may cut the ſame Degree and Minute of Latitude in the two outſide Meridians; the Degree and Minute marked by the Thread in thoſe Meridians, is the Latitude of the Place.

PROB. 2. *To find the Longitude of a Place in a Map.*

Stretch a Thread over the Place, ſo that it may cut the ſame Degree and Minute in the two extreme Parallels of Latitude; the Degree and Minute cut by the Thread is the Longitude required: But on this Occaſion we may uſe a long flat Ruler, and then the Edge thereof will be inſtead of the Thread.

PROB. 3. *To find a Place in a common Map, its Latitude and Longitude being given.*

Stretch one Thread over the Degree and Minute of Latitude, in the extreme Meridians, and another Thread over the Degree and Minute of Longitude, in the two extreme Parallels of Latitude; the Point, where theſe Threads croſs one another, will ſhew the Place required. This Method will alſo ſerve to inſert Places in a Map, their Latitudes and Longitudes being given, and will be a pretty Amuſement for a young Gentleman, if he chuſes to lay down a County, or any particular Part of a Kingdom.

What has been ſaid in theſe three Problems, is only applicable to ſuch common Maps as have the Meridians and Parallels on the Globe repreſented by ſtrait Lines, for if they are repreſented by Circles, or Parts of Circles, they require other Directions.

PROB. 4. *To find the Latitude of any Place in a Map, where the Parallels are expreſſed by Curves.*

Imagine a Parallel to be drawn through it, and the Degree which is cut in the bounding Lines of the Map, is the Latitude of the Place required.

PROB. 5. *To find the Longitude of a Place in a Map, where the Meridians are expreſs'd by Curves.*

Imagine a Meridian to be drawn thro' the given Place, and the Degree where the imaginary Meridian cuts the Equator, or one of the bounding Lines, ſhews the Longitude required.

PROB. 6. *To find the Bearing of one Place from another.*

If both Places lie on one Parallel, the Bearing is either Eaſt or Weſt; if on the ſame Meridian, they lie North and South of each other; but if in any other Direction, the Point of the Horizon is eaſily diſcovered by the Eye, and ſufficiently accurate to ſatisfy one's Curioſity.

PROB. 7. *To find the Diſtance of two Places on a Map.*

If the Maps exhibit Provinces, or Diſtricts of a large Scale, the Diſtance of Places may be obtain'd accurately enough, by extending the Compaſſes to the given Places, and applying that Diſtance to the Scale in the Map, which will ſhew the Miles *&c.* contain'd between thoſe Places.

But the true Diſtance of Places is not to be found in a *general Map*, by uſing the Compaſſes in this Manner; however, if the two Places are either both on the Equator, or have the ſame Meridian, the Number of Degrees of their Diſtance, being reduced to Miles, by allowing 60 Geographical or computed Miles to a Degree, will give the Diſtance requir'd.

PROB. 8. *Knowing the Hour of the Day at any Place, to find the Longitude from* London.

Suppoſe we have a Watch that keeps excellent Time, and ſet to a particular Place; by ſeeing the Difference of Time, when we return to *London*, we have the Longitude of that Place. For 4 Minutes in Time make a Degree of Longitude, ſo that if a Perſon finds his Watch too faſt by 4 Minutes, he concludes the Place he came from to be 60 Miles, or one Degree, to the Eaſt of the Meridian of *London*; but if on his Return to *London*, the Watch is too ſlow by 4 Minutes, he concludes the Place is 60 Miles to the Weſt of the Meridian of *London*.

Prob. 9. *Knowing the Hour of the Day at* London, *to find the Longitude of all other Places.*

If ſuch a Watch will keep Time exactly, and ſet to the Meridian of *London*, by knowing the Latitudes of the Places come to, we have the Longitudes, and which may be inſerted in a Map accordingly, by proportioning 4 Minutes in Time to one Degree of Longitude, and one Hour to 15 Degrees of Longitude. This Problem is of the utmoſt Conſequence to Navigators; for if ever they are bleſſed with a true Timekeeper, they may always, by having the Latitude they are in, know the Ship's Place; as all they aim at is to know exactly the Latitude and Longitude they are in, which cannot be obtain'd without both theſe being properly known.

Prob. 10. *To find the preſent Hour of the Day in every Capital throughout the World.*

Obſerve by the general Map the Longitude of the City, and turn it into Time, by allowing one Degree of Longitude to 4 Minutes of Time; as for Inſtance, ſuppoſe the City has 15 Degrees Eaſt Longitude from *London*, then it is One o'Clock in the Afternoon there when it is juſt Twelve with us, as every 15 Degrees of Longitude make juſt one Hour, and 5 Degrees make 20 Minutes in Time. But if the City required lies on the Weſtern Side of the Meridian of *London*, then 15 Degrees of Longitude ſhew that it is Eleven o'Clock in the Morning with them when it is Noon with us.

Prob. 11. *To find by a general Map the Meridian Altitude of the Sun, at* London, *throughout the Year.*

Reckon on the Meridian from Latitude 38 Northwards $\frac{1}{4}$ of a Degree for every Day after the Middle of *Maroh* to the Middle of *June*, and that Place ſhews nearly how high in the Sky the Sun is that Day at Noon: And for every Day from the Middle of *June* reckon on the Meridian from Latitude 62 Southward, and that likewiſe gives the Meridian Altitude that Day. After the Middle of *September* reckon $\frac{1}{4}$ of a Degree for every Day from Latitude 38 Southward, and that likewiſe gives the Altitude; but from the Middle of *December* to the Middle of *March*, reckon $\frac{1}{4}$ of a Degree for every Day on the Meridian, from Latitude 15 Northward, and that alſo gives the Meridian Altitude requir'd, near enough for the preſent Purpoſe.

Prob. 12.

Prob. 12. *To determine the Extent of the viſible or ſenſible Horizon.*

Let there be a Mountain whoſe Height is three Miles, and it is requir'd to find out, at what Diſtance from the Sea, the Top of it can be diſcovered. If we add three Miles to the Earth's Diameter, which call 8000 meaſur'd Miles for a round Number, and multiply this by the Height of the Mountain, and then find the Square Root, we ſhall find that the Top of the Mountain is viſible at the Diſtance of about 155 Miles. But from Surface to Surface, where no Eminence is conſider'd on either Side, the viſible Ray is about 90 Miles in Length; for here we only take the Root of the Earth's Diameter, without conſidering the Height of the Eye above the Water, or the Height of the Object.

Prob. 13. *To conſtruct a True Plain Map, that preſerves the true Diſtances and Bearings among themſelves.*

If a young Gentleman will take Notice of this Problem, and will be at the Trouble to conſtruct the following double Triangle, by having the Latitudes and Longitudes of any two Places, he may be of Service to himſelf in drawing a *Map of a County*, or any larger Part of a Kingdom, *&c.*

Thus, ſuppoſe one of the two Places, which are to come within the Map, lies in Latitude 50 Degrees, and in Weſt Longitude 5 Degrees 14 Minutes; and the other Place in 39 Degrees Latitude, and 9 Degrees 50 Minutes Weſt Longitude; theſe two Places are the *Lizard* and *Liſbon*, as an Example, as it is all one whether it is a Map of the Sea or Land, as it may do here for an Illuſtration. Here then the *Difference of Latitude* is 50—39=11 *Deg.* or 660 Miles, which is to ſtand on the Side or Edge of the Map for a Meridian, running North and South; and *the Departure* to be found as hereafter deſcribed, is 172 ½ Miles, which is to be drawn at a Right-angle, running Weſt and Eaſt from the Bottom or Top of the Meridian; and now if you form the Parallelogram, or long Square, from theſe two Lines drawn, you will have the Limits of the Map requir'd; and the Departure, or Line running Weſt and Eaſt, becomes the Limit of Longitude, as the other running North and South becomes the Boundary of Latitude. Theſe two Lines being divided into Miles and Parts, all other Parallels and Meridians may be drawn from theſe Diviſions, at the Diſtance of every 60 Miles, being one Degree, or nearer or further off as you think proper: You muſt graduate the outward Lines, repreſenting the Latitude and Longitude of Places, in what Proportion you pleaſe, and run it to the deſigned Largeneſs of the Map. And now mark the *Lizard* and *Liſbon* in the Draught, according to their Latitudes and Longitudes (the Departure ſtanding now for the Longitudes of Places); but you muſt remember to draw the Map longer and wider than the real Dimenſions of the intended Space you would lay down; and this is done by only continuing the Diviſions on the outward Lines more to the North and South, Eaſt and Weſt, than the Limits the *Lizard* and *Liſbon* require: And, laſtly, inſert every Place within your Limits, by having its Latitude and Longitude, according to *Problem* 3.

The Manner of conſtructing the Double Triangle, for the above uſeful Purpoſe, is as follows: By the Table of *Meridional Parts*, in any Book of Navigation, find the Number agreeing with Latitude 50, which is 3474; then look for the Number againſt Latitude 39, which you will find to be 2544; the Difference of theſe is 930, call'd the *Meridional Difference of Latitude*, which make the Baſe of the Triangle. The Difference of Longitude is 4 Degrees 36 Minutes or 276 Miles; raiſe this perpendicularly on the End of the Baſe, and draw the Hypothenuſe when you will have form'd the Right-angle Triangle. Now ſet off, from the Angle at the Baſe, the Difference of the two Latitudes 11 Degrees, or 660 Miles, on the Baſe; and where the Reckoning ends, raiſe another Perpendicular on that Point in the Baſe, that ſhall touch the Hypothenuſe. This laſt Perpendicular forms the ſecond Triangle, which is in Proportion to the greater, formed from the Difference of Longitude and Meridional Parts. The Baſe and Perpendicular of this leſſer Triangle are the two Sides, by which you are to conſtruct a true plain Map, or Sea Chart. I ſhall give you no Drawing to guide you in this uſeful Part of Knowledge, but ſhall leave you to try your own Abilities, as you have been taught how to lay down Triangles. To ſay the Truth, there cannot be a more effectual Method of fixing the rational, genteel, and engaging Sicence of Geography on the Mind, than thus actually laying down Maps. The diligent Youth, that will determine to lay down all the Counties in *England*, in the Manner we have deſcrib'd, will have ſhewn a noble Spirit, and excellent Taſte; and it will be no Surprize to Men of Learning, if he then proceeds to do the ſame by the various States of Europe, nay of the whole World.

THE SUPPLEMENT *to* GEOMETRY.

The different Meaſures of ARTIFICERS *in eſtimating their* WORK.

By Decimals.] TO what has been ſaid of *Menſuration*, muſt here be added, as its moſt convenient Place, the Manner of caſting up the Work done for your Employer. We muſt ſuppoſe your Rule to be divided into *Decimals*; that is, every Inch divided into *ten equal Parts*. I take it for granted the Youth has, at leaſt, made himſelf Maſter of what has been ſaid of the four firſt Rules in *Decimal Fractions*, and the requiſite Parts of Geometry; otherwiſe he cannot be ſaid to be qualified to underſtand what ſhall be here delivered concerning this neceſſary Part of Knowledge, which his deſtined Trade may require. Now, by his Rule, in meaſuring a Piece of Mahogany Work, he finds the Length 24 Inches and 9 Parts, and the Breadth 7 Inches and 6 Parts; that is, when properly read, 24 Inches 9 Tenths, and 7 Inches 6 Tenths, and then they will ſtand as in the Margin. Theſe Dimenſions are, you know, to be multiplied together, in order to find——What? Before, when you meaſured a Superficies, you multiplied the Length of a Parallelogram into its Breadth, and the Product gave you the ſquare Inches, Feet, Yards, &c. So here you have a Parallelogram, or Rectangle, as that will ever be the Caſe when two Numbers are multiplied together, unleſs the two Dimenſions are alike. Your Multiplication Table is a pretty Inſtance of this; for 3×3 is a Square, 3×4 is a Parallelogram; 8×8 is a Square; 8×7 is a Parallelogram; and ſo on, in all Numbers and Fractions, as above.

Inch. Pts.
24 . 9
7 . 6
1494
1743
189.24

Now take your Scale and Compaſſes, and lay down your Dimenſions; that is, draw a Parallelogram, whoſe Length is 24.9 Inches, and whoſe Breadth (7.6 Inches.) You know you may draw a Line at Pleaſure and ſet off 24.9; then raiſe a Perpendicular at the End of it, and ſet off 7.6, and compleat your Parallelogram, as before taught you, and it will incloſe a Space containing 189.24 Inches; that is, the Work meaſured is 189.24 ſquare Inches. And now vary the Meaſure from the laſt Example, and form of it ſeveral others, and conſtruct the Figures by your Scale and Compaſſes, in order to be very ready at this Part of Knowledge, that you may be a Pleaſure, and ſoon of Uſe, to the Maſter you are to ſerve.

When you have ſound the Content of your geometrical Figure, for ſuch every Product is, you muſt then find what your Employer is to pay you for the Work done; ſuppoſe the above 189.24 ſquare Inches, and at the Rate of 4½*d. per* Inch. The Manner of finding the Value is this; you muſt ſay thus, as 1 : 4½*d.* : : 189.24 : the whole Value. You know you will have your fourth Number, or Anſwer, in Farthings, if 4½*d.* is reduced into Farthings, and that if you divide by 4, 12, and 20 you will have the Pounds, Shillings, Pence, and Farthings. You will do well to vary the Price of the Mahogany Work, with the Dimenſions, in order to be very ready at finding the Value of the Work done in this valuable Wood.

But let us ſuppoſe the Length to be 3 Feet 7 Inches and .8, and the Breadth 2 Feet 5 Inches and .6: How will you multiply theſe two Dimenſions together, ſo as to find the ſquare Inches, as we will ſuppoſe you are to be paid by the Inch, and not by the Foot? You know if you multiply the 3 Feet by 12 and take in 7, you will have 43, to which put the Decimal of an Inch, and the Length will be 43.8 Inches; and in the ſame Manner you will have for the Breadth 29.6 Inches. Theſe two Numbers are to be now multiplied together, and if we ſuppoſe the Price to be at 4½*d. per* ſquare Inch, the Work will ſtand as in the Margin.

Inches, Pts.
43.8
29.6
2628
3942
876
1296.48
Proportion.
1 : 4½ :1296 .48

Now let us imagine the Mahogany Work is to be paid for by the Foot, and let the Dimenſions be as in the laſt Example. You are now to conſider you want a ſquare Foot that will contain ſquare Inches; that is, there will be 12×12= 144 ſquare Inches in a ſquare Foot, becauſe there are 12 Inches in the Side of the Square: Then you have this Proportion, as 144 ſquare Inches is to 1 ſquare Foot, ſo is 1296.48 ſquare Inches, as above, to the ſquare Feet required; that is, $\frac{1296.48}{144} = 9$ ſquare Feet, and a Fraction of no Value over. You may ſuppoſe the Foot at any Price, and find the Value accordingly.

Again, let us imagine the Length of a Piece of Work to be 3 *Yds.* 2 *F.* 7 *Inch.* and .8; and the Breadth 2 *Yds.* 2 *F.* 8 *Inch.* and .6. Here you are to ſuppoſe you are to be paid by the Yard; that is, a ſquare Yard, or 36 Inches by 36 Inches, which you know contains 3 Feet in each Side, and each Foot 12 ſquare Inches. Now you perceive you muſt bring both the Dimenſions into Inches for the Sake of the Decimals in your Meaſure; and when you have done this, your Work will ſtand as in the Margin. In order to find the ſquare Yards in the whole Content in Inches, you ſee your Diviſor is 1296, which are the ſquare Inches in a Yard; that is, 144×9=1296, for a ſquare Yard contains 9 ſquare Feet. I have ſo far ſhewn you how to be very exact in bringing out your Anſwers, which you will not be ſorry for knowing.

Inches.
139.8
104.6
8388
5592
13980
1296) 14623.08
Yds. 11.283

But to keep cloſer to the common Cuſtom in taking the Dimenſions, we will now omit the Decimals of an Inch, and let the Length be only 11 Feet 7 Inches, and the Breadth 8 Feet 8 Inches, to find the ſquare Yards contained in ſuch a Piece of Work, which is to be paid for by the ſquare Yard. You are now to turn you Inches into Decimals, as was formerly taught you; that is, 7 Inches is $\frac{7}{12}$=.583, and 8 Inches is $\frac{8}{12}$=.666; ſo then all Things will ſtand as in the Margin. You perceive your Diviſor is 9; the Reaſon is, there are 3 Feet in Length and 3 in Breadth in one ſquare Yard, ſo that there are 9 ſquare Feet in a ſquare Yard; and then the Proportion is, as 9 *ſq. F.* : 1 *ſq. yd.* : : 100.378278 *ſq. F.* here you perceive 9 becomes a Diviſor to the ſquare Feet, in order to bring them into ſquare Yards: But you perceive there is a Difference of .13 of a ſquare Yard between this Way and uſing the Decimals of an Inch.

11.583
8.666
69498
69498
69498
92664
9)100.378278
11.153 *ſq.yds.*

But in order to eaſe you in caſting up your Work, I ſhall, in the Margin, give you the Decimals of every Inch, that when you have taken your Dimenſions in Feet and Inches, you have the Advantage of only multiplying the Length by the Breadth

Inches	*Decimals.*
1	.083
2	.166
3	.25
4	.333
5	.416
6	.5
7	.583
8	.666
9	.75
10	.833
11	.916

to find the Content in ſquare Feet and Decimals of a Foot: Nay, I ſhall even ſuppoſe you know nothing of the Nature of Decimals, and yet you may find the Anſwers to your Queſtions: As thus, ſuppoſe you find the Length of the Work is 24 Feet 7 Inches, and the Breadth 15 Feet 11 Inches, and you are to have 3*s.* 6*d.* for every ſquare Yard of your Work; look in the Table for the Decimal of 7 Inches, then the Length is 24.583; then look for the Decimal of 11 Inches, and the Breadth is 15.916. You are to multiply theſe together as if they were whole Numbers; and when you have done ſo, always reckon how many Decimals you have in the Length and Breadth, which you ſee, in the preſent Caſe, are ſix; then point off ſix in the Product, as you ſee in the Margin, and you will have the ſquare Feet and the Decimals of a Foot. But as you are to be paid by the ſquare Yard, you muſt divide by 9 to bring them into ſquare Yards, becauſe 3 ſquare Feet make a ſquare Yard. You muſt remember, that if the Price of the Yard is pretty high, you ſhould uſe all the Decimals in the Table, but if it s not, you may be content with two, as 24.58, and 15.91. Now, as you are to have 3*s.* 6*d. per* Yard, your Proportion is, as 1 ſquare Yard is to 3*s.* 6*d.* ſo is 43.4736 to the Value of the Work; that is, bring the Price into Pence, and multiply them by the ſquare Yards, thus 43.4736×42*d.*=1825.8912*d.* but you ſee I have only taken .4736, which are only four of the ſix Decimals; and if you only uſe 43.47×42, the Product is 1825.74*d.* which comes near enough, as there is only a ſmall Part of a Farthing Difference. Now bring your Pence into Pounds, rejecting the Decimal .74, (tho'=¾*d.*) and you have 7*l.* 12*s.* 1*d.* the Value of your Work.

```
Feet.   Parts.
        24.583
        15.916
     ---------
        147498
        24583
      221247
     122915
     24583
   ------------
   9)391.262028
   ------------
   Yds. 43.473669
```

Now take your Dimenſions in Feet, and the Decimals of a Foot as before, and ſuppoſe you are to be paid by the Square of 100 Feet, whoſe Side is 10. Let the Length be 97 Feet 9 Inches, and the Breadth 45 Feet 5 Inches, at 20*s.* the Square. Here turning the Inches into Decimals by the Table, we have 97.75×45.416=4439.4, ſquare Feet; but as you are to be paid for a geometrical Figure, which contains 100 ſquare Feet, therefore you are to divide by that Square, that the Quotient may be the ſame Sort of Squares, for one of which you are to charge at the Rate of 20*s.* You may reject the Decimal .414 as inſignificant, and divide 4439 by 100, and this is done by pointing off two Figures, as you ſee in the Margin, where the Quotient is 44.39; that is, 44 Squares of 100 Feet, and 39 ſquare Feet over. Now then, to find the Value; you know the Squares will come to 44 *l.* and for the Value of the 39 ſquare Feet, multiply by 20 and 12, and point off two Figures each Time, and you will have the Shillings and Pence on the Left Hand of the Point each Time. This is nothing more than working this Stating, 100 *pts.* : 20*s.* : : 39 *pts.* : 7*s.* and as there will remain 80, it will be, 100 *pts* : 12*d.* : : 80 *pts.* : 9*d.* and as there is ſtill a Remainder of 60, it will then be as 100 *pts* : 4 *qrs.* : : 60 *pts.* : ½ *d.* if ſuch Exactneſs was requir'd.

45.416
97.75
4439.414
44.39

Now again, take your Dimenſions in Feet and Inches, and ſuppoſe you are to be paid by the ſquare Rod of 272.25 Feet, whoſe Side is 16½ Feet. Let the Length be 72 Feet 6 Inches, and the Height 19 Feet 3 Inches. Turning the Inches into Decimals, by the Table, we have 19.25×72.5=1395.625 ſquare Feet, which we are to reduce to ſquare Rods by this Proportion, 272.25 : 1 : : 1395.625. But, in dividing by the Rod, reject more than two Decimals, in the Product of the Dimenſions, multiplied together; and then the Work will ſtand as in the Margin. Now reject the Decimals in both the Dividend and the Diviſor, in order to ſee the Difference. You ſee the Anſwer is in the ſecond Way 5 Rods 35 Feet more, by .63 of a Foot, than by the firſt Operation, and which Difference is very inſignificant, as 272 Feet is the Meaſure you are paid by, ſuppoſe at 1 *l.* 2 *s.* 8 *d.* *per* Rod; then 1 Foot is only 1 Penny, and the Fraction is little more than Half that; however it is a good Thing to be exact, by uſing the firſt Way. But to value the odd Feet, the ſhorteſt Way is to multiply 35 by the Price, and divide by 272; if you have a Mind to be very exact, you muſt multiply the Remainder 34.37 Feet by the Price, and divide by 272.25, which you may do as in whole Numbers, only minding to keep the Points in their Places when you multiply; that is, pointing off two Figures in the Product, when you multiply by the Pence in the Price; and then when you divide by 272.25, the Quotient will come out whole Numbers, which are to be brought into Shillings and Pounds.

272.25) 1395.62
5 *Rods.* 34.37
272)1395
5 *Rods.*
35

Once more, take your Dimenſions, and ſuppoſe you are to charge ſo much for every Rood; that is, a Parallelogram 7 Yards in Length, and 1 Yard in Breadth. Let there be a Wall built, the Length whereof is 245 ¾ *Yds.* and the Height 2 ¼ *Yds.* at 5 *s.* 9 *d. per* Rood. Firſt turn the odd Meaſure into their Decimals; $\frac{3.00}{4}=.75$, and $\frac{1.00}{4}=.25$; and then the whole will ſtand, as in the Margin, to be multiplied together; that is, to reduce the Dimenſions to a Parallelogram, whoſe Length is 245.75 *Yds.* and Breadth 2.25 *Yds.* and Area or Content 552.9375 *Yds.* Now this geometrical Figure muſt be divided by a Parallelogram or Rood of 7 ſquare Yds. that is, 7 in Length and 1 in Breadth, and you will have 78.99 Roods (reject more Decimals than two) at 5 *s.* 9 *d. per* Rood. The Rule is, mutiply the Roods and Decimals by the Pence in the Price; that is, 78.99×69=5450.31 *d.* reject the Decimal, as it is little more than a Farthing, and can never reach a Penny, and bring 5450 *d.* into Pounds, and you will have 22 *l.* 14 *s.* 2 *d.* for your Work done. But now reject the ¾ and ¼ Yard, when you meaſure the Wall, and ſee the Difference. Then $\frac{245\times2}{7}=70$ Roods; here you find there will be near 9 Roods loſt in your Work, and there will be near 2 *l.* 11 *s.* 6 *d.* ſhort of what you ought to have, and therefore this ſhould make you careful to meaſure exactly.

```
      245.75
        2.25
    --------
      122875
      49150
     49150
    --------
   7)552.9375
    --------
Roods. 78.9910
```

Let us perform the ſame Queſtion by Feet inſtead of Yards; that is, 245 ¾ Yards=737 Feet 3 Inches, and 2 ¼ Yards=6 Feet 9 Inches, and theſe brought into a Parallelogram, by multiplying the Dimenſions after you have taken the Decimals of 3 and 9 Inches from the following Table, will make 4976.4375 ſquare Feet; and as your Rood is 21 Feet by 3=63 ſquare Feet, ſo dividing by this Parallelogram you have 78 Roods, and 99 Parts of 100 as before, and plainly ſhews you which Method you ought to uſe in computing the Work, provided you can turn Quarters, Half Yards, or Three Quarters into Decimals: If you ſhould ever forget the Way of doing it, you may eaſily remember what you ſee in the Margin, as you will prefer meaſuring and computing very large Dimenſions.

```
      737.25
        6.75
    --------
     3686 25
    51607 5
   442350
    --------
63)4976.4375
    --------
Roods. 78.99.
```

ſions by the Yard; for no ingenious Workman will refuſe the ſhorteſt Method, or to be very exact in valuing his Work done, whatever he afterwards abates of his Bill. In order to this, I have alſo given the Decimals of Inches from 1 to 9, which you may uſe or omit as you think proper. If your Work requires you to be exact in making your Bill, you may, by the Table, find any Decimal Part of a Yard thus; ſuppoſe you want the Decimal of 26 Inches, you know 18 is .5, and 8 is .222, therefore add theſe together, and you have .722, the Decimal of 26 Inches. Only take Care of the Points, and let the Figures ſtand as you ſee they do in the Table, and not as in common Addition; for if you ſhould add the .5 to the firſt 2 on the Right-Hand, you will make the Decimal .225, which is quite of a different Value.

Qrs.	*Decimals. Yds.*
¼	.25
½	.5
¾	.75

And ſo for Quarters of every Thing.

Inches.	*Decimals. Yds.*
1	.027
2	.055
3	.083
4	.111
5	.138
6	.166
7	.194
8	.222
9	.25

We will now ſuppoſe ſome Sort of Work requires to be computed even by the Decimals, into which an Inch is divided on your Rule, and the Dimenſions taken in Feet and Inches, as thus: Let the Length be 4 Feet 8 Inches, and .7, and the Breadth 1 Foot 4 Inches, and .2. Here then you are to bring the Meaſures into Inches, and they will ſtand thus; $\frac{56.7\times16.2}{144}=6.378$ ſquare Feet, at ſuppoſe 15 *d.* a Foot; then 6.378 × 15 = 95.67 *d.* reject the Decimal, and bring the 95 *d.* into Shillings, and you have the Value 7 *s.* 11 *d.* for the Work done. You know the Reaſon of dividing by 144; for the Dimenſions multiplied together make 918.54 ſquare Inches; and to bring theſe into ſquare Feet, you muſt divide by the ſquare Inches in a Foot, as you ſee in the Margin. Now reject the Decimal of an Inch, and ſee what Difference it will make in your Bill. Turn your Inches into Decimals by the Table, and your Dimenſions will be 4.666 × 1.333 = 6.2197 ſquare Feet; multiply

```
      56.7
      16.2
     ------
      1134
     3402
     567
     ------
144)918.54 Sq. Feet.
     ------
Feet.  6.378
     ------
      Feet.
      4.666
      1.333
     ------
Square 6.2197 Feet.
```

multiply this by the Price *per* Foot, 15*d.* and we have 93.29*d.* reject the Decimal, as it is only about a Farthing, and you have 93*d.* ſo you find there is only a Difference of 2*d.* but you muſt conſider your Dimenſions are but ſmall; ſuppoſe them 100 Times larger, and the Difference, in omitting ſo ſmall a Part, will be 16*s.* 8*d.*

From what has been ſaid of two Dimenſions you can never be at a Loſs to form your Judgment how to reduce them either to Squares or Parallelograms; but the diligent Youth muſt now be ſhewn the Method of reducing *three* Dimenſions to Solids, as in the Courſe of the World this muſt needs be as neceſſary among Workmen as the other.

Let there be three *cubical Dimenſions*, each Side whereof being, by the Rule, expreſſing the Decimals of an Inch, 17.5: Then 17.5×17.5×17.5=5359.375 at 4*s.* 6*d.* the ſolid Foot: In order to find the ſolid Content, we are to conſider how to reduce the ſolid Inches to this Meaſure. You know when the Dimenſions multiplied together made ſquare Inches, you were content with 12×12=144 ſquare Inches, to reduce the given geometrical Figure to ſquare Feet; but now it muſt be 12×12×12=1728, in order to make the Quotient ſolid Feet of the ſame Length in the three Dimenſions.

$$\begin{array}{r} 17.5 \\ 17.5 \\ \hline 306.25 \\ 17.5 \\ \hline 1728)5359.375 \\ \hline \textit{Cubes}\ 3.101 \end{array}$$

Therefore, $\frac{5359.375}{1728}=3.101$ the ſolid Feet in the Work done, and 3.101×54*d.*=167.454*d.* and rejecting the Decimal .101×54*d.*=5.454*d.* the Bill would be only 162*d.* or 13*s.* 6*d.* but reſerving the Decimal .101, the Bill is 13*s.* 11*d.* rejecting .454*d.* and this Error, in 100 Times the Dimenſions of the ſmall Piece of Work done, is 545*d.* or 2*l.* 5*s.* 5*d.*

Now, ſuppoſe your Dimenſions of a Solid, taken by your Rule, an Inch whereof being divided decimally, are 25 Inches ·3 by 9 Inches ·1 by 25 Feet; required the Content of 5*s.* 6*d.* the ſolid Foot. Here obſerve that we include all Bodies, admitting of three Dimenſions, whether the Ends be Squares, Triangles, Circles, &c. Parallelogram, under the general Name of Solids. Then 25·3×9·1×25=5755·75, and this divided by 144, we have 39·97 ſolid Feet, at 5*s.* 6*d.* Here 39·97×66=2638.02*d.* reject the Fraction .02; and the Bill will be 10*l.* 19*s.* 10*d.* but rejecting .97, the Value is too little by 5*s.* 4*d.* If we reject the Decimals of an Inch in meaſuring, the Bill falls 5*s.* 2*d.* ſhort of the true Value. You obſerve in the Margin, that 144 is the Diviſor, inſtead of 1728; but you are to conſider, we have multiplied by 25 Feet, which ſhould have been 25 × 12 Inches, and omitting the Multiplier 12, we alſo omit the Diviſor 12; for inſtead of 144, it ſhould have been 144 × 12 = 1728, to have a Quotient in ſolid Feet; you may be convinced that this Omiſſion brings out the ſame Anſwer; for $\frac{144\times2\times12}{1728}=\frac{144\times2}{144}=$ 2 ſolid Feet both Ways, and leaves no Room for a Doubt, and is a ſhorter Method of caſting up the Work.

25.3
9.1
230.23
Feet 25
144)5755,75
Solid Feet 39.97

Let your three Meaſures taken be 2 Feet 9 Inches by 1 Foot 7 Inches by 16 Feet 9 Inches. Look into your Table for the Decimals of the Inches, and your Dimenſions will be 16.75 × 2.75 × 1.583 = 72.9169 ſolid Feet, at 5*s.* 6*d.* *per* Foot. Then 72.9169 × 66*d.* = 4812*d.* or 20*l.* 1*s.* rejecting the Decimal of a Penny. Here you may find, that if you multiply only 72.9 into the Price 66*d.* you will fall ſhort 1*d.* and where there are a great many of ſuch Dimenſions, the Omiſſion would be too conſiderable to be overlooked. But let us do this ſame Queſtion another Way; 2 Feet 9 Inches = 33 Inches; 1 Foot 7 Inches = 19 Inches; and then 33 × 19 = 627 Inches; and this 627 × 16.75 = 10502.25 Inches, as you may call them; and this laſt Number divided by 144 gives the ſame Anſwer; that is, $\frac{33\times19\times16.75}{144}=72.932$ Feet; and this Quotient, multiplied into the Price, makes your Bill to be 4813*d.* which is 1*d.* more than by the other Way; but this Difference ariſes from the Decimal of 7 Inches, which, if carried out to .5833, would come nearer the Value. You ſee in the Margin your Work is again ſhortened by not multiplying 16 Feet 9 Inches by 12 Inches, as you can thereby omit 12, which ſhould be multiplied into 144 = 1728, if you had brought 16 Feet 9 Inches into Inches, and had multiplied 627 by it.

16.75
2.75
46.0625
1.583
Feet. 72.9169

33
19
627
16.75 *Feet.*
144)10502.25
Feet. 72.932

It is needleſs introducing any more Inſtances of working the Queſtions in Menſuration *by Decimals*, as what has been done is only to ſhew how very accurately a Bill may be made out, and not to repeat the Rules of meaſuring Superficies and Solids, which were particularly taught before in this Treatiſe.

By Aliquot Parts.] This is another Method of caſting up the Dimenſions: But you are now to conſider the Inches on a Rule, as divided each into Eight Parts, or Half Quarters of an Inch. Thus, ſuppoſe with your Rule you find the Length of your Work is $25\frac{3}{8}$ Inches, and the Breadth $9\frac{5}{8}$ Inches, to find the Content in ſquare Inches at 5 *d. per* ſquare Inch. Here firſt multiply $25\frac{3}{8}$ by 9, ſaying 9 Times 3 Eights is 27, and this divided by 8 is 3 and carry 3; 9 Times 5 is 45 and 3 is 48, 8 and carry 4; 9 Times 2 is 18, and 4 is 22. Now you have done with 9; you are now to conſider that $\frac{4}{8}$ is the Half of 1 Inch, and to multiply by $\frac{1}{2}$ is to divide by 2; therefore ſay, the Half of 25 is 12 and 1 Inch over, which is 8 Parts and 3 is 11, the $\frac{1}{2}$ of 11 is 5 Eights; and then $\frac{1}{8}$ is the fourth of $\frac{4}{8}=3\frac{1}{8}$. Now add up what you have, and it will be 244 ſquare Inches at 5 *d. per.* Inch; and multiplying by 5 *d.* you will have 1220 *d.* or 5 *l.* 1 *s.* 8 *d.* Now as 1 ſquare Inch is 5 *d.* $\frac{1}{8}$ will be $\frac{1}{2}$ *d.* ſo that the Value is 5 *l.* 1 *s* $8\frac{1}{2}$ *d.*

	$25\frac{3}{8}$
	$9\frac{5}{8}$
	$228:\frac{3}{8}$
	$12:\frac{5}{8}$
	$3:\frac{1}{8}$
	$244:\frac{1}{8}$
	5 *d.*
	1220 *d.*
$\frac{1}{8}$	$\frac{1}{2}$
	5 *l.* 1 *s.* $8\frac{1}{2}$ *d.*

Now prove this Queſtion, by doing it decimally; thus, $\frac{3}{8}=.375$, and $\frac{5}{8}=.625$; then $25.375\times9.625=244.23$ ſquare Inches, and this multiplied by 5 *d.* = 1221 *d.* ſo that by Decimals it is $\frac{1}{2}$ *d.* more occaſion'd by the Remainders in the aliquot Parts. If you are Maſter of Vulgar Fractions, you may make the Anſwer come out the ſame as by Decimals; thus,

$$25\frac{3}{8}=\frac{203}{8}, \text{ and } 9\frac{5}{8}=\frac{77}{8}; \text{ then } \frac{203}{8}\times\frac{77}{8}=\frac{15631}{64}=244.23$$

ſquare Inches, as by Decimals.

Next let the Decimals be 75 Feet 6 Inches by 9 Feet 9 Inches, to find the Meaſure of the Work done at 6 *d. per* ſquare Foot. Firſt multiply 75 Feet 6 Inches by 9 Feet, ſaying 9 Times 6 is 54 Inches, 6 and carry 4 Feet; 9 Times 5 is 45 and 4 is 49, 9 and carry 4; 9 Times 7 is 63 and 4 is 67; Then, becauſe 9 Inches is no even Part of a Foot, divide it into 6 and 3 Inches, 6 is the Half; ſay the Half of 75 is 37, and 1 Foot over, which is 12 and 6 is 18 Inches, the Half of 18 is 9 Inches; then

F.	*I.*
75	6
9	9
679	6
37	9
18	$10\frac{1}{2}$
2)736	$1\frac{1}{2}$
36,8 *Shil.*	
18 *l.* 8 *s.*	$0\frac{3}{4}$

then 3 Inches is the Half of 37 : 9, which is 18 Feet $10\frac{1}{2}$ Inches. Add theſe all up together, and you will have 736 ſquare Feet, and $1\frac{1}{2}$ ſquare Inch over at 6*d.* *per* Foot. Then 6*d.* is the Half of a Shilling; that is, the Half of 736 is 368*s.*=18*l.* 8*s.* and $1\frac{1}{2}$ Inch is the $\frac{1}{8}$ of 6*d.*=$\frac{3}{4}$. Now try the Event in Decimals; that is, $75.5\times9.75=736.125$ ſquare Feet, and this by 6*d.*=4416.75*d,*=18*l.* 8*s.* $0\frac{3}{4}$*d.* the $\frac{3}{4}$*d.* ariſe from .75*d.* or $\frac{75\times4}{100}=\frac{3}{4}$.

Now ſuppoſe the Dimenſions are 95 Yards 2 Feet 11 Inches, by 2 Yards 1 Foot 8 Inches, at 5*s.* 6*d.* *per* ſquare Yard. Firſt multiply the upper Line by 2 Feet, ſaying twice 11 is 22, 10 and carry 1; twice 2 is 4 and 1 is 5, 2 and carry 1; twice 5 is 10 and 1 is 11, 1 and carry 1; twice 9 is 18 and 1 is 19: Then 1 Foot is the third of the upper Line; the threes in 9 will go 3 Times, the threes in 5 will go 1, and 2 over, which is 6 and 2 is 8 Feet; the threes in 8, 2 and 2 over, which is 24 and 11 is 35; the threes in 35, 11 and 2 over, which is $\frac{2}{3}$: Then 6 Inches is the Half of 1 Foot; the Half 31 : 2 : $11\frac{2}{3}$=15 : 2 : $11\frac{2}{3}$; and 2 Inches is the third of 6 Inches; then $\frac{1}{3}$ of 15 : 2 : $11\frac{2}{3}$ = 5 : 0 : $11\frac{2}{3}$. Now add all theſe up together, and you have 245 ſquare Yards 9 Inches, at 66*d.* the Price: Then 245×66*d.*+$\frac{1}{4}$ of 66*d.* for 9 Inches=67*l.* 8*s.* $10\frac{1}{2}$*d.* for the odd Meaſure, 9 Inches is the $\frac{1}{4}$ of the Price=$16\frac{1}{2}$*d.*

Yds.	*F.*	*Inch.*
95	2	11
2	1	8
191	2	10
31	2	$11\frac{2}{3}$
15	2	$11\frac{2}{3}$
5	0	$11\frac{2}{3}$
245	0	9

Now let us try the Iſſue of the laſt Queſtion in *Decimals*, that you may form your Judgment of both Methods of Operation. The Dimenſions, when properly prepared, will be $\frac{287.916\times7.666}{9}=245.24045$ ſquare Yards, and this multiplied by the Price 66*d.*=16185*d.* or 67*l.* 8*s.* 9*d.* You may obſerve that by aliquot Parts the Value is 67*l.* 8*s.* $10\frac{1}{2}$*d.* and by Decimals 67*l.* 8*s.* 9*d.* but if you make the Decimals .9166, and .6666, the Value will be 67*l.* 8*s.* 11*d.* which now exceeds the Way by aliquot Parts.

Let the Dimenſions of Work done be 2 Feet 9 by 1 Foot 6 by 3 Feet 3, at 15*d.* *per* ſolid Foot. Then once 9 is 9, once 2 is 2; and becauſe 6 Inches is the $\frac{1}{2}$ of 1 Foot; therefore the $\frac{1}{2}$ of 2 Feet 9 Inches is 1 : $4\frac{1}{2}$; theſe added together is the Total 4 : $1\frac{1}{2}$; then 3 Times $1\frac{1}{2}$ is $4\frac{1}{2}$, and 3 Times 4 is 12; and as 3 Inches is the $\frac{1}{4}$, therefore we have 1 Foot;

and theſe added together is 13 ſolid Feet, and $4\frac{3}{4}$ Inches, at 15*d.* *per* ſolid Foot. Let the 13 Feet ſtand for Shillings, and 3*d.* will be the $\frac{1}{4}$ of it $=$ 3*s.* 3*d.* and 4 Inches is $\frac{1}{3}$ of 15*d.* $=$ 5*d.* and $\frac{1}{2}$ Inch is the $\frac{1}{8}$ of 4 Inches, and $\frac{1}{4}$ of an Inch is the $\frac{1}{2}$ of that, making in the whole 16*s.* $8\frac{3}{4}$*d.* But try this Queſtion by Decimals, and you will be the better enabled to judge of both Methods of computing. Now you may obſerve that, *by Decimals*, it comes to $\frac{1}{4}$*d.* more than by aliquot Parts, which you ſee will neceſſarily loſe by uſing Diviſors, which may be avoided by Multiplication. The Work indeed is ſooner performed by aliquot Parts; but when the Dimenſions run very high, rather than charge ſhort of the Value in the Bill, you will be well paid for your Time and Trouble if you uſe the Computation by Decimals. You will do well to remember, that ſome Workmen, ſuppoſe in the ſolid 13 Feet $4\frac{3}{4}$ Inches, call the 4 Inches a Part of a cubical Foot: But you need no other Proof that this is a Miſtake againſt themſelves, than by viewing the Queſtion done decimally, where 13 ſolid Feet $4\frac{3}{4}$ Inches by aliquot Parts $=$ 13.406 by Decimals. When I ſay they are equal, you muſt underſtand $4\frac{7}{8}$ Inches, becauſe in taking the Parts, $\frac{1}{8}$ of an Inch was dropt, as being of no Value in the Queſtion. You know the Decimal of 4 Inches is .3333, &c. 4 Inches remain as $\frac{1}{3}$ of a Foot; then $\frac{4}{8}$ of an Inch is $\frac{1}{8}$ of .3333; $\frac{2}{8}$ is the $\frac{1}{2}$ of this; and $\frac{1}{8}$ is the $\frac{1}{2}$ of this: Add them up and you have the Decimal $=$ $4\frac{7}{8}$ Inches. Let us ſuppoſe the Product of the three Dimenſions had been 1 ſolid Foot and 6 Inches, at 20*s.* a Foot; thoſe who would have 6 Inches to be Part of a cubical Foot muſt expreſs it thus $1\frac{1}{4}$; that is, one ſolid Foot, and

	F.	*I.*
	2	9
	1	6
	2	9
	1	$4\frac{1}{2}$
Sq.	4	$1\frac{1}{2}$
	3	3
	12	$4\frac{1}{2}$
	1	$0\frac{1}{4}$
Solid.	13	$4\frac{3}{4}$ *at* 15*d.*
Shill.	13	0
	3	3
	0	5
	0	$0\frac{1}{2}$
	0	$0\frac{1}{4}$
Shill.	16	$8\frac{3}{4}$

2.75	*Length.*
1.5	*Breadth.*
4.125	*Squares.*
3.25	*Height.*
13.406	*Solid.*
15*d.*	
12)201.093*d.*	
Shill. 16 : 9	

Proof.

	Feet.
4 *Inches* $=$	.3333
$\frac{4}{8}$ *is* $\frac{1}{8}$ $=$	.0416
$\frac{2}{8}$ *is* $\frac{1}{2}$ $=$	.0208
$\frac{1}{8}$ *is* $\frac{1}{2}$ $=$	.0104
$4\frac{7}{8}$ *is* $=$	.4061

and $\frac{1}{4}$ of a ſolid Foot, at 20*s.*=25*s.* the Value of the Work. If the three Dimenſions are 1.14×1.14×1.154. we ſhall have nearly 1 Foot, .5 the ſolid Meaſure at 20*s.*=30*s.* the Value of the Work done; and therefore you muſt reſt the Truth of this Affair upon what the decimal Operation offers to your Conſideration, and in all Doubts try it by working the Dimenſions by Decimals.

By Duodecimals, or Croſs Multiplication

Workmen are extremely fond of caſting up their Work by *Croſs Multiplication*; and therefore I ſhall endeavour to make you Maſter of this Method of Operation. And here I muſt ſuppoſe your Rule divided *duodecimally*, that is, every Inch divided into 12 Parts; whereas, by *Decimals*, the Inch ought to be divided into 10 Parts, and by *aliquot Parts* into 8. Why Rules are not divided *duodecimally* is hard to ſay; but we may venture to determine, that thoſe Workmen who always take their Dimenſions in Feet, Inches, and to a Quarter of an Inch, are Loſers, by not being able to take the Meaſure to a *Twelfth of an Inch*. Here then the Youth may be told, that when he meaſures to a Quarter of an Inch he may in Part ſupply the Want of a Rule *divided duodecimally*; for $\frac{1}{4}=\frac{3}{12}=$; $\frac{1}{2}=\frac{6}{12}=$; $\frac{3}{4}=\frac{9}{12}$; thus 3 Feet 7 Inches $\frac{3}{4}$ by 2 Feet 8 Inches $\frac{1}{4}$ will be 3 Feet 7 Inches 9 Parts, by 2 Feet 8 Inches 3 Parts. But beſides, ſuppoſing the Inches on the Rule divided into 12 Parts, you muſt imagine one of theſe Parts again divided in the ſame Manner, and alſo one of theſe laſt divided again into other 12 Parts. Now when the Meaſures are taken by the Rule, you are then to find your ſquare Feet, &c. which you may eaſily do by getting off the Table in the Margin. Let the Meaſures be 75 Feet 7 Inches by 9 Feet 8 Inches; here begin and multiply 75 : 7 by 9 Feet firſt; but you are undoubtedly at a Loſs to gueſs the Reaſon of what you are directed to do, as it ſeems ſo contrary to any Thing you have learned before in Figures. However work it firſt, and we will ſee if you can receive any Aſſiſtance from the Nature of abbreviating Vulgar Fractions. Now then ſay 9 Times 7 is 63 Inches, (*Tab.* No. 2.) 3 and carry 5 Feet, 9 Times 5 is 45 and 5 is 50, &c. as in the Margin. Then take the 8 Inches, and ſay 8 Times

The Croſs Table.

1. *Feet into Feet, give Feet.*
2. *Feet into Inches, give Inches.*
3. *Feet into Parts, give Parts.*
4. *Inches into Inches, give Parts.*
5. *Inches into Parts, give Seconds.*
6. *Parts into Parts, give Thirds.*

F.	I.	P.
75	7	0
9	8	0
680	3	0
50	4	8
730	7	8

8 Times 7 is 56 Parts, *(by Table,* No. 4.*)* 8 and carry 4 Inches; 8 Times 5 is 40 Inches, *(by Table,* No. 2.*)* and 4 is 44, 4 and carry 4; 8 Times 7 is 56 and 4 is 60; in all 604 Inches=50 Feet 4 Inches. Now add both the Products together, and the Anſwer is 730 Feet 7 Inches and 8 Parts. Conſider a little; ſuppoſe your Dimenſions were 3 Feet, 9 by 2 Feet 6; then, from the Nature of Reduction, your firſt Product, by multiplying 9 Inches by 2 Feet, will ſtand thus, $\frac{2\times12\times9}{12\times12}$=1 Foot 6 Inches. Here you ſee you have 12 a *Multiplier* in the Numerator, and if you omit this, and omit likewiſe 12 a *Multiplier* in the Denominator, your improper Fraction (as it ſtands, and is in the Form of one) is not altered in Value; therefore it will be $\frac{2\times9}{12}=1$ Foot 6 Inches; and ſo the Rule *(Tab.* No. 2.*)* is eſtabliſhed and explained. Then again we have, by the Nature of Reduction and Vulgar Fractions, $\frac{3\times12\times6}{12\times12}+\frac{9\times12\times6}{12\times12}$ = 1 *F* : 10 *I* : 6 *P*. Here then, by throwing out the *Multiplier* 12 in all, we have, $\frac{3\times6}{12}+\frac{9\times6}{12}$ = 1 *F* : 10 *I* : 6 *P*; $\frac{3\times6}{12}$ eſtabliſhes the Rule, *(Tab.* No. 2.) and $\frac{9\times6}{12}$ eſtabliſhes alſo the Rule, *(Tab.* No. 4.*)*

F.	*I.*	*P.*
3 :	9 :	0
2 :	6 :	0
7 :	6 :	0
1 :	10 :	6
9 :	4 :	6

Now let us ſuppoſe Parts were taken in the Dimenſions, and let us ſee what Alterations they make in the whole Meaſure of the Work when caſt up. You may obſerve, that as the Figures ſtand in Feet, Inches, and Parts, they muſt follow the Rules in multiplying them; therefore, beginning with Parts in the Multipliers, or ſecond Line, ſay 9 Times 3 is 27 Thirds, *(Table,* No. 6.*)* 3 and carry 2; 9 Times 9 is 81 and 2 is 83 Seconds, *(Tab.* No. 5.*)* 11 and carry 6; 9 Times 3 is 27 and 6 is 33 Parts, 9 and carry 2: Then taking 6 Inches, another Multiplier, ſay 6 Times 3 is 18 Seconds, 6 and carry 1; 6 Times 9 is 54 and 1 is 55 Parts, 7 and carry 4; 6 Times 3 is 18 and 4 is 22 Inches, 10 and

F.	*I.*	*P.*	″	‴
3 :	9 :	3 or $\frac{1}{4}$		
2 :	6 :	9 or $\frac{3}{4}$		
	2 :	9 :	11 :	3
1 :	10 :	7 :	6	
7 :	6 :	6		
9 :	7 :	11 :	5 :	3
9 :	4 :	6		
0 :	3 :	5 :	5 :	3

carry 1. Then, laſtly, uſing the Multiplier 2, ſay twice 3 is 6 Parts; twice 9 is 18 Inches, 6 and carry 1; twice 3 is 6 and 1 is 7 Feet; and adding theſe all up together, you have 9 *Feet* 7 *Inches* 11 *Parts* 5 *Seconds* and 3 *Thirds*; whereas before you had 9 Feet 4 Inches and 6 Parts; ſo that you have gained 3 *Inches* 5 *Parts*, *&c.* by meaſuring ſo exactly as to take in the Parts, when you took the Dimenſions of the *Superficies* or Work done.

Let us now work 3 Feet 9 by 2 Feet 6 *decimally*, then we have 3.75 by 2.5=9.375 ſquare Feet. In order to turn 4 Inches and 6 Parts into a Decimal, you know 4 Inches is .33333, *&c.* and 6 Parts is the $\frac{1}{8}$ of this Decimal, or .04166, *&c.* and both together is .375. Now let the Price for the Work done be 12*d.* *per* ſquare Foot; then 9.375 × 12*d.*= 112$\frac{1}{2}$*d.* or 9*s.* 4$\frac{1}{2}$*d.* Then if 4 Inches is conſidered as a Part of a *geometrical Square*, it is the $\frac{1}{9}$th Part of 12*d.*=1$\frac{1}{4}$*d.* and 6 Parts is the $\frac{1}{64}$th Part of 4 Inches; but you ſee the *decimal Method* forbids you to reaſon in this Manner, to your own Loſs; therefore the Price by Duodecimals muſt be reckoned thus; 9 Feet at 12*d.* is 9*s.* and 4 Inches is the $\frac{1}{3}$ of 12*d.* and 6 Parts is the $\frac{1}{8}$ of 4*d.* all which will ſtand as in the Margin. And now you perceive, if you value the Work from the Inſtance of meaſuring Parts, you have 3 Inches and 5 Parts more to reckon up and charge in the Bill. Here then 3 Inches is the $\frac{1}{4}$ of 12*d.*=3*d.* 4 Parts $\frac{1}{6}$ of 3 Inches=$\frac{1}{2}$*d.* and 1 Part is the $\frac{1}{4}$ of $\frac{1}{2}$*d.* = $\frac{1}{8}$*d.* All which makes a Difference of 3*d.* $\frac{5}{8}$, too much to loſe for Want of Accuracy.

	F.	*I.*	*P.*
	9 :	4 :	6 *at* 12*d.*
Shill.	9 :	0	
	0 :	4	
	0 :	0$\frac{1}{2}$	
Shill.	9 :	4$\frac{1}{2}$	
	0 :	3	
	0 :	0$\frac{1}{2}$	
	0 :	0$\frac{1}{8}$	
Diff.	0 :	3$\frac{5}{8}$	

Let us look upon 9 Feet, 7 Inches, 11 Parts, 5 Seconds, and 3 Thirds, as a Superficies, to be multiplied by 6 Feet, 5 Inches, and 8 Parts, another Dimenſion of a ſolid Body, to find the Content. In the laſt Queſtion I directed you to begin with the firſt Multiplier 9 on the Right Hand; but to give you another Form, in order to avoid Confuſion, begin

F.	*I.*	*P.*	″	‴	⁗
9 :	7 :	11 :	5 :	3	
6 :	5 :	8			
57 :	11 :	8 :	7 :	6	
4 :	0 :	3 :	9 :	2 :	3
	6 :	5 :	3 :	7 :	6
62 :	6 :	5 :	8 :	3 :	9

begin with 6 Feet the Multiplier on the Left Hand, and with the 3 in *Thirds*, ſaying 6 Times 3 is 18, 6 and carry 1; 6 Times 5 is 30 and 1 is 31, 7 and carry 2; 6 Times 11 is 66 and 2 is 68, 8 and carry 5; 6 Times 7 is 42 and 5 is 47, 11 and carry 3; 6 Times 9 is 54 and 3 is 57: Then 5 Times 3 is 15, 3 and carry 1; 5 Times 5 is 25 and 1 is 26, 2 and carry 2; 5 Times 11 is 55 and 2 is 57, 9 and carry 4; 5 Times 7 is 35 and 4 is 39, 3 and carry 3; 5 Times 9 is 45 and 3 is 48, 0 and carry 4: Then 8 Times 3 is 24, 0 and carry 2; 8 Times 5 is 40 and 2 is 42, 6 and carry 3; 8 Times 11 is 88 and 3 is 91, 7 and carry 7; 8 Times 7 is 56 and 7 is 63, 3 and carry 5; 8 Times 9 is 72 and 5 is 77, 5 and carry 6; and adding all theſe up together there will be 62 ſolid Feet, 6 Inches, 5 Parts, 8 Seconds, 3 Thirds, and 9 Fourths. You need not be troubled in having Examples, where the third Dimenſion contains *Seconds* and *Thirds*, as no ſuch *Meaſures* can be taken. You ſee it was taking the two firſt Dimenſions in Parts, which threw Seconds and Thirds into your Superficies. If the Feet in the Multiplicand are more than 12, the Work becomes troubleſome by Duodecimals; in this Caſe *aliquot Parts* will relieve you, which I ſhall leave to your own Ingenuity, and ſhall work this Queſtion for you by Decimals, and find the Value at 12*d.* *per* ſolid Foot, that you may know to which you ought to give the Preference.

When you have prepared the Numbers in Inches to work them *decimally*, they will be $\frac{45.25 \times 30.75 \times 77.666}{1728} = 62.539$ ſolid Feet × 12*d.* = 3*l.* 2*s.* 6½*d.* nearly. And by aliquot Parts, the ſolid Content, as found by *Croſs Multiplication*, will ſtand as in the Margin: I have rejected all lower than Parts, as being of no Value. Here 4 Inches = $\frac{1}{3}$ of 12*d.* 2 Inches the $\frac{1}{2}$ of that; and, for a round Number, 6 Parts is the $\frac{1}{4}$ of 2 Inches, in all 3*l.* 2*s.* 6½*d.* the ſame as by *Decimals*.

	F.	*I.*	*P.*	
	62	6	5	*at* 12*d.*
Shill.	62	0		
I.				
4, $\frac{1}{3}$	0	4		
2, $\frac{1}{2}$	0	2		
P.				
6, $\frac{1}{4}$	0	0½		
Shill.	62	6½		

Thus much of the three different Ways of eſtimating any Work done, I preſume, is very ſufficient to make a Youth expert at caſting up his *Meaſures*, and valuing them by any Price; and if he has made himſelf Maſter of finding the Contents of all the Geometrical Figures contain'd in this little Treatiſe, he may now be ſaid to be a *Compleat Meaſurer*, except in what relates to the Properties of the *Elipſis*, *Parabola*, and *Hyperbola*, their Solids and Parts, which, to inſert in this ſmall Volume, would ſwell it out too far, and therefore they muſt be reſerv'd for the ſmall Volume to contain the Third and Fourth Parts, belonging to the whole Plan for the Inſtruction of Youth. I ſhould here conclude, but that I am willing to inſert ſomething relating to the Subject of this *Supplemental Claſs*, and which I receiv'd from an experienced Meaſurer at *York*. His Words are as follow:

Of the Menſuration of ſeveral Artificers' Work, concerned in Building, &c. *with the Manner of taking Dimenſions of the ſame, which in general are in Feet and Inches.*

" THE Superficies of ſuch Work are moſtly in the Form " of Squares or Parallelograms, or muſt be reduc'd to " ſuch a Form; and then the Length and Breadth, taken in " Feet and Inches, and being multiplied one by the other, " they call ſquaring the Dimenſions; and this is uſually per" formed by Duodecimal Arithmetic, commonly call'd Croſs " Multiplication: In which you are to note, that one ſuper" ficial Foot contains 12 Inches call'd long Inches, one long " Inch 12 Parts, one Part 12 Seconds, &c. as may be ſeen by " Inſpection, in *Fig.* 26, 27: For, ſuppoſe *abcd*, *Fig.* 26. " repreſents a ſquare ſuperficial Foot, which being divided " into 12 Parallelograms, theſe repreſent the 12 long Inches " in one ſuperficial Foot. *Fig.* 27, *ABC*, is a Parallelogram " whoſe Length *CD* is 3 Feet 4 Inches, and the Breadth *CA* " 2 Feet 2 Inches; and thoſe Dimenſions multiplied together " make 7 Feet 2 Inches 8 Parts, as may be ſeen by the Fi" gure; the *a*'s being Feet, the *b*'s Inches, and the 8 little " Squares, at the Angle *B*, are Parts; that is ſix *a*'s, four" teen *b*'s, make 7 Feet 2 Inches, add to which the 8 little " Squares at the Angle *B*, making in all 7 Feet 2 Inches " and 8 Parts.

Carpenters' Work.] " Roofs, Fram'd Floors, &c. are mea" ſured by the Square, containing 100 ſuperficial Feet, or " 10 Feet every Way: The Dimenſions are taken in Feet and " Inches and ſquared, then divided by 100 ſhews the Con" tent in Squares. To take the Dimenſions and meaſure a " Roof, you are to conſider whether the Roof is *True Pitch*, " as it is called; that is, if *AC Fig.* 28 be 3 Fourths of the " Breadth of the Building *BA*, it is called *True Pitch*, and

“ the Content of the Roof is equal to the Area and Half of “ the Building, whether it has *Gable Ends* or is hipped in; “ that is, you take the Length of the Building, and multiply “ it by the Breadth, and this Product, and Half this Product “ added together, is the Content of the Roof. Example; “ ſuppoſe the Length of the Building is 40 Feet, and the “ Breadth 20 Feet; then 40×20=800, and 800+400= “ 1200, and this divided by 100=12 Squares, each 100. “ But if the Roof is of any *other Fitch*, either higher or low- “ er, that is, ſteeper or flatter, this Rule will not hold good; “ but you muſt multiply the Length of the Roof by the “ Length of the Rafter, and twice that Product is the Con- “ tent, as the *Pitch Ba A* in *Fig.* 28 is flatter, the Rafter “ being but 2 Thirds of the Breadth of the Building. The “ meaſuring of *Framed Floors* and *Framed Partitions* are the “ ſame; but in taking the Dimenſions of the *Floors* you “ muſt allow for the Length of the Joiſts going into the Wall. “ *Boarded Floors* are done by the Square in ſome Parts of “ *England*, and by the Yard in others; that is, 9 ſuperficial “ Feet; in taking the Dimenſions in Feet and Inches, you “ muſt take them to the very extreme Parts, out of which “ you are to deduct the Vacancies, if there be any, as Stair- “ Caſes, Chimney-Hearths, *&c.* Suppoſe a *Floor* 20 Feet “ long, and 15 Feet broad; then 20×15=300, and 300 “ divided by 100=3 Squares; or 20×15=300, and 300 “ divided by 9=3 Yards 3 Feet.

Bricklayers' Work.] “ That is, *Walling*, *Tiling*, &c. Of “ Brick Walls there are ſeveral Cuſtoms in *England*; as in “ *London*, and many Parts of *England*, Brick Work is mea- “ ſured by the Rod of 16 ½ Feet ſquared, whoſe Content is “ 272 ¼ Feet for one Rod of Brick Work, at the *Standard* “ *Thickneſs* of 1 ½ Brick thick. The Dimenſions are taken “ in Feet and Inches, and if the Wall is *Standard Thickneſs*, “ this is the Rule; multiply the Product of the Length and “ Breadth ſquared by 16, the ſquare Quarters in a Foot, “ and divide the Product by 4356, the ſquare Quarters in a “ Rod ſquare, and the Quotient is the Number of Rods “ contained therein. Or thus, by Decimals, which is “ ſhorter, ſquare the Dimenſions, and divide by 272.25, “ and the Quotient is the Content. For Inſtance, ſuppoſe “ a Brick Wall, *Standard Thickneſs*, of 1 ½ Brick thick, 42 “ Feet long, and 20 Feet high; then 42×20=840, and “ 840 divided by 272.25=3 Rods 23.25 Feet. But if the “ Brick Walls are more or leſs than the *Standard Thickneſs*,

“ they are reduced by the following Rule; multiply the Number of Feet in the ſuperficial Content of the Wall, by the Number of Half Bricks the Wall is in Thickneſs, and divide the Product by 3; then proceed as before. Example; ſuppoſe a Wall is 2 Bricks in Thickneſs, 42 Feet long, and 20 Feet high; then 42×20×4; and this Product divided by 3, and this Quotient divided by 272.25=4 Rods 31 Feet *Standard Thickneſs.* *Tiling* at *London*, and the greateſt Part of *England*, is meaſured by the Square of 100 Feet, as the Carpenters' Work. *Bricklayers' Work* at *York*, and all the North Part of *England*, is done by the Rood of 7 ſuperficial Yards, making 63 ſuperficial Feet, and the Thickneſs of *One Brick*. Thus, ſuppoſe a Wall 42 Feet long and 20 Feet broad, and the Thickneſs of one Brick, that is, the Length of the Brick of about 9 Inches; then 42×20 and divided by 9 ſquare Feet, and this Quotient divided by 7 Yards=13 Roods 2 Yards 3 Feet. Suppoſe a Wall of the ſame Length and Breadth, and 1 ½ Brick thick; then 42×20=840, and $840+\frac{840}{2}=1260$ Feet; and theſe divided by 9, and this Quotient divided by 7=20 Roods. If it had been 2 Bricks thick it would have been double to the firſt; that is, 26 Roods 4 Yards 6 Feet. Tiling is alſo meaſured by the ſame Rood.

Plaiſterers' Work.] “ This is done by the Yard ſquare, and the Dimenſions are taken in Feet and Inches; but Mouldings, Ornaments, *&c.* are meaſured and valued by the ſuperficial Foot; and in taking the Dimenſions of ſuch Work, all Mouldings, Ornaments, *&c.* are *girt*.

Joiners' Work.] “ Wainſcotting is done by the Yard ſquare, and the Dimenſions are taken in Feet and Inches. Chimney-Pieces, and other Work that conſiſts of Mouldings, are done by the ſuperficial Foot; and in taking the Dimenſions they *girt* the Mouldings with a ſmall String, as thus; ſuppoſe *Fig.* 29 to be the Profile of a Chimney Cornice, they begin at the Top, *a*, and preſs the String into all the Turnings of the Moulding sto *b*, *c*, *d*, and *e*, which is called the Girt or Height of the Cornice.” To what he ſays I ſhall add the following Trades.

Glaziers' Work.] They do their Work by the ſquare Foot, and take their Dimenſions in Feet and Inches, and to a Quarter of an Inch; but it would be better for them, could they take it in Parts inſtead of Quarters. Example; a Window is 4 Feet 8 Inches $\frac{3}{4}$ high, and 1 Foot 4 Inches $\frac{1}{4}$ broad; then 4 Feet 8 Inches 9 Parts × 1 Foot, 4 Inches, 3 Parts = 6 Feet 4 Inches 10 Parts. Now as this Square ariſes from multiplying the Dimenſions together, the 4 Inches is not to be conſider'd as Part of a Square Foot, but as Part of a Foot in Length, and then 4 Inches is $\frac{1}{3}$ of the Price, and 6 Parts the $\frac{1}{8}$ of 4 Inches, and 4 Parts the $\frac{1}{12}$ of what 4 Inches come to. If you ſhould aſk a Glazier what he will let you have two Pieces of finely painted Glaſs for, one exactly a ſquare Foot, and the other exactly half a ſquare Foot; if he ſhould ſay, for a Penny a ſquare Inch; and that you muſt therefore pay 12*s.* for the Piece a Foot ſquare, and 6*s.* for the Piece ſix Inches ſquare, becauſe it is the Half of 12 Inches. Now you know there are 144 ſquare Inches in the greater Piece, and only 36 ſquare Inches in the leſſer; therefore $\frac{144}{36}$ = 4th Part of 12*s.* = 3*s.* ſo that you ought to pay 15*s.* only for both Pieces. Let *AEFC* be the larger Square, *Fhqp* the leſſer = *rghF*, which is the 4th Part of the greater Square; and conſequently, when the Inches are conſider'd as a Square, and not as a Part of a Foot in Length, it makes a wide Difference.

Painters' Work.] They take their Dimenſions in Feet and Inches, and then reduce them to ſquare Yards; but, like the Joiners, they girt over the Mouldings and ſwelling Pannels in taking the Height, and charge their Work by the ſquare Yard, according to the Times it is colour'd over; for Window-Lights, Window-Bars, Caſements, and the like, they do at ſo much a Piece. Now again, the Youth is to obſerve, that when the Dimenſions are ſquar'd, and brought into ſquare Yards, the Feet remaining are not to be conſider'd as Squares, but as Heights, whoſe Breadths are one Foot; that is, as Part of the Squares in a Yard Thus, 6 Feet 6 Inches by 7 Feet 6 Inches = 48 Feet 9 Inches; and this Product divided by 9 = 5 Yards 3 Feet 9 Inches. Here then 5 × 9*d.* = 48*d.* and 3 Feet in Height and one in Breadth is $\frac{1}{3}$ of 9*d.* = 3*d.* and 6 Inches is $\frac{1}{6}$ of 3 Feet = $\frac{1}{2}$*d.* and 3 Inches is $\frac{1}{2}$ of 6 Inches; and all make 48$\frac{3}{4}$*d.* So $\frac{6.5\times7.5}{9}=5.416$; and $5.416\times9d.=48\frac{3}{4}d.$ But a ſingle ſquare Yard being painted for 9*d.* 4$\frac{1}{2}$*d.* is not the Price for a ſingle half ſquare Yard; for if you call *AEFC Fig.* 27. a ſquare Yard, *Fhqp* = *rghF*, is only the $\frac{1}{4}$ Part of the Colouring = $\frac{9}{4}$ = 2$\frac{1}{4}$*d.* for painting $\frac{1}{2}$ a ſquare Yard at 9*d. per* Yard.

Maſons' Work.] They differ in their Cuſtoms of meaſuring their Work; in ſome Parts it is done by the cubical Foot, in other Places by the ſuperficial Foot; but in moſt Parts they meaſure their *Walling*, and charge by the Rood of 7 Yards in Length, and one in Breadth. Let us ſuppoſe a Marble Hearth to be 3 Feet 4 Inches long and 1 Foot broad, for our preſent Purpoſe, as a Superficies, at any Price; ſuppoſe at 18*d.* *per* ſquare Foot: Here 3 Feet $\times 18d.=54d.$ and 4 Inches is $\frac{1}{3}$ of $18d.=6d.$ then the whole is 60*d.* or 5*s.* Now look at *Fig.* 27. wherein *tqpC* repreſents 3 Feet, and *uwDp* alſo repreſents the little Square of $4\times4=16$ Inches; then $\frac{144}{16}=$ 9th Part of a Square Foot, *Fhqp*; that is, $\frac{1}{9}$ of $18d.=2d.$ and $54d.+2d.=56d.$ the whole Price of the Marble Superficies, $tqpC+uwDp=3$ ſquare Feet 16 ſquare Inches. So, here again, the Youth perceives the Difference between charging 4 Inches, as $\frac{1}{3}$ of 18*d.* and the Square of 4 Inches, as $\frac{1}{9}$ of 18*d.* Now let a Marble Hearth, *Fig.* 27. be *ABDC*, $AB=3$ Feet 4 Inches, and $DB=2$ Feet 2 Inches; then 3 Feet 4×2 Feet $2=7$ Feet 2 Inches 8 Parts, at $18d.=130d.$ by *Duodecimals*; and 3.333, &c. $\times$ 2.166, &c. $\times 18d.=130d.$ nearly, by *Decimals*; and 3 Feet 4×2 Feet $+\frac{1}{6}$ of 3 Feet 4, at $18d.=130d.$ by aliquot Parts: But if you look upon 2 Inches and 8 Parts as the Sides of Squares, then $2\times2=4$, and $\frac{144}{4}=36$ Part of $18d.=\frac{1}{2}d.$ and 8 Parts ſquared $=64$, and $\frac{144\times2}{64}=4\frac{1}{2}$ Part of $\frac{1}{2}d.$ So that you ſee there is the Difference of $3\frac{1}{2}d.$ in the two Ways of taking the Remainders.

Timber-Meaſure.] But firſt, all regular *Boards* are meaſured as Squares or Parallelograms; that is, by multiplying the Length and Breadth together, by any of the Forms before taught you. *Squared Timber* are ſuch as have equal Baſes, and the Sides ſtrait and parallel, and the Content is found in the ſame Manner you did that of the *Parallelopipedon*, which is to multiply the Area of the Baſe into the Height of the Timber for ſolid Feet. You muſt guard againſt the common Method in adding the Depth and Breadth together; and taking Half for the Side of a Square, whoſe Area, it is imagined, is equal to that of the Parallelogram: Thus, if the Depth is 12 Inches, and the Breadth 18 Inches, the Parallelogram will be 216 Inches; but $\frac{18+12}{2}=15$, and 15 ſquared 225 Inches; and here you ſee are in theſe ſmall Dimenſions, 9 Inches too much in the Area of the Baſe; and if this Error is multiplied by the Length of the Piece of ſquared Timber, ſuppoſe only 36 Inches, it will be 9×36=324 ſolid Inches too much: But when it is *unequal ſquared Timber*; that is, the Fruſtum of a Pyramid, you muſt meaſure it by that Rule, and not as in the uſual Way, to take a Square near the Middle of the Piece, and imagine this, when multiplied into the Length of the Timber, is the ſolid Content; that is, to imagine this Method has reduced the *Fruſtum of a Pyramid* to a *Parallelopipedon,* whoſe Baſe conſiſts of that middle Square. The Error will be more conſiderable in large Concerns than you perhaps think; for, to inſtance in leſs Affairs, if the Square, at the greater End, be 25 Inches, and the leſſer End 9 ſquare Inches, and the Length of the Piece of Timber is 20 Feet, the Error will be near 3 Feet too little for the true Menſuration. *Round Timber,* with equal Baſes, are Cylinders, and muſt be meaſured as ſuch, and not by the uſual Way; which is, to take the 4th of the *Girt* for the Side of a Square, whoſe Area is to be multiplied into the Height; but this cannot be done without doing an Injuſtice to Somebody; for if you are the Buyer of a round Piece of Timber of this Sort, whoſe Girt at the Baſe is 96 Inches, and 18 Feet long, the true Meaſure is 91.67 ſolid Feet, and by uſing the Girt Method you pay only for 72 Feet; ſo that the Owner is injured in the Error of near 20 ſolid Feet, unleſs he bought by Girt Meaſure. *Round Timber, with unequal Baſes,* are Fruſtums of Cones, and ought to be meaſured as ſuch, and not by committing Error upon Error in taking the 4th Part of a middle Girt for the Side of a Square; and then imagine, when this Area is multiplied into the Length of the Timber, that it is reduced into a Cylinder, equal in its ſolid Content to the given Fruſtum of a Cone. It is to no Purpoſe giving Rules, deduced from Demonſtration, to meaſure by, if you ſhould hereafter fall into theſe groſs Errors in finding the Contents of Bodies.

THE SUPPLEMENT *to* GEOGRAPHY.

The two Motions of the Earth.

AS I would omit no Method, which occurs to me, that may tend to illuſtrate any Point under Conſideration, and fix the Attention of Youth, I ſhall here introduce the following Dialogue of the Globes, debating about the Motions of the Earth. What has hitherto been taught of Geometry, and of the aſtronomical Principles of Geography, will abundantly enable the worthy Youth to underſtand what is herein deliver'd. This may prove of Conſequence in guarding him againſt the Prejudices ariſing from the apparent Motions of the Frame of the Univerſe, give him enlarged Views of Nature, and inſpire him with becoming and awful Sentiments of that glorious Being the Author of ſuch a Profuſion of Power and Magnificence, viſible in the Motions of his greater Works.

Let us ſuppoſe the whole Machine of the vaſt Expanſe, the fix'd Stars, call'd ſo from their preſerving an immutable Diſtance from each other, and the Sun, Moon, Planets, Comets, and the Earth, to be all perfectly motionleſs, and all ready to perform their reſpective Functions, but to do nothing in vain; on the contrary, to preſerve a moſt noble and beautiful Simplicity and Harmony for the Good of the whole Society of Nature, and the Benefit of the particular Syſtems thereof. Here then let us conſider all the vaſt Globes of the Univerſe going to debate how to perform the two Motions, which are neceſſary to effect the happy Viciſſitude of Day and Night, Winter and Summer, ſo abſolutely neceſſary to the Inhabitants of the Earth. But becauſe there are many Parties concern'd in this Conſultation, we will let one *Fix'd Star* ſpeak for all his Party, and be the Repreſentative of that auguſt and glorious Part of Nature; and as there are ſeveral Planets, we will chuſe that majeſtic Globe, *Jupiter*, to repreſent all the reſt of the Planets; and as the *Sun* is of ſuch vaſt Importance to our Solar Syſtem, and has none to repreſent him, or he them, being ſingular in our Neighbourhood, therefore he muſt be allow'd the Liberty, and that very juſtly, of ſpeaking for himſelf. And becauſe the *Earth*, according to the *Ptolomeans*, expects, nay commands, ſuch vaſt Attendance, and is not pleaſed unleſs the whole Creation waits upon it with amazing Toil and Labour, and unleſs all the complicated Wheels, and perplexed Contrivances of the *Ptolomeans*, only grind and tear themſelves to Pieces for its Service, when it may eaſily do its own Buſineſs, therefore the *Earth* muſt manage its own Debate, as it is more ſingular than the *Sun*, upon the Account of its unreaſonable Expectations. And becauſe there are ten ſecondary *Planets* or *Moons* revolving about their reſpective Primaries, ſo we will name the *Moon*, which attends the *Earth*, to manage the Debate in Behalf of all the reſt.

And, laſtly, as there are many *Comets*, who are eminently concern'd in this Adjuſtment of the noble Simplicity to be obſerv'd throughout the material World, therefore we will admit *one* to repreſent all the reſt of thoſe wandering Globes.

Sun. As we are now to debate upon theſe two Motions neceſſary to bring about the Seaſons, Day and Night, on the *Earth*, and as this can only be done by either my moving for that Purpoſe, or the *Earth*, for the Benefit of its Inhabitants, moving round me; therefore I muſt deſire the Opinion of the fix'd *Stars*, as you ſtand ſo nearly related to me both in Dignity and Office, in the Commonwealth of created material Beings; and I hope you will have no other Motive in View than the Utility of the Whole and all its Parts, the Preſervation of a noble Simplicity, and whatever may tend to the Glory of our *Firſt Cauſe*, who has ſhewn ſuch expreſs Marks of his infinite Power, Wiſdom, and Goodneſs, in our Exiſtence and Office, as Inſtruments under him.

Earth. I am very ready to concur in any Thing which may tend to thoſe great Ends you have pointed out, according to my Station, and conſiſtent with the Rank of Beings I have the Charge of; but I do not ſee why the *Sun* ſhould make any Difficulty of performing his Motions round and about me, as he is appointed only to give me Light and Heat; neither can I think it equitable to have the Cauſe tried by the *Stars*, as I expect they will conſider my Dignity, and pay their Homage to me accordingly, by keeping their immutable and moſt regular Diſtances, and yet revolve about me every 24 Hours. And what I ſay to the *Sun* and the *Stars* I hope will be taken into immediate Conſideration by all here preſent; and thus, putting an End to all Diſputes, the Frame of Nature may again begin the Motions, for the Utility, Delight, and Entertainment of my Inhabitants, for whom ye, and your Motions, were commanded into Being.

Star. We acknowledge Heat and Light neceſſary for you and your Inhabitants; but the glorious Firſt Cauſe, whom all Nature adores, and to whom we all owe our Exiſtence, has remov'd us to ſuch a Diſtance from you, that we cannot, as we think, be of any Sort of Service to you; and therefore we cannot help thinking it ſomewhat unreaſonable in you to put us upon ſuch Toil and Labour, only to pleaſe your Inhabitants; however, before we give our Opinion to our Relation, the *Sun*, as he aſked it of us, we deſire to hear what he and you have to offer, and to give your Reaſons in a Manner that becomes you both.

Sun. Waving any Reply to what the *Earth* is pleaſed to expect from the Whole, and from me in particular, I ſhall only obſerve to her, that Modeſty becomes us all, and that the Merits of the Cauſe in Hand ought alone to be attended to, by which we all may, perhaps, be the better enabled to execute our Duty, when we know how, and in what Manner, it is requir'd, according to the Wants of every particular Globe. The Earth ſtands in great Need of my Aſſiſtance in more Caſes than my Agency of diſpenſing Light and Heat to her, which I chuſe not to mention at preſent, but ſhall again put the Queſtion, whether ſhe or I ought to move, to obey the Commiſſion given us, to bring about the Change of Day into Night, Summer into Winter.

Earth. I am ſtill of the Opinion, that all valuable Ends will be fully anſwer'd if I ſtand ſtill; and I am the more confirmed in it from the Expreſſions of unerring Wiſdom, and therefore here I reſt the Point, as upon a ſolid Rock: Beſides the Violence done to the Senſes, it is ſhocking to think of my performing two ſuch Motions; and therefore, tho' I put it upon the general Iſſue of a Vote, when all Parties are fairly heard, yet I hope it may be remembered, that there lies a great Objection to my moving; and that is, the Impoſſibility of the Inhabitants keeping their Legs, or any Thing on my Surface ſtanding upright, as my diurnal Motion, in particular, will diſturb their Buildings, and leave them and all looſe Bodies ſo thrown down behind me.

Star. We deſire the *Sun* will proceed to give his Reaſons, that all preſent may he prepar'd, from what he ſays, to give theirs in their Turn. We are all but created Beings, and conſequently limited, and very unable to form a Judgment of what is, or what is not beſt; only we know, that what is, where ever we find it, is beſt; and therefore we are only to judge from Reaſon, where theſe Motions ought to reſide, from the Maxims we have laid down to ourſelves; and to purſue what ſhall appear to us to be the Reſult of a modeſt Inquiry into our Duty, in Regard to the Subject of our Debate. But we ought to remember that our Votes cannot limit the Firſt Cauſe in his Manner of Operation, as we may poſſibly be miſtaken in our Concluſions, tho' there may be the ſtrongeſt Probabilities, which amount almoſt to the ſtricteſt Demonſtrations; and, on the other Side, the ſtrongeſt Improbabilities amounting to almoſt Impoſſibilities, or groſs Abſurdities. We are only Agents, and are to do our Duty in the beſt Manner that Reaſon can ſuggeſt to us, for the orderly conducting of our reſpective Parts: We are only of Yeſterday, and know but little, and yet that little ought to be well executed.

Sun. I ſhall firſt obſerve, that whether I take theſe Motions upon myſelf, or whether the *Earth* will ſave me that Trouble, and perform them herſelf, the Effects, in all Circumſtances, will be the ſame; that is, ſhe will have my Light and Heat, and the Changes of Day into Night, and Summer into Winter, in either Caſe; and ſurely if ſhe can perform this Buſineſs, and prevent my annual Journey, and perpetual going out of my Road in ſo ſtrange a Manner to ſerve her, I think ſhe ought, for the Kindneſs I do her, not to ſay for the vaſt Conſequence I am of to her. Beſides it ought to be remember'd, that my Bulk is amazing to hers; and then ſurely it is more agreeable to Reaſon that the *Earth*, which is only a Pin's Head, when compar'd to me, ſhould move round me, than that I ſhould travel about her. If you conſider, the Journey is very great in both my Motions, and tho' my annual Motion is not difficult to be conceiv'd, yet it muſt puzzle you all very much to know how the diurnal Duty can be performed, as there is nothing in Nature which can give you a Precedent of Bodies going backwards and forwards in a Circle, and yet is oblig'd to go forwards in another Circle. I do not ſay the two Motions are impoſſible to be performed by me at one and the ſame Time; but, according to our Maxim, it does not point out that Simplicity of Motion which will appear if the Earth will undertake her own Buſineſs.

Earth. You are pleas'd to ſpeak of me in a very diminutive Strain, but you muſt remember I am big enough for the Office aſſign'd, in providing for thoſe I have under my Care; and therefore you ought not to ſpeak of me in that contemptible Manner, eſpecially when you conſider the Importance of my Charge, and for whom you were, by the Firſt

Cauſe, originally appointed. I acknowledge that you are abſolutely neceſſary to continue my Happineſs, and for which I am ſincerely thankful to the Author of our Being, who has ſo plentifully provided for me in my needy State, in all the Particulars of my Wants; but ſetting aſide the great Diſproportion of our Bulk, and all Obligations I lay under to you, where is the Difficulty of your performing the diurnal Motion, as well as the annual; cannot you take a little Trouble upon you, in ſtepping out of your Road, it will not retard your other Journey in the leaſt; you will perform that in the very ſame Time, notwithſtanding theſe perpetual Excurſions, in mounting up every Day from the Eaſtern Sides of all my Horizons, and deſcending at Night on the Weſtern Parts.

Sun. I cannot help charging you with being unreaſonable; but I muſt, however, impute this to that Fondneſs which you have expreſs'd for yourſelf, and all your Concerns; to ſome Prejudices which you have received from your Infancy; to that Pride which you have ſhewn, ariſing from the Importance of your Charge; and to an imprudent Attachment to ſome Expreſſions in the Sacred Records, where Mention is made of my Motions, and your Stability. I again ſay, it appears to me ſomewhat unnatural that you ſhould deſire me to run backwards and forwards in this Manner; I do not think you capable of performing theſe two Journeys yourſelf in the Manner you expect of me; you ſurely do not conſider the Vaſtneſs of the daily Journey; you certainly would withdraw your Importunity, did you know this is not much ſhort of the annual Revolution.

Star. The Sun was ſaying juſt now, that the *Earth* could perform this Buſineſs, and that a noble Simplicity will be preſerv'd; *Jupiter* ſeems a little uneaſy at the Conſequence of this Debate, concerning the diurnal Motion of the *Sun* or *Earth*, as it will again involve him and his Brethren, and indeed us all, in much Toil, if the Votes go for the *Earth* to ſtand ſtill; and therefore we ſhall be glad to have it well explain'd; and we hope, if Reaſon declare for our Brother to be at Reſt, that the *Earth* will make no Difficulty of giving up the Point; and this is no more than a plain Duty, for Truth is only an Image of the glorious Firſt Cauſe; and ſurely we ought to pay great Homage and Attention to it, wherever we meet with it. It muſt be owned indeed, that Falſhood too often puts on the Appearance of Truth; but it muſt be our Buſineſs to be cautious, and to take great Care that plain Truth be not cover'd with the gaudy Mantle of Falſhood;

and, above all, that Pride, the Deſtruction of Men and Angels, does not engroſs too great a Share in our Debates; for if that Hell-born Vice rules us, then Deluſion, and all its Train of Infelicities, ſeizes us at once.

Jupiter. As I repreſent a Part of Nature, whoſe Uſe to the *Earth* has not been ſo much as once mention'd by her, and as ſhe will have a Sight of us whether the Motions reſide in her or not, which is, I ſuppoſe, all ſhe deſires to have, we hope the *Sun* will undertake the common Cauſe, and explain how the *Earth* can perform theſe two Motions with more Eaſe than himſelf. She cannot be inſenſible of our great Deſire to perform our Duty; but then we have Reaſon to expect that no unmeaning Motions ſhould be extorted from us; beſides, we beg it may be taken into due Conſideration, by all preſent in theſe Debates, that we having been endued with two Motions already, one periodic, and the other what we reſpectively call diurnal, we do not ſee how we can, conſiſtent with Reaſon and Propriety, perform a third Motion round her in 24 Hours, which means nothing to her: We acknowledge the two former Motions neceſſary to our Well-being, as by that Means we enjoy, with the *Earth*, the ſame Benefits of Light and Heat from the *Sun*, that Repreſentative of the glorious Firſt Cauſe, whether the two Motions in Debate are to reſide in the one or the other of the contending Parties.

Comet. When you take our tedious long Journeys into Conſideration, and remember how ſeldom we are ſeen by the *Earth*, which can only be in the latter End of our aſtoniſhing Travels, and the Nature of the Path we deſcribe in the Execution of our Duty, we hope that, whatever Reaſons may appear in Favour of the illuſtrious *Jupiter* and his Conſtituents againſt their third Motion, thoſe Reaſons will be greatly heightened in Favour of our wandering Race of Globes, almoſt Strangers to the Earth, from the Nature of our Commiſſions. Neither do we acknowledge any Dependence on her; on the Sun we do, as from that Source of Heat and Light we derive what is ſufficient to our Wants, and are, in order to receive his Favours, obliged to travel in a dependent State on him, and at the ſame Time to perform our ſecond Motion for good Ends. The *Earth*, in the Opinion of my Brethren, is too high in her Demands, as ſhe will ſee us in either Caſe, whether the Votes decide for or againſt her, when we make our Viſits into thoſe Regions, to pay our Homage to the *Sun*, purſuant to his Command, and the Nature of the Commiſſion which he has receiv'd from the mighty Firſt Cauſe, to whom we all owe our Exiſtence, and acknowledge our Power to be deriv'd from him: But the *Earth* ſhould conſider what a ſtrange Affair this third Motion of 24 Hours perpetual Excurſion from the Periphery of our Orbits would be to us; beſides, were the illuſtrious *Jupiter* and his Brethren to expect the ſame hard Terms from us, we muſt alter our Speed every Time we come near each of them, for that Uniformity of 24 Hours will not fit the Horizons of *Jupiter* (and his Excellence has a much better Title from his State and Dignity, conſider'd with his chearful Attendants, his *Moons*, than the *Earth* taken merely as a *Globe*, to our exact Performance of this third Motion on his Horizons in half that Revolution, which cannot be done, unleſs we greatly haſten our Pace when we come within Sight of him): Beſides, in the Courſe of our long Journeys, from one Corner of the Syſtem to another, we do not ſee how we can keep from daſhing againſt ſome of the primary *Planets* and their *Moons*, and from running foul on ourſelves by this third Motion, as we are ſo wonderfully ſpread about the Univerſe; and, like a well-appointed cruizing Fleet, this departing every Inſtant from our regular Courſe, and returning in ſo ſhort a Time to it again, muſt take up an amazing Breadth of the vaſt Ocean we ſail on, and conſequently threaten the Deſtruction of what, by our Office, we are appointed to preſerve. We are now a well-regulated Squadron, and our reſpective Courſes are ſo happily adapted to ſuccour our Friends and Allies, in the great Society of Nature, whoſe Harmony we are appointed to promote, that there is not the leaſt Danger of our running foul of any of them, by any Deviation from our Courſe. And even when there ſhould happen to be together ſeveral Globes in one Part of the Heavens, which muſt neceſſarily ſometimes come to paſs, yet our Journeys are ſo fix'd and order'd, that when we pay our Viſits round the *Sun*, theſe Maſſes of Matter, by a wiſe Foreſight, are always out of our Way; but they know we cannot be blam'd, if they ſhould be found in our Paths, and linger about till they receive a rude Concuſſion from us, as they know we can neither give ourſelves Motion, nor alter what we have; but indeed all Nature is ſenſible theſe Things cannot happen; the general and particular Plans of Operation are too well laid to admit of any Danger, unleſs we are forc'd into this third Motion, which we hope, from the declar'd Maxims of this Aſſembly, will not be the Caſe, and therefore here we reſt the Point.

Jupiter. To the Danger expreſs'd by this awful Stranger, the *Comet*, of doing great Harm to the Syſtem, if the *Earth* expects this third Motion from them, I muſt beg Leave to ſay a Word or two on that Head concerning ourſelves, as it will be with great Reluctance that we go into this Motion, if this Aſſembly ſhould determine in Favour of the *Earth's* Stability. You all very well know what a Compaſs in the Heavens, for Inſtance, *Saturn* and I take up, by being adorn'd with *Satellites*, ſituated at different Diſtances from our reſpective Surfaces; you were told before in what Manner we perform our two Motions, neceſſary to our own Well-being in particular; but if this third Motion is to take Place in all Globes that have a different Motion, when the *Comets*, for Inſtance, are performing this third Motion, and in our Neighbourhood, I do not ſee how we in particular, that have *Satellites*, can anſwer for their Safety, when all Nature, as it were, is in this ſtrange Motion, and in ſo much imminent Peril of deſtroying the Parts of the great Machine of the Univerſe. Therefore, in Behalf of my Conſtituents, and myſelf in particular, I reſt the Point on the Needleſſneſs of this aukward departing from a regular and well-appointed Courſe, and the great Danger in performing it; two Conſiderations which will, I make no Doubt, have their due Weight in this illuſtrious Aſſembly.

Moon. Tho' the two contending Parties both ſtand nearly related to me, from the Benefits I receive from each, yet as I am intruſted by my Brethren, the ſecondary Planets, or *Moons*, to manage their Intereſts in this Debate, I ſhall ſpeak my Mind with the utmoſt Impartiality, and nothing ſhall bias me but Truth itſelf, if I can by any Means diſcover on which Side it lies. I receive, it is true, the kindeſt Influences from that magnificent Fountain of Light and Heat, the *Sun*, but I likewiſe derive much Benefit from the *Earth*, who lends me moſt generouſly what ſhe only borrows from the ſame Source; ſo that I am doubly happy in a Supply of Light. And yet, from what has been urg'd for and againſt the Point in Debate, I am inclin'd, from the many Inconveniencies ariſing from this perplex'd Method propos'd by the *Earth*, to bring about the Viciſſitudes of Day and Night, Winter and Summer, to lean to the greateſt Simplicity propos'd by the *Sun*, in effecting theſe Phænomina. And therefore muſt at preſent expreſs my Diſlike of the *Earth's* Stability, by gueſſing what my Situation will then be, and that of my Brethren, who you all know are very far from making ſo contemptible a Figure in the lovely and mighty Machine of the Univerſe, as the *Earth* is but too apt to imagine: For we *Satellites*, tho' our Bulk be inferior to the Planets, are much better ſupplied with Light than the *Earth* is willing to think; ſhe is a moſt glorious *Moon* to me, anſwering this Purpoſe better, by fifteen Times the Advantage, than I do to her, for which Bleſſing I am thankful, as are all my Conſtituents for their manifold Benefits far exceeding mine, which they receive from their Primaries, about which they perform their periodic Motions, as I faithfully perform mine round the *Earth*: But we all wiſh ſome Method may be propos'd to ſave us the Journey in Queſtion, as it will be a moſt troubleſome, and, as was formerly and juſtly obſerv'd, a moſt dangerous one. I muſt obſerve that I can perform my monthly Travels very well, but I cannot reliſh this Agitation to and fro in the Periphery of a Circle from one Point of my Orbit to another, inſtead of going on regularly; and what I ſay of myſelf muſt be underſtood of my Brethren alſo, who are all apprehenſive that great Toil and Diſorder will be the Reſult of the *Earth's* Stability, and therefore here we reſt the Point, unleſs the Apprehenſion of theſe Hazards and needleſs Toils can be remov'd.

Earth. From the Tendency of what has been urged in this Aſſembly, relating to the Diſpute which is ſubſiſting between me and the magnificent *Sun*, I infer, that I am likely to do my own Buſineſs for the Purpoſes of procuring to myſelf Light and Heat. And in order to this, and that he may enjoy perfect Repoſe in the Center of the mighty concave Frame of Nature, I ſuppoſe the Way you will unanimouſly vote for will be, that while I am travelliug round him in a Year, I muſt at Six o'Clock in the Morning ſtart up from the firſt Point of *Aries*, deſcribe a Circle, like the Celeſtial Equator, and at Six in the Evening be with *Libra*, deſcend below the Ecliptic, and the next Morning ſtart fair again a few Miles to the North of the firſt Point of *Aries*, and riſe above my annual Path, deſcend again, and ſo on every Day to deſcribe a Parallel of the Equator, to give the *Sun* an Opportunity of doing his kind Office to every Part of me; and I ſuppoſe my Axis muſt be kept parallel to itſelf and inclin'd to the Ecliptic in an Angle of 66 ½ Degrees. I am ready to conform to the Deciſion of this auguſt Aſſembly, if the Votes go againſt me, as I ſuppoſe they will: But, if I miſtake not, the *Moon*, who is my Attendant, and who faithfully performs her monthly Travels at a ſmall Diſtance round me, muſt be ſenſible now of her Error, in being apparently againſt my Opinion; for ſhe muſt partake with me of this my ſecond Motion, which will be a third Journey to her, and that a very troubleſome one too; however ſhe may thank herſelf for it.

Moon. I hope you will only conſider my Opinion as being the Reſult of my Judgment, to promote wiſe and good Ends in the moſt ſimple and eaſy Manner, where that can be done; but I muſt own, tho' your deſign'd Compliance to the Determination of this awful Aſſembly pleaſes me, yet the Manner you have pointed out of performing your annual and diurnal Motions perplexes me more than I was before; for your Motions not only partake of that ſtrange Oddity which you expected from the illuſtrious *Sun* in his perpetual Excurſions backwards and forwards in a Circle, from one Point of his annual Path to the oppoſite Side; but my additional Journey makes the Affair in you ſtill more complicated and unnatural, difficult and hazardous, than if the Motions were to depend on the *Sun*; and therefore, in Behalf of myſelf, I vote for the Reſt of the *Earth*, but for my Conſtituents I am of my former Opinion, if this Aſſembly will indulge me the Liberty of making this Diſtinction, and of having two Votes.

Star. Were not the Intereſt of my Brethren too much concern'd in this Debate, I ſhould make no Reply to what the *Moon* has advanc'd; neither do I ſpeak ſo ſoon after her to throw any Difficulty in her Way; I make no Doubt of her being allow'd the Freedom of a ſeparate Vote in what relates to her own immediate Intereſt, and of her being indulg'd in whatever Reaſon can expect from us; but I am ſorry to ſee her go ſo ſoon into another Opinion, on the Apprehenſion of her own imaginary Evils, which (could no other Way be found out to bring about the Happineſs of the Seaſons, Light and Darkneſs, by the Motions of the *Earth)* to be ſure would be a moſt difficult and a very ſtrange Affair for her: But then, it were much better for her to undergo Difficulties and Hardſhips, than for every individual Globe of the mighty Univerſe to be empowered to perform ſuch vaſt and needleſs Journeys; and ſuch an Opinion have I of her Benevolence to the whole Circle of Nature, tho' I and my Brethren have not had the Honour of ever ſeeing her before this Time, and perhaps never ſhall again, that, rather than torture and diſtreſs the goodly and well-regulated Fabrick of the Univerſe, ſhe will undertake this Commiſſion, tho' it may be attended with great Aukwardneſs, Toil, and Hazard: Not but our kind Brother the Sun, as he is of ſuch Conſequence to that Neighbourhood, may, in his Wiſdom, hit upon ſome Method to relieve both the *Earth* and her faithful Attendant from this ſtrange Motion, to procure Day and Night, as the annual may, by the Earth, be performed with great Eaſe and Regularity, in the Manner executed by the Planets and Comets.

Sun. The *Earth* and *Moon* have given themſelves unneceſſary Uneaſineſs, in raiſing up to themſelves imaginary Evils. I hope the *Earth* will not be above doing what that illuſtrious Globe, *Jupiter*, does; he, with great State and Dignity, performs his Journey round me, with all his dutiful Train of Moons, to bring about the Change of Seaſons; and, in order to procure Day and Night, he does not perform his double Motion in that ſtrange irregular Manner as the *Earth* has pointed out, in deſcribing ſuch a Number of Circles, parallel to each other, and oblique to what he calls his annual Path about me, as the Center of his Orbit: And I leave the *Earth* to judge whether *Jupiter*'s Moons would not be in the ſame perplexed Situation with the Terreſtrial Moon, were the two Motions of the *Earth* to be performed in the Manner ſhe has pointed out to this Aſſembly, as the Plan of her intended Operation, and deſigned Willingneſs to comply with the Determination of the Majority of all here preſent. No, the majeſtic Globe of *Jupiter* does not perform this ſecond Motion in ſuch a laborious Manner; he enjoys my Influences in a more eaſy Method; he, while he brings about his Seaſons by his periodic Journey round me, turns with Eaſe on his own Axis, and thereby effects that Happineſs, without either troubling me or any Part of Nature. And I muſt here again leave it to the *Earth* to judge, whether *Jupiter*'s Attendants do not effect their appointed Viciſſitudes by travelling round me, from the Nature of their Attendance on *Jupiter*, and by turning on their own Axis, in ſome Method or other, and ſo by one or both Motions, alternately, enjoy my Influences, and the Light of a moſt glorious *Moon*, as they call their Primary. I leave it to the Conſideration of this auguſt Aſſembly to judge of the reſt of the Planets and Syſtems of Bodies, from this Specimen of *Jupiter*; and if, upon Inquiry, there is no Inſtance of one of theſe *Moons* performing a diurnal Circle, but only an annual Circle, or periodic Revolution, reſpectively round *Jupiter* and *Saturn*, I hope the general Motion on Axis, and not in diurnal Circles, will be readily credited by the *Earth*, and the leſs Difficulty raiſed in doing, at leaſt, what renowned *Jupiter* does for the Well-being and grand Simplicity of the whole. And here I reſt the Point.

Star. Illuſtrious Brother, you have hit upon a wiſe Method, and kept up to the Dignity of your Rank; and in aſſerting your Prerogative you aſſert ours, as the Reaſon of Things, and our immutable Diſtance from each other, ſeem to require; for, according to what was ſaid before, Reaſon is given us to be our Guide when we look into the Works of the Eternal Firſt Cauſe; I mean ſuch Parts of the Operation of his Almighty Will, as he has been pleaſed to preſent to our View, or within the Limits of our Faculties; beyond which all are Depths unfathomable, where we have no Bottom to ſtand on: And I hope both the benevolent *Earth*, and her faithful Attendant and Companion the *Moon*, will readily concur in this Compromiſe, that the Buſineſs of our reſpective Agencies may be no longer retarded by our Debates; and I hope that we ſhall all of us conſpire with the greateſt Harmony to execute a moſt regular and beautiful Simplicity, and ſhew to whom we belong, and from whom we have received our Commiſſions.

Sun. As to my Part I am ready, nay even impatient, to perform the Duties of my Station, and to bring about thoſe noble Purpoſes which my Brethren the *Stars* have mentioned, and I make no Doubt of the Willingneſs of all here preſent to do the ſame. But I think it juſt that the *Earth* ſhould be again heard before we collect the Votes, which I could wiſh may be all on one Side, that by this Unanimity the Buſineſs may go on the better, and that we may return to our reſpective Stations, perfect Friends, and with an intire Confidence in each others Affection. There is one Affair, indeed, which I cannot well help obſerving to the beloved *Earth*, in order to court her Conſent to this happy Simplicity of her two Motions, the annual round me, and the diurnal upon her own Axis, and that is the Kindneſs I do her, in keeping her in my Neighbourhood, inſtead of her rambling from me into the boundleſs Regions of the Univerſe, in which unmeaning Excurſion ſhe may do herſelf great Harm, and other Globes likewiſe, by daſhing againſt them, if any ſhould happen to be in the Way of her endleſs Journey. It is true, I myſelf ſhould do her Harm, nay compleat her Deſtruction, from my great Strength, while I am thus ſerving her, had not the Firſt Cauſe, who gave me my Power, given her alſo a Power to act againſt mine; by the happy Proportion of which two Powers the *Earth* is kept from this Deſtruction, and preſerved in her annual Journey round me. I do not mention this to heighten her Obligation to me; I am only a Servant, and have nothing to boaſt of, though the *Earth* is often pleaſed to allow me to be a ſplendid Orb of Light and Glory, and of the utmoſt Conſequence in my Neighbourhood, and to the Society to which I belong, and of which I have the Honour to be a conſtant Preſident. I ſay yet I have nothing to be proud of; I am not the Author of this Refulgency: He that ſaid, Let the Sun appear in all its State and Glory, can as eaſily ſay, Let the Sun diſappear; let his Light be put out, and then I am a poor, uſeleſs, blind Maſs of Matter. And what I ſay of my keeping the *Earth* from Deſtruction, by her leaving me, I ſay likewiſe of the other Planets; and thoſe that have *Moons* have a like Power with me, differing only in Degree, to keep their Attendants from rambling through the wide Expanſe; ſo likewiſe the *Earth* ſhews her Kindneſs to her *Moon:* And as this is happily effected by the different Strength or Size of the Globes, and at well-regulated Diſtances from me, and from themſelves; I hope the *Earth* will conſider the Conſequence, if I leave the Center of the Syſtem to perform the two Motions in Debate; and my Brethren, the *Stars*, are the beſt Judges whether there is not ſuch a Proportion of Power, in their Parts of the World, between each of them and other Globes, which we call Planets and Comets, if any ſuch there be, to keep them from running off into the wide Heavens: But as Modeſty becomes them and me, I ſhall ſay no more; and only hope to hear the *Earth* decline all Pretenſions to her enjoying a State of Reſt in the Center of the Syſtem, to the Prejudice of the whole.

Moon. From what has been ſaid, I intirely agree for myſelf in particular, and my Brethren, the ſecondary Planets, in general, that the two Motions be in the *Earth*, as I humbly conceive Things cannot be better appointed. I am thoroughly ſatisfied and convinced, that I ought to travel round the *Earth*, and while I do this, to take Care of my Motion on my own Axis, conformable to my Conſtituents, who are of more Conſequence to their Primaries, than I am to the *Earth*; and therefore here I finally reſt the Point.

Earth. Since the illuſtrious Examples of *Jupiter* and his Conſtituents are urged againſt me, I ſhould agree to move, and the more, as I find myſelf miſtaken in the Manner I propoſed to perform my ſecond Motion to bring about a Succeſſion of Day and Night; according to their Practice, I find the Buſineſs eaſy to be executed, and ſhould ſet about it with the greateſt Pleaſure, as I now conſider it in the Light of ſaving a great deal of Toil to the whole Frame of Nature in general, and to the particular Syſtems and Parts thereof. To be ſure, I have been often confounded with that Infinity of Motions, as it were, round me in 24 Hours; and I have as often reflected, with a ſecret Pride, on my Conſequence in the Univerſe, when I conſidered the Number, Magnitude, and Diſtances of thoſe reſplendent Orbs, perpetually rolling over and about me, and paying their Court to me, in that diligent Attendance on me. But I muſt now follow the Example of my intimate Friend and Companion, the *Moon*, in her Caution to vote ſeparately for herſelf, and to reſerve another Vote for her Conſtituents; therefore I hope, like her, to be indulged the Liberty of putting a Negative againſt my performing either the annual or diurnal Motion, in Behalf of one Part of thoſe whom the glorious Firſt Cauſe has graced me with; and for myſelf and the other Part of thoſe who honour my Surface with their Preſence, I vote for the Stability of the magnificent *Sun*.

Moon. I make no Sort of Doubt of your being permitted by this auguſt Aſſembly to uſe the Caution you mention, if it is of any Service; but I do not apprehend that the Negative you have put can be of any Conſequence here; for we are to conſider of what is beſt for the whole, and moſt fitting for the Parts, and not barely to govern ourſelves by Appearances, which may deceive us; which has been the Caſe with you and me, and may be ſo with thoſe for whom you have put a Negative; and they, like us, may be ſatisfied, if they will with Caution ſeek after Truth only, and embrace it wherever they find it. I remember, my dear Friend, during this Meeting, you whiſpered a few general Hints to me of your Inhabitants; and by what I can find they are Beings of Conſequence; therefore I commend you for the great Care you take to give no Offence to any of them; but I am heartily ſorry to find that ſome of them are ſunk ſo far below their Natures, as to aſcribe our Creation to ourſelves.

Star. As it is agreeable to this noble Aſſembly that I ſhould have the Honour to ſay a Word or two before we break up, I ſhall with more Pleaſure undertake it, as I perceive there is now a perfect Harmony of Opinions relating to the Cauſe of our Meeting. And now thou *Sun*, delightful Source of Light and Heat, ſhed the kindeſt of thy Influences throughout thy Syſtem, and particularly on Man, and with thy lovely Agency cheriſh him, and make his Pilgrimage a pleaſant Journey to Realms of perfect Happineſs. And thou highly-favoured *Earth*, with the reſt of thy Kindred the Planets and the Comets, with all the Train of ſecondary Agents, the faithful Moons, do you all lovingly diſcharge your reſpective Truſts, while we of the boundleſs ſtarry Regions act in Concert with you, to proclaim the Power, Glory, and Majeſty of Him, who is the Cauſe of all our Splendor.—— *Farewell.*

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Fig. 1

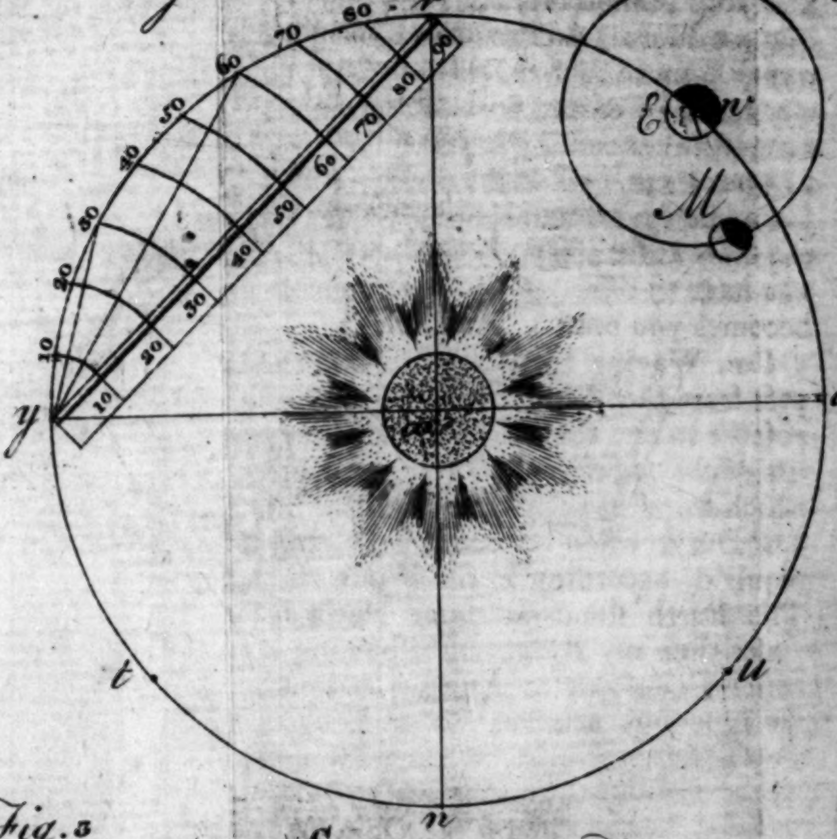

Fig. 3

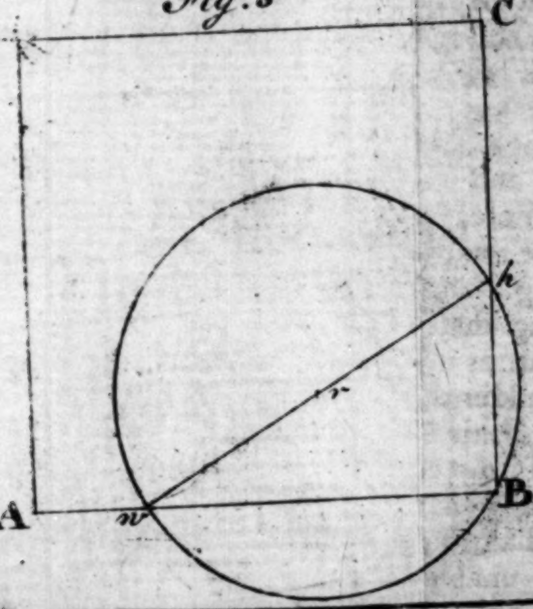

Fig. 2

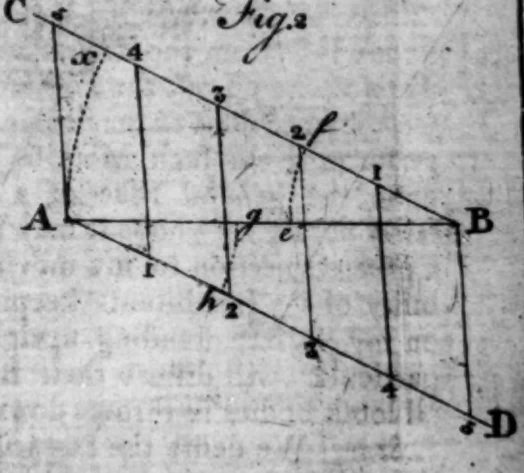

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Fig: 4

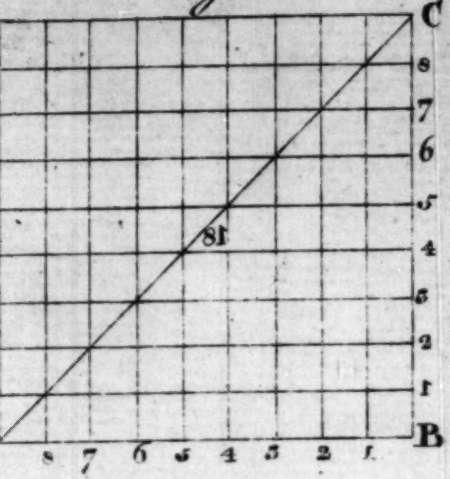


Fig: 5

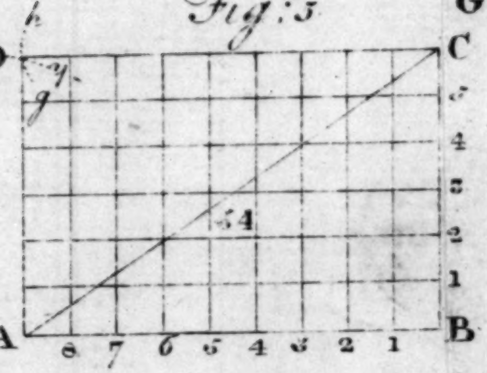
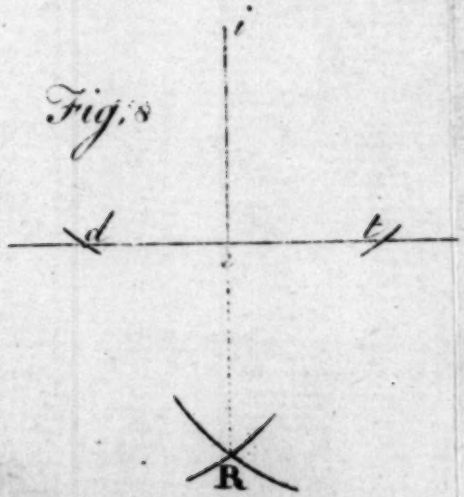


Fig: 8



Fig; 6

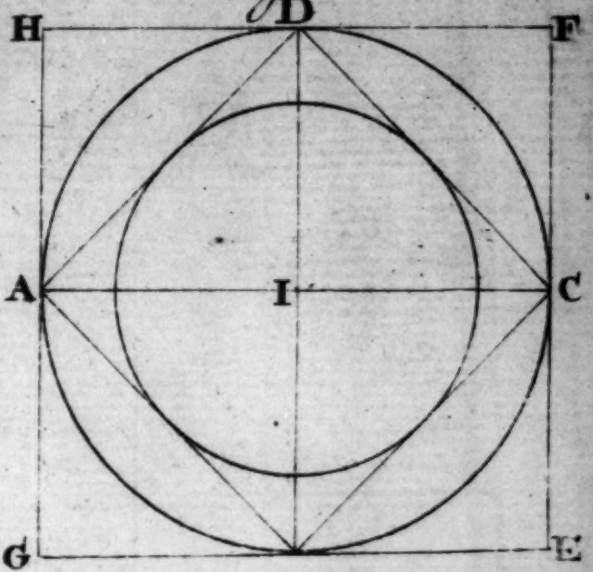
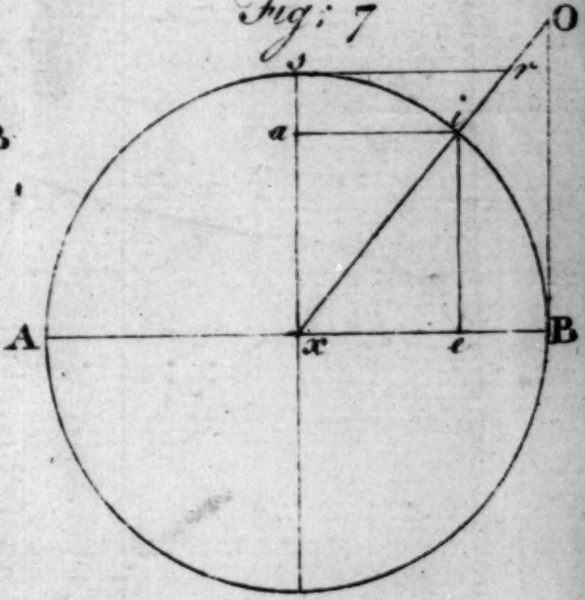
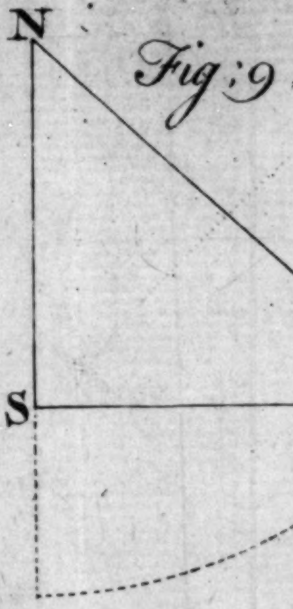


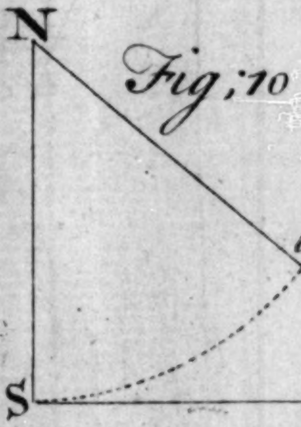
Fig: 7



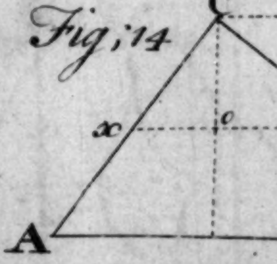
Fig;9



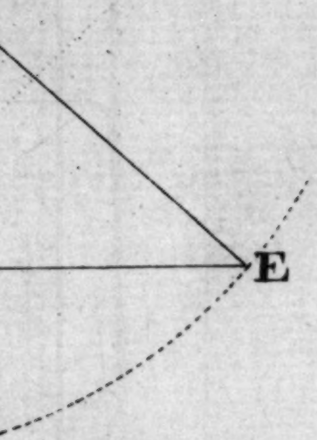
Fig;10



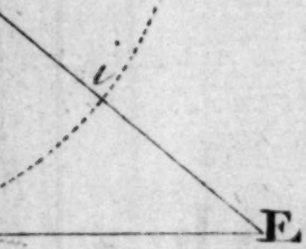
Fig;14



g;9



g;10



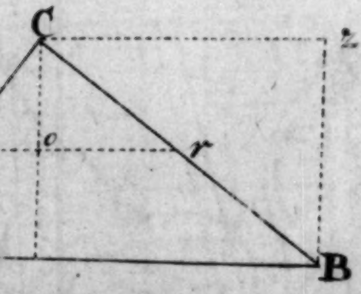


Fig 11

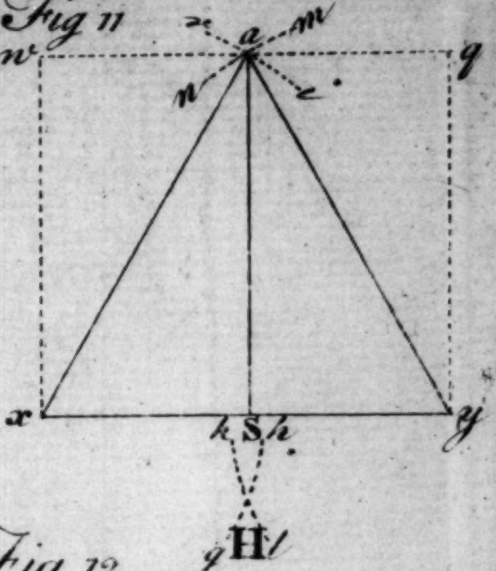
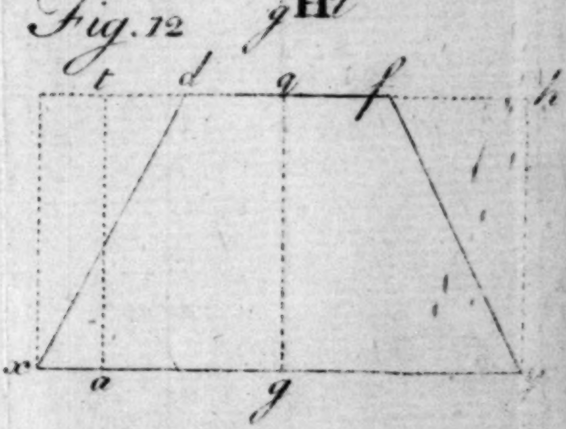


Fig.12



Fig;13

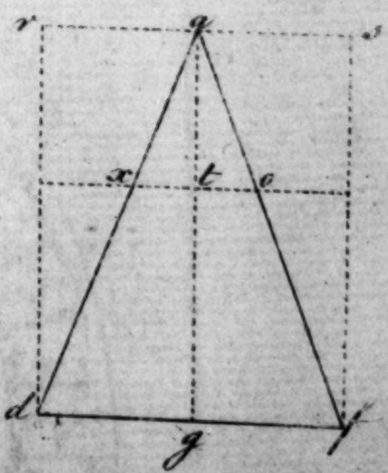


Fig.

Fig. 18

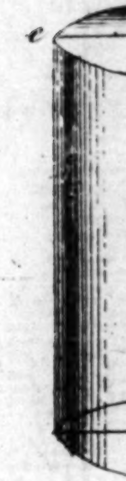


Fig. 15

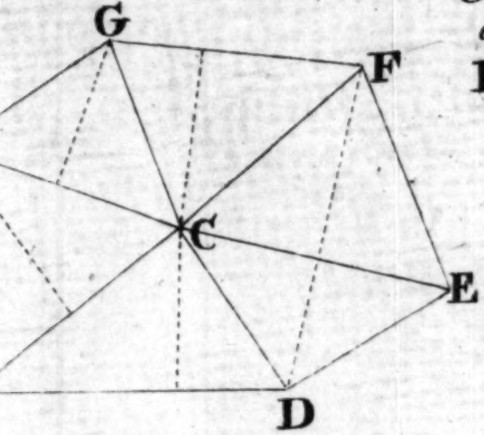


Fig. 16

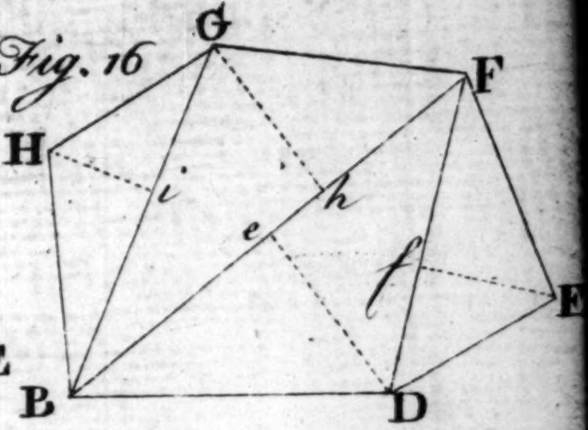
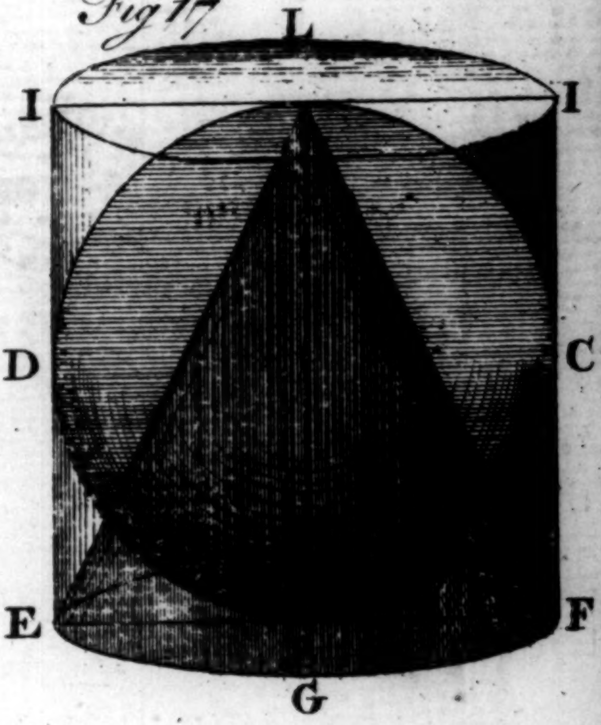


Fig 17



ig.18

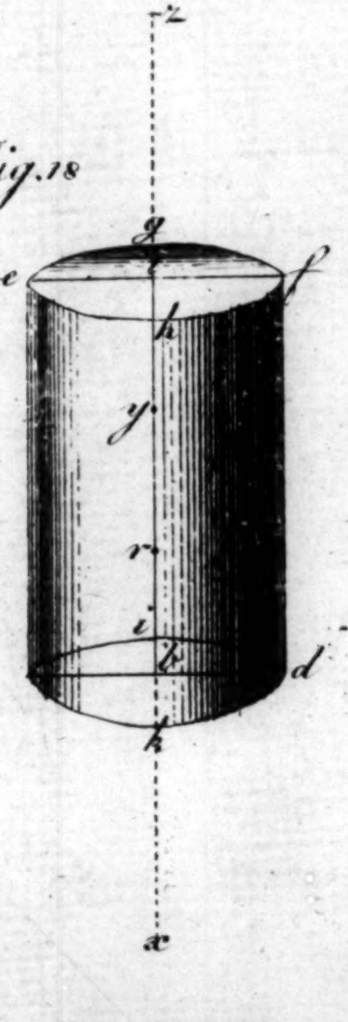


Fig.19
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Fig.19

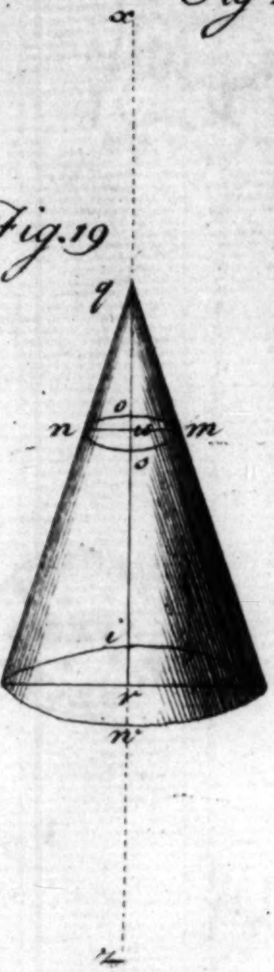


Fig 21

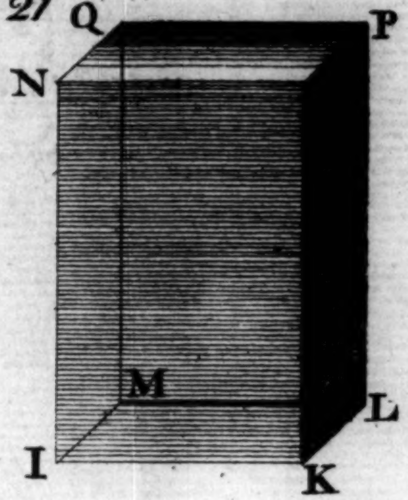


Fig.22

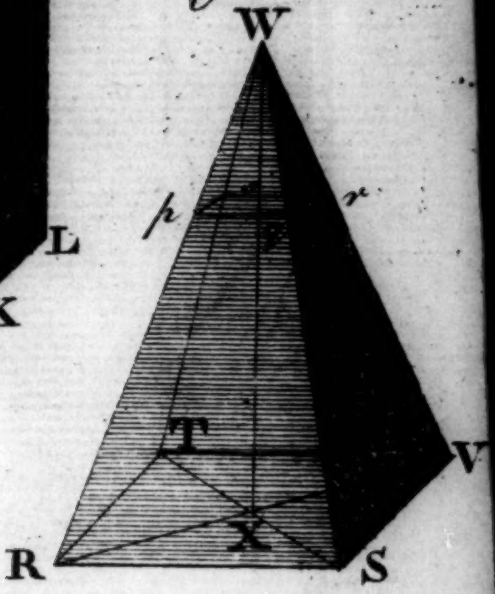


Fig.20

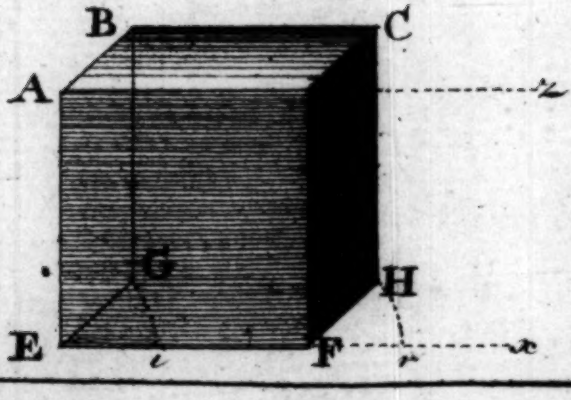
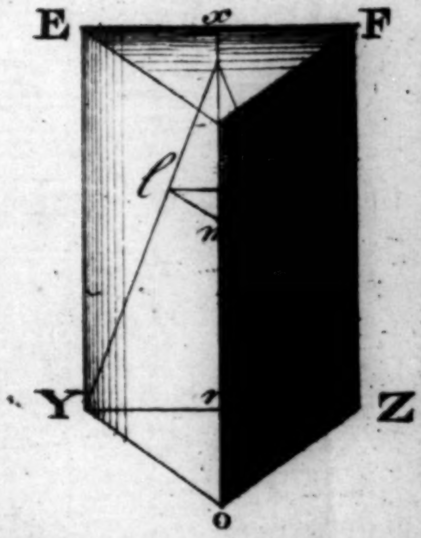


Fig.23



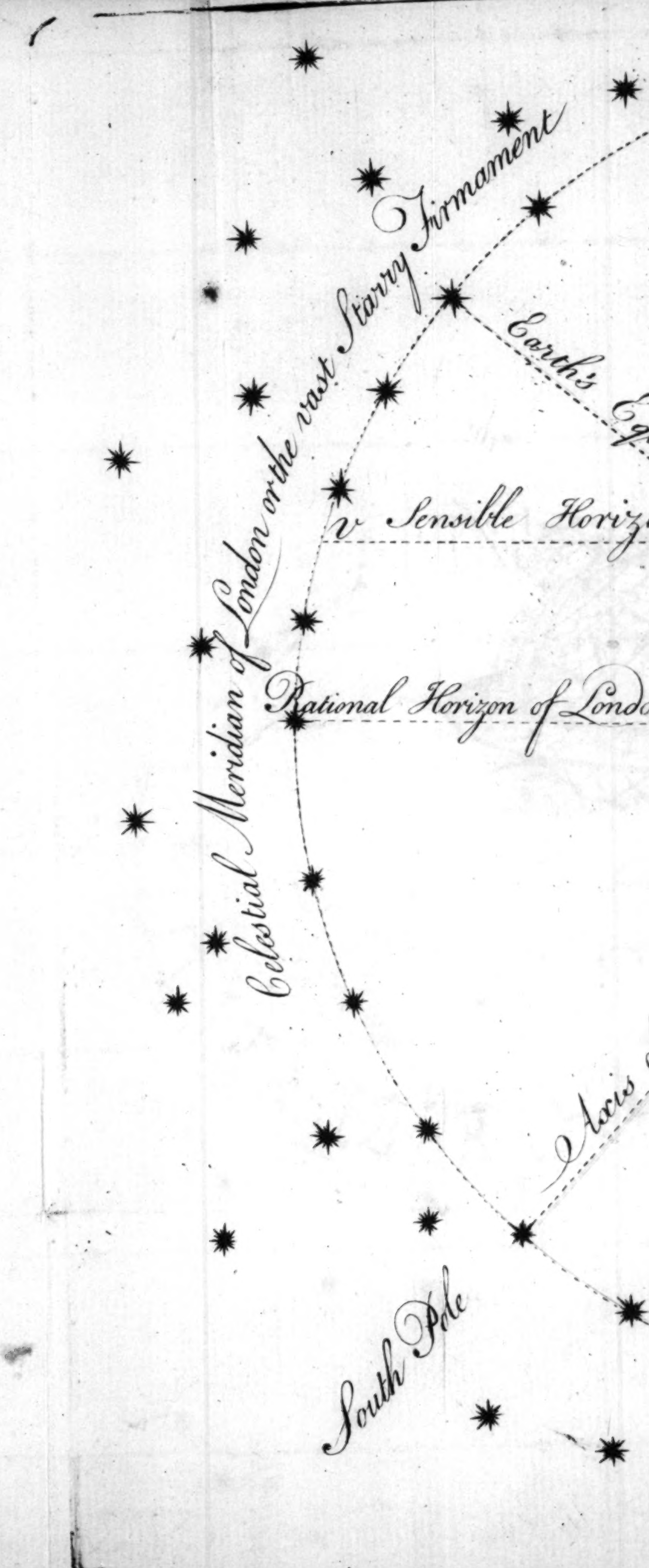
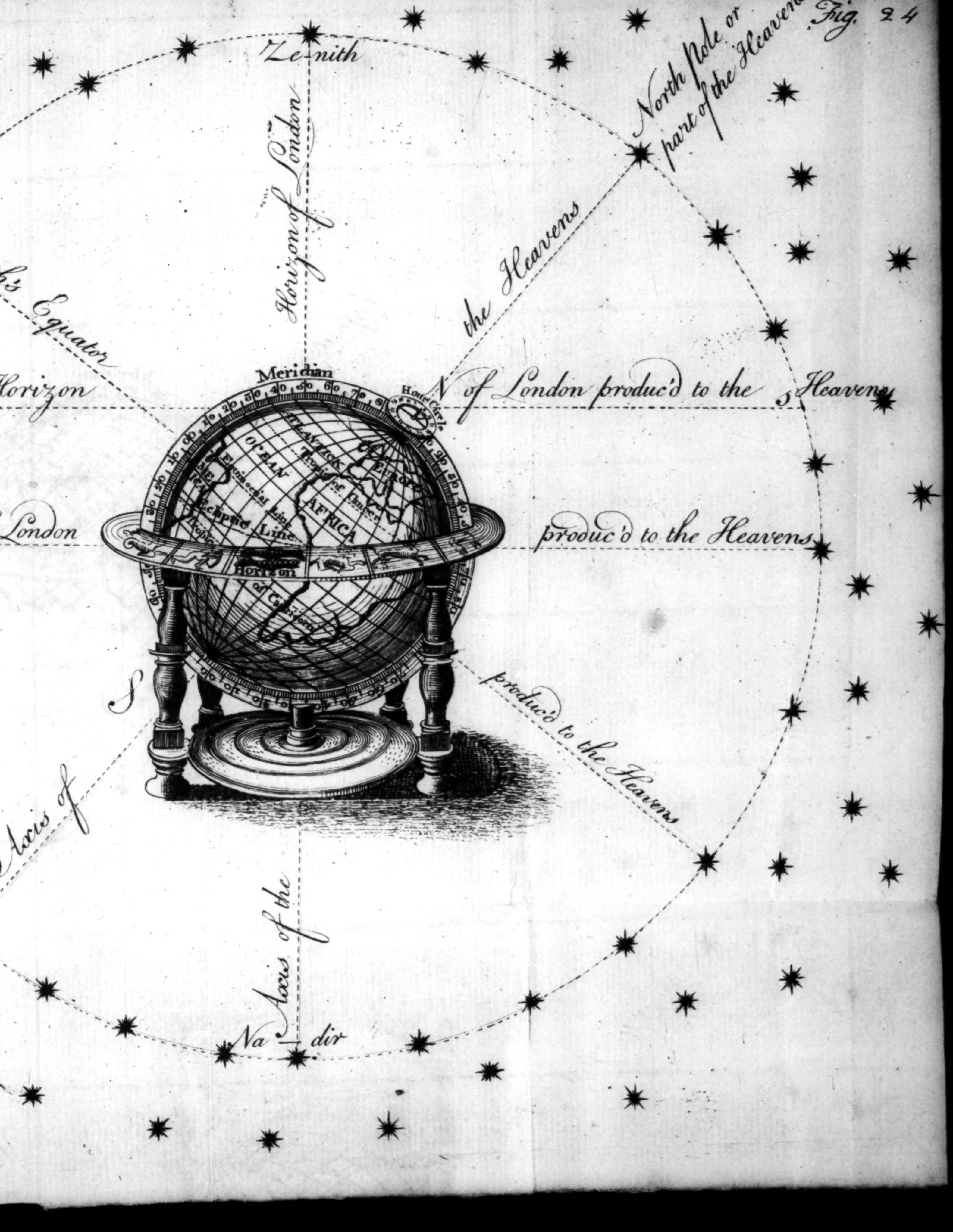

Celestial Meridian of London or the vast Starry Firmament
Earth's Eq
v Sensible Horiz
Rational Horizon of Londo
Axis
South Pole

Fig. 24


Ze-nith
North Pole or part of the Heavens
Horizon of London
the Heavens
h's Equator
Horizon
Meridian
Hour Circle
N of London produc'd to the S Heavens
London
produc'd to the Heavens
ATLANTICK OCEAN
EUROPE
Tropic of Cancer
Equinoctial Line
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Tropic of Capricorn
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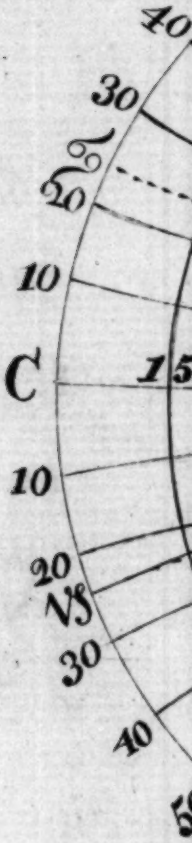
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C
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Fig. 25

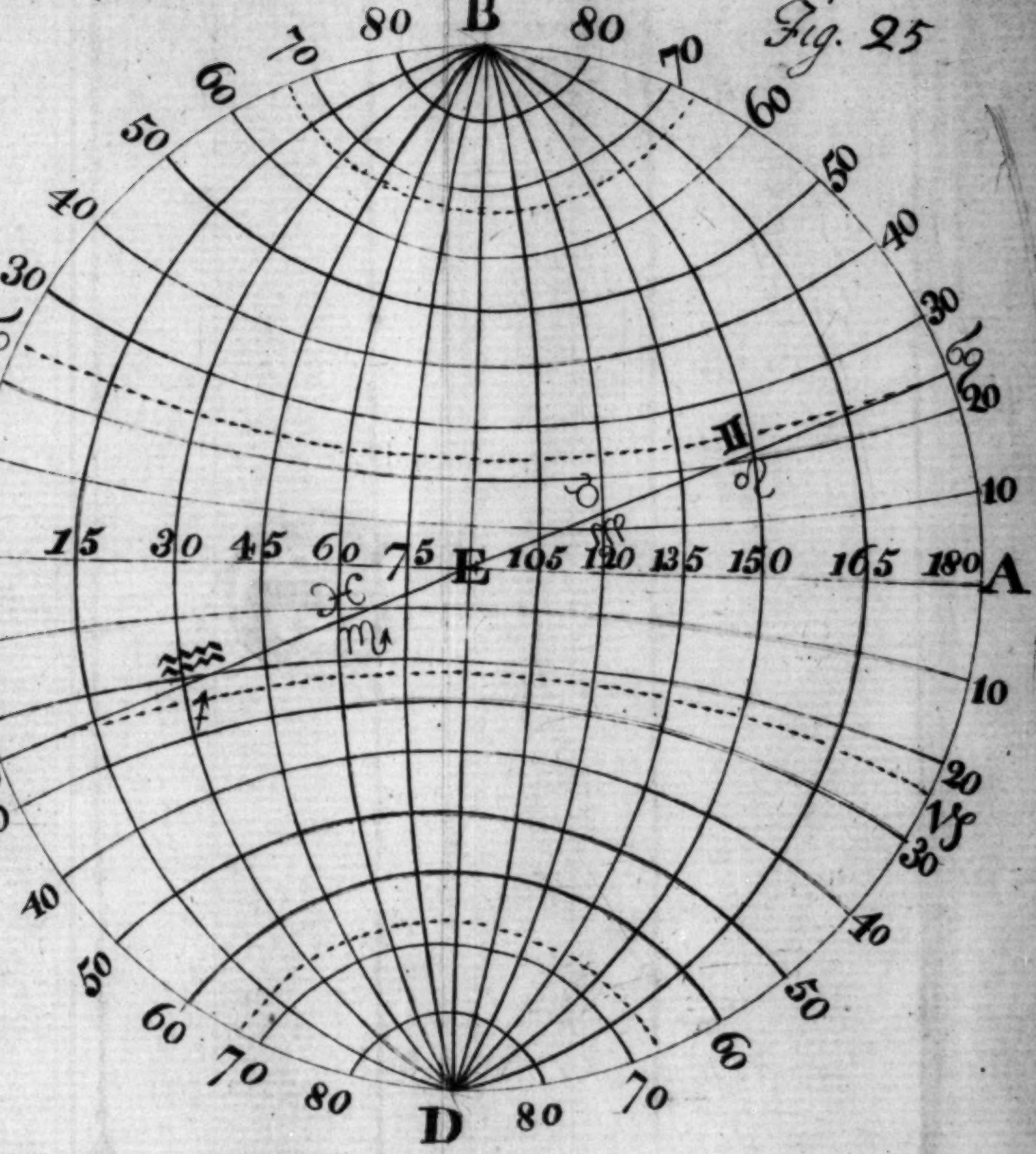
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Fig. 26

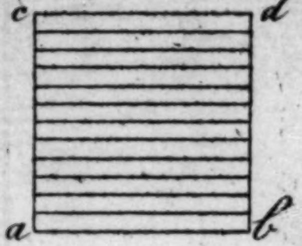

Fig. 28

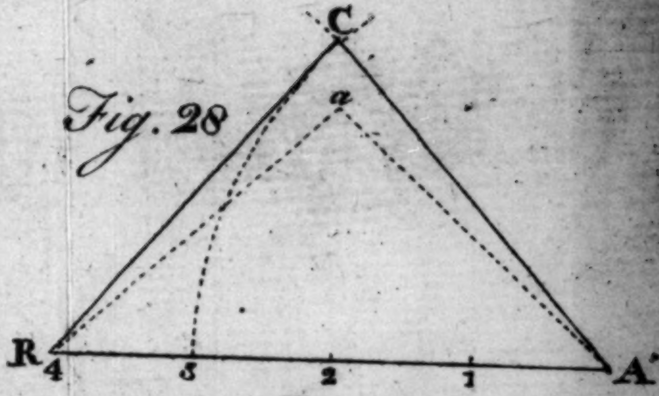

Fig. 27

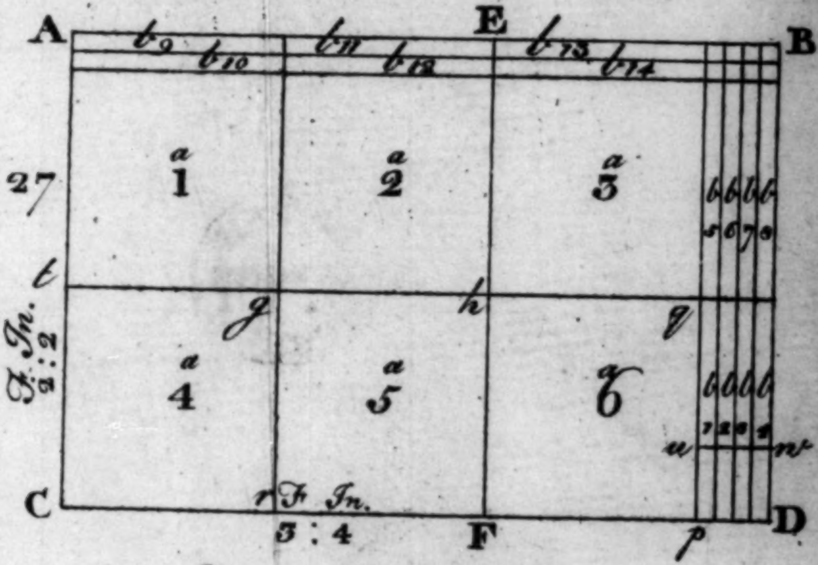

Fig. 29